

# Professional Development that Supports Teachers' Use of Learning Trajectories

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# Presentation overview:

## Designing practice-based professional development

1. Dev-TE@M materials background: Four "core elements" of professional development content
2. Examples: Activities for developing teachers' knowledge of learning trajectories
3. Insights into this professional development approach for learning trajectories
4. Becoming involved: Using Dev-TE@M materials
5. Questions

# 1. Background:

*Four "core elements" of professional development content*

# Dev-TE@M project

- Developing web-based professional development modules
  - Representing and comparing fractions (in use)
  - Reasoning and explanation (broad pilot)
  - Geometric measurement & spatial reasoning (initial pilot)
- Constructing a set of robust resources for facilitators

# Materials features

## Quick facts:

- Ten 1.5-hour sessions
- Professional development for *practicing* elementary mathematics teachers
- Integrated content
- Practice-based design
- Accessible via the web with multimedia components
- Facilitated sessions conducted in real-time

**Dev-TEAM**

Session 5-4: Analyzing others' narrations of the number line to compare  $\frac{3}{4}$  and  $\frac{4}{3}$

**Narrating the construction and use of a representation**

Which fraction is larger –  $\frac{3}{4}$  or  $\frac{4}{3}$ ?

With a partner:

- One person talks through the use of a number line to solve this problem.
- The other person notes phrases or ideas that are shared during the "narration."
- When the problem is complete, discuss the narration and think about which parts seem to be important when doing this kind of work.

**Overview**  
This part analyzes sample narrations of the number line to compare  $\frac{3}{4}$  and  $\frac{4}{3}$ . This analysis helps further articulate the work entailed in narrating representations.

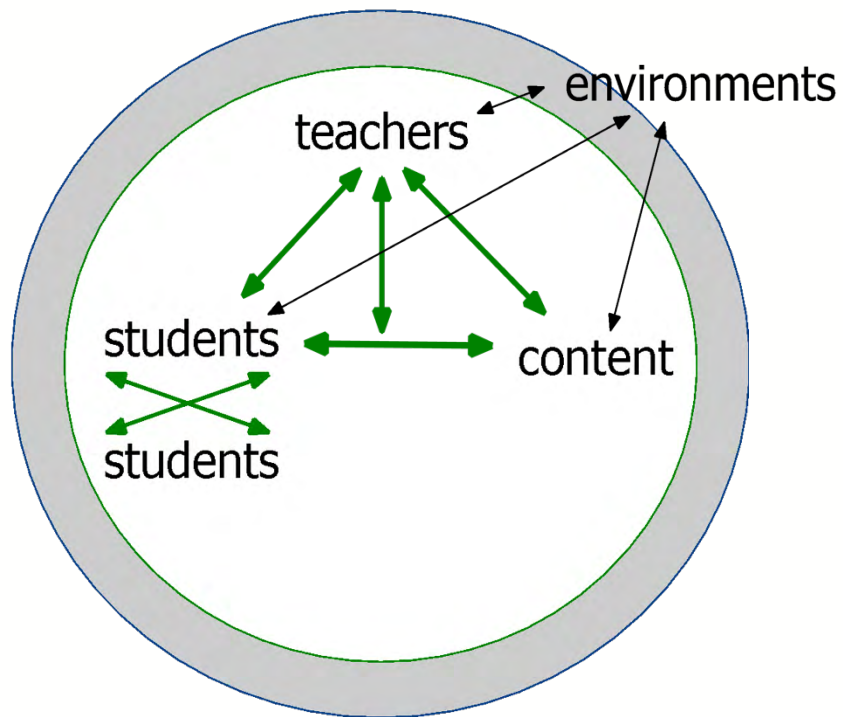
**Resources**  
Handout: Blank number lines

Select a video:

Video A: Using one number line with number sense	Video B: Using two number lines	Video C: Using one number line and different colors	Video D: Using one number line with common denominators
Video E: Using one number line with number sense			

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# Introduction to the core elements



(Cohen, Raudenbush, and Ball, 2003;  
Lampert, 2001)

Dev-TE@M materials  
integrate attention to four  
core elements

1. Mathematical knowledge for teaching
2. Students' thinking about mathematics
3. Essential teaching practices that support student learning; and
4. Routines for learning in and from practice

# Core elements in the “Geometric Measurement and Spatial Reasoning in Elementary Mathematics Teaching” module

- **Mathematics:** concepts, methods, and tools used in linear, 2D, and 3D measurement
- **Student thinking:** understanding trajectories of children’s reasoning about linear, 2D, and 3D measurement
- **Teaching practice:** using learning trajectories in assessing and curriculum analysis
- **Learning from practice:** studying learning trajectories of one’s own students through the use of video and anecdotal notes

## 2. Examples:

*Approaches to developing teachers' understanding of learning trajectories*





# Organization of Module Content

- Three parts of a learning trajectory
  - a) Mathematical goal
  - b) Developmental progression of student thinking
  - c) Instruction and teaching practices
- Three topics within geometric measurement
  - Linear measurement
  - Area measurement
  - Volume measurement

# Organization of Module Content

Linear measurement: Sessions 1-3

- a) Mathematical goal (session 1)
- b) Developmental progression of student thinking (session 2)
- c) Instruction and teaching practices (session 3)

Area measurement: Sessions 4-6

- a) Mathematical goal (session 4)
- b) Developmental progression of student thinking (session 5)
- c) Instruction and teaching practices (session 6)

Volume measurement: Sessions 7-9

- a) Mathematical goal (session 7)
- b) Developmental progression of student thinking (session 8)
- c) Instruction and teaching practices (session 9)

Connecting linear, area, and volume measurement (session 10)

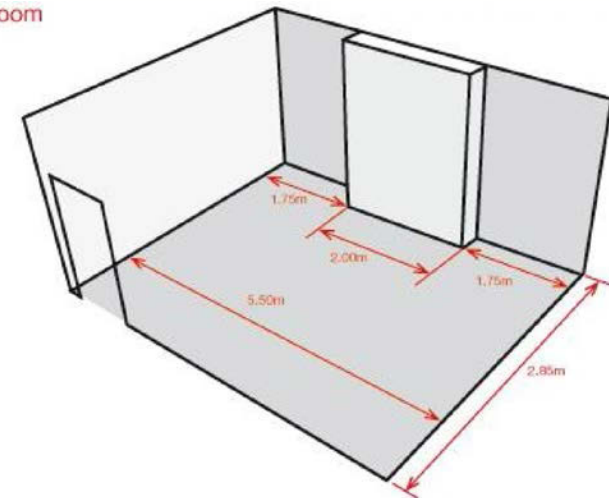
# Activity #1: Solving mathematics problems

## Measuring the room sequence

### How Long is the Room?

- Really...how long? How wide?
  - Estimate. Write it in your notebooks.
  - Think: How did you estimate?
  - What did you have to know and be able to do?
  - Write a brief summary in your notes.
  - Brief sharing.
- Now let's measure...

Example Room



## Activity #1: Solving mathematics problems

# Measuring the room sequence

### How Long is the Room?

- Structure: Individual, pairs, whole group
- Choose a personal ruler\* to measure the length of the room

\*any object you have with you, including...  
any part of your body!

Then, with partner, measure your personal rulers using standard ruler and compute the length of the room in standard units

- Go measure!





# How Long is the Room?

- Why did we get different answers?
  - How did different personal ruler selections and methods affect the results?
  - What differences are or are not acceptable?
- How did you deal with partial units?
- Errors—did they propagate multiplicatively?
- How did the results compare with your initial estimate? Why? How did you estimate?



# How Long is the Room?

- 2 min. write in notebooks
  - What are the implications for students' measurement activity?
  - What did you notice about your and others' use of language, tools, representations, and structure to justify or critique solutions?
- How about mathematical practices?...

# CCSS' Practices



2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

# Advancing knowledge of learning trajectories through solving mathematics problems

- The three parts of a learning trajectory
  - Goal
  - Developmental Progression
  - (Correlated) Instruction
- Such activities focus strongly on the goal: Profound knowledge of early and elementary *mathematics*.
- However, they also motivate reflection on thinking and learning—strategies, concepts, procedures, and so forth (the 2<sup>nd</sup> part of a learning trajectory)

## Activity #2: Video analysis

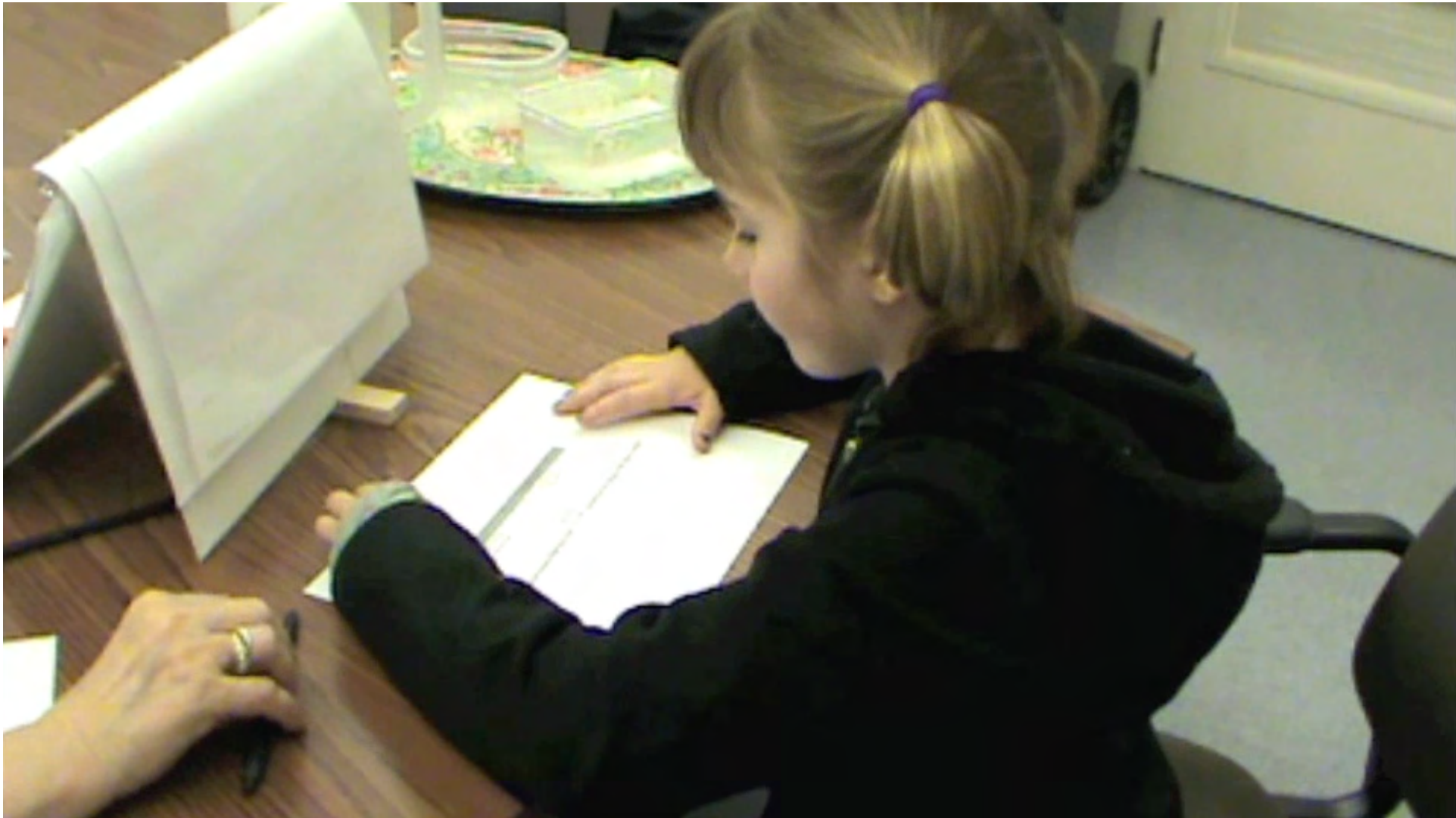
# Student performance examples

Focusing fully on the *Developmental Progression*, we study the thinking and learning of students: Those on video and those in teachers' classrooms...



# End-to-End Length Measurer

- Lays units end-to-end. May not see the need for equal-length units.
- Another example:
  - Lays 9 inch cubes in a line beside a book to measure how long it is.



# Length Unit Relater and Repeater

- Relates size and number of units
  - “If you measure with centimeters instead of inches, you’ll need more of them, because each one is smaller.”
- Repeats or iterates a single unit to measure. Sees need for identical units. Uses rulers with guidance.
  - Measures a book’s length well with a ruler.



# Advancing knowledge of learning trajectories through video analysis

- Analyses of these videos helps teachers build integrated concrete and abstract knowledge of the learning
- Each level—see video, think-pair-share, describe, then see the research-based name and description.
- Sets a framework for their observation of their own children, as they repeat the classic task and conduct that and other tasks (level-eliciting tasks) the following week.

Approach #3: Engaging teachers' own students with a “classic activity”

## Classroom Connection Activities

Classroom connection activities are “professional homework” that:

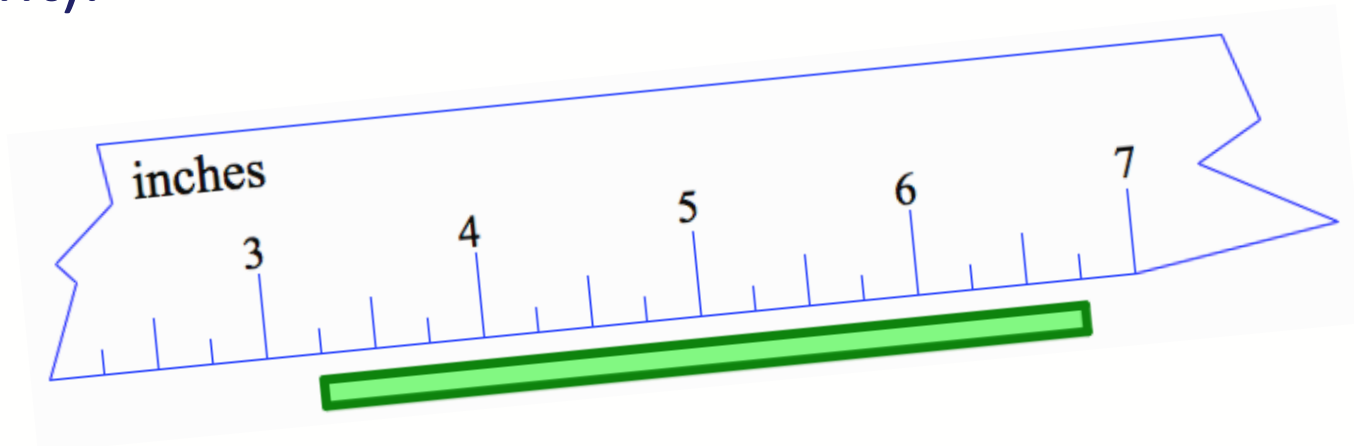
- Connect professional development content with classroom teaching
- Extend thinking about the content of the present and previous sessions

Approach #3: Engaging teachers' own students with a “classic activity”

# Classroom Connection Activities

## The Broken Ruler Task

The task is to tell the length of an object measured with a “broken ruler” (one with no origin, or zero point).

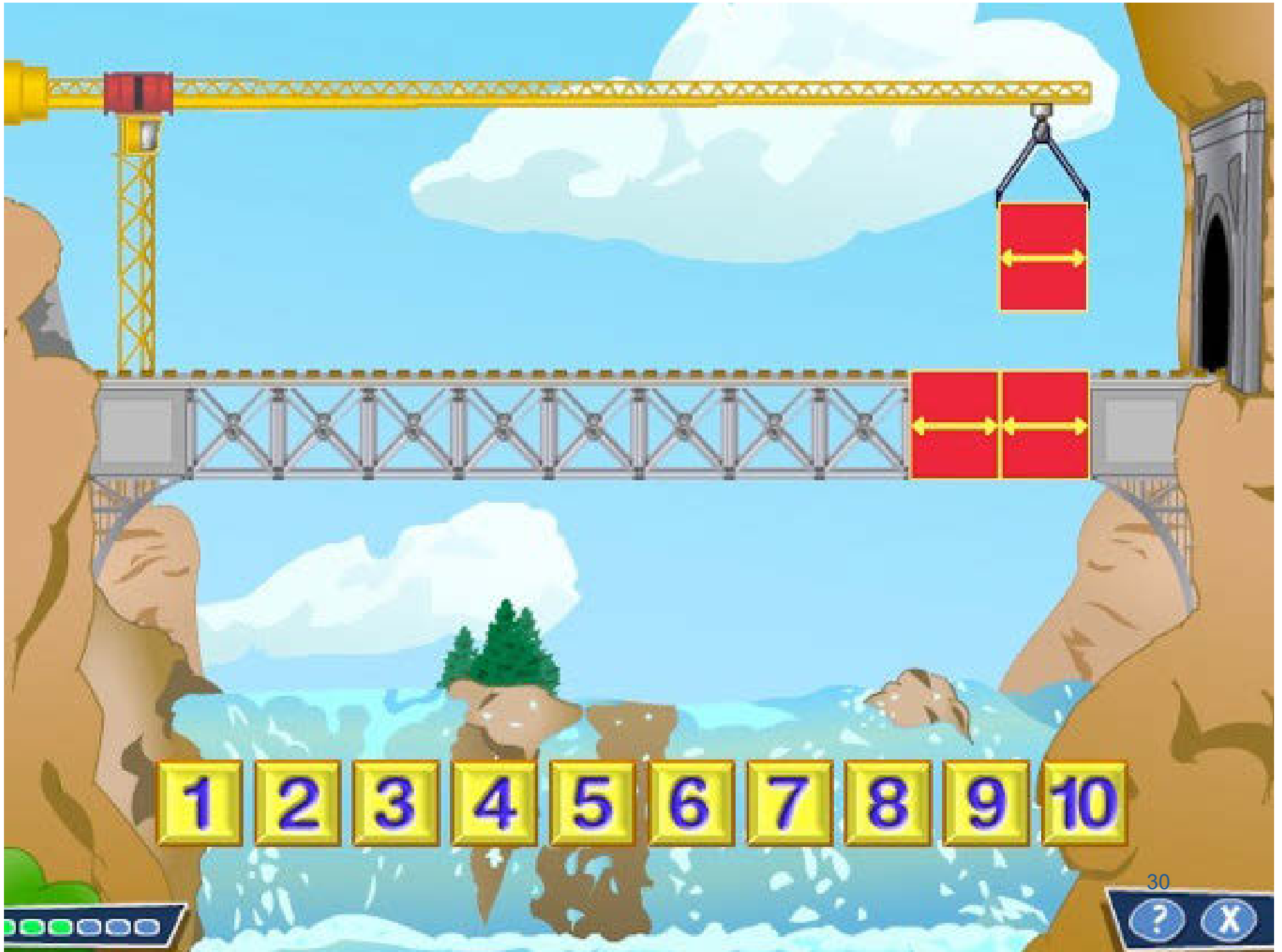




# Learning trajectories, part 3

## The three parts of a learning trajectory

- Goal
- Developmental Progression
- **(Correlated) Instruction**

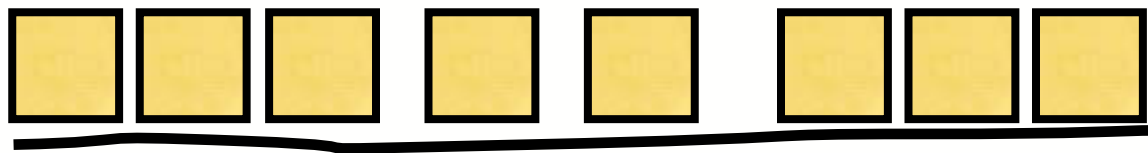




# Mr. Mixup's Measuring Mess

Mr. Mixup measures string with connecting cubes

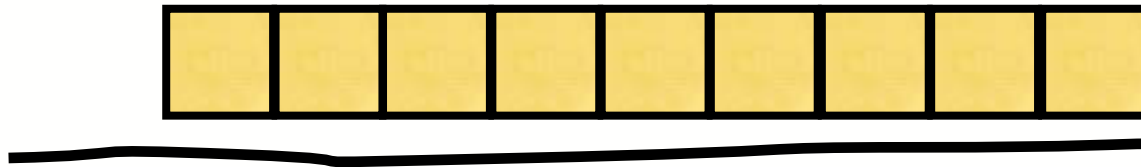
- Gaps between cubes



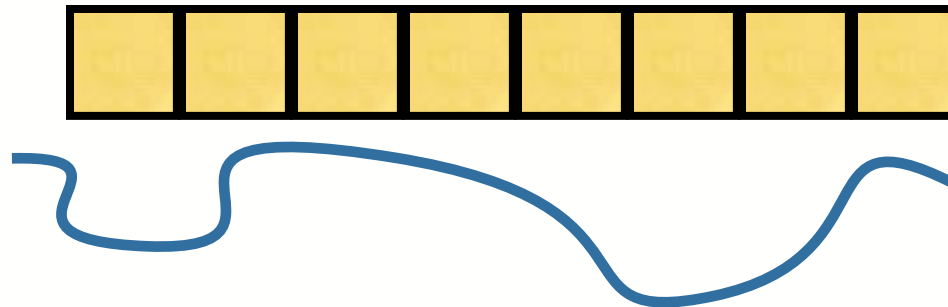


# Mr. Mixup's Measuring Mess

- String end misaligned with cubes



- String or cubes not in a straight line



# Connect with instruction...

- The key is to \*connect\* the tasks to the developmental levels.
- Then teachers modify their own tasks, video them, and discuss.

# A glimpse inside

- Videos of –
  - Classroom interactions
  - Student interviews
  - Experienced facilitation of professional development
  - “Virtual colleagues”
- Slides
- Resources for sessions
- Classroom Connection Activities
- Supplements

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# Insights into this PD approach for learning trajectories

## Advantages:

- Connecting PD content with teachers' own students and curriculum
- Using familiar activities to introduce new dimensions of measurement
- Leveraging “virtual colleagues” to support professional learning

## Challenges:

- Recruiting districts and participants willing to engage in extended PD experiences

and at the same time...

- Needing more depth of experience to connect learning trajectory levels to particular instructional activities

# 3. Becoming Involved:

*Using the Dev-TE@M materials*



# “Geometric Measurement and Spatial Reasoning in Elementary Mathematics Teaching” module

## Fall 2015 pilot

We are seeking teacher educators doing real-time professional development with practicing elementary teachers in:

- School/district professional development
- University courses
- Teacher institutes or workshops
- Teacher study groups

Visit our website to learn more:

<http://www.umich.edu/~devteam/>

# Dev-TE@M Modules

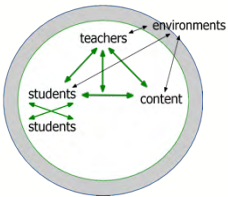
**Module 1:** *Representing and comparing fractions*

**Module 2:** *Reasoning and explanation*

**Module 3:** *Geometric measurement & spatial reasoning*



# Credits



Graphic on slide 6:

Cohen, D. K., Raudenbusch, S., & Ball, D. L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis*, 25 (2), 119-142.



Image on slide 9:

Watterson, B. (1986). *Calvin and Hobbes* [Cartoon]. Retrieved from <http://www.gocomics.com/calvinandhobbes/2011/06/20>



Image on slide 15:

IKEA. (2013). *Shoe measurements* [TV commercial].



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Image on slide 30:

Clements, D. & Sarama, J. (2006). Triad/Building Blocks. *Workin' on the Railroad*. Retrieved from <http://triad-research.du.edu>