## Lecture 24

Chemical Reaction Engineering (CRE) is the
field that studies the rates and mechanisms of chemical reactions and the design of the reactors in which they take place.

Web Lecture 24
Class Lecture 20-Thursday 3/28/2013

Review of Multiple Steady States (MSS)

Reactor Safety (Chapter 13)

- Blowout Velocity
- CSTR Explosion
- Batch Reactor Explosion


## Review Last Lecture

## CSTR with Heat Effects



## Review Last Lecture

## Energy Balance for CSTRs

$$
\begin{gathered}
\frac{d T}{d t}=\frac{F_{A_{0}}}{\sum N_{i} C_{P_{i}}}[\boldsymbol{G}(\boldsymbol{T})-\boldsymbol{R}(T)] \\
G(T)=\left(r_{A} V\right)\left[\Delta H_{R_{k}}\right] \\
R(T)=C_{P_{s}}(1+\kappa)\left[T-T_{C}\right]
\end{gathered}
$$

$$
\kappa=\frac{U A}{F_{A 0} C_{P 0}} \quad T_{C}=\frac{T_{0}+\kappa T_{a}}{1+\kappa}
$$

## Review Last Lecture

## Energy Balance for CSTRs


$5 \quad$ Variation of heat removal line with inlet temperature.

## Review Last Lecture

## Energy Balance for CSTRs



## Review Last Lecture

## Multiple Steady States (MSS)



Variation of heat generation curve with space-time.

## Review Last Lecture

## Multiple Steady States (MSS)



Finding Multiple Steady States with $T_{0}$ varied

## Review Last Lecture

## Multiple Steady States (MSS)



Temperature ignition-extinction curve

## Review Last Lecture

Multiple Steady States (MSS)


Bunsen Burner Effect (Blowout)

## Review Last Lecture

Multiple Steady States (MSS)


Bunsen Burner Effect (Blowout)

## Reversible Reaction

## Gas Flow in a PBR with Heat Effects

$A \leftrightarrow B$


Reversible Reaction Gas Flow in a PBR with Heat Effects
$A \leftrightarrow B$


Reversible Reaction
Gas Flow in a PBR with Heat Effects
$A \leftrightarrow B$
$K_{C}=\frac{C_{B e}}{C_{A e}}=\frac{C_{A 0} X_{e} y T_{0} / T}{C_{A 0}\left(1-X_{e}\right) y T_{0} / T}$
(8) $X_{e}=\frac{K_{C}}{1+K_{C}}$

Reversible Reaction Gas Flow in a PBR with Heat Effects
$A \leftrightarrow B$
Exothermic Case:


Endothermic Case:


## Adiabatic Equilibrium Conversion

Conversion on Temperature
Exothermic $\Delta H$ is negative
Adiabatic Equilibrium temperature ( $\mathrm{T}_{\text {adia }}$ ) and conversion ( $\mathrm{X}_{\mathrm{e}, \text { adia }}$ )


$$
\begin{gathered}
\mathrm{T}=\mathrm{T}_{0}+\frac{\left(-\Delta \mathrm{H}_{\mathrm{Rx}}\right) \mathrm{X}}{\mathrm{C}_{\mathrm{PA}}} \\
\mathrm{X}_{\mathrm{e}}=\frac{\mathrm{K}_{\mathrm{C}}}{1+\mathrm{K}_{\mathrm{C}}}
\end{gathered}
$$

## Gas Phase Heat Effects

Trends:
Adiabatic:
$X$ exothermic $X$ endothermic


## Gas Phase Heat Effects



## Gas Phase Heat Effects

Effect of adding inerts on adiabatic equilibrium conversion
Adiabatic:



Adiabatic Equilibrium Conversion
$\Theta_{\mathrm{I}}=0$
T
$\mathrm{T}_{0}$
$X=\frac{\left(T-T_{0}\right)\left[C_{P_{A}}+\theta_{I} C_{P_{I}}\right]}{-\Delta H_{R x}}, T=T_{0}+\frac{\left(-\Delta H_{R x}\right)}{C_{P_{A}}+\theta_{I} C_{P_{I}}}$



## Adiabatic Exothermic Reactions

$$
\mathrm{A} \longrightarrow \mathrm{~B} \quad \Delta \mathrm{H}_{\mathrm{Rx}}=+15 \frac{\mathrm{kcal}}{\mathrm{~mol}}
$$

The heat of reaction for endothermic reaction is positive, i.e.,

## Energy Balance :

$$
\mathrm{T}=\mathrm{T}_{0}-\frac{\Delta \mathrm{H}_{\mathrm{RX}} \mathrm{X}}{\mathrm{C}_{\mathrm{P}_{\mathrm{A}}}+\Theta_{\mathrm{I}} \mathrm{C}_{\mathrm{P}_{\mathrm{I}}}} \quad \text { and } \mathrm{X}=\frac{\left(\mathrm{C}_{\mathrm{P}_{\mathrm{A}}}+\mathrm{C}_{\mathrm{P}_{\mathrm{I}}} \Theta_{\mathrm{I}}\right)\left(\mathrm{T}_{0}-\mathrm{T}\right)}{\Delta \mathrm{H}_{\mathrm{Rx}}}
$$



We want to learn the effects of adding inerts on conversion. How the conversion varies with the amount, i.e., $\Theta_{1}$, depends on what you vary and what you hold constant as you change $\Theta_{1}$.

## A. First Order Reaction

$$
\frac{\mathrm{dX}}{\mathrm{dV}}=\frac{-\mathrm{r}_{\mathrm{A}}}{\mathrm{~F}_{\mathrm{A} 0}}
$$

Combining the mole balance, rate law and stoichiometry

$$
\frac{\mathrm{dX}}{\mathrm{dV}}=\frac{\mathrm{kC}_{\mathrm{A} 0}(1-\mathrm{X})}{v_{0} \mathrm{C}_{\mathrm{A} 0}}=\frac{\mathrm{k}}{v_{0}}(1-\mathrm{X})
$$

Two cases will be considered
Case 1 Constant $v_{0}$, volumetric flow rate Case 2: Variable $v_{0}$, volumetric flow rate

## A.1. Liquid Phase Reaction



For Liquids, volumetric flow rates are additive.

$$
v_{0}=v_{\mathrm{A} 0}+v_{\mathrm{I} 0}=v_{\mathrm{A} 0}\left(1+\Theta_{\mathrm{I}}\right)
$$

## Effect of Adding Inerts to an Endothermic Adiabatic Reaction

What happens when we add Inerts, i.e., vary Theta I??? It all depends what you change and what you hold constant!!!






## A.1.a. Case 1. Constant $v_{0}$

To keep $v_{0}$ constant if we increase the amount of Inerts, i.e., increase $\Theta_{1}$ we will need to decrease the amount of A entering, i.e., $v_{A 0}$. So $\Theta_{1} \uparrow$ then $v_{A 0} \downarrow$

$$
\mathrm{T}=\mathrm{T}_{0}-\frac{\Delta \mathrm{H}_{\mathrm{Rx}} \mathrm{X}}{\mathrm{C}_{\mathrm{P}_{\mathrm{A}}}+\Theta_{\mathrm{I}} \mathrm{C}_{\mathrm{P}_{\mathrm{I}}}}
$$

A.1.a. Case 2. Constant $v_{\mathrm{A}}$, Variable $v_{0}$

$$
\frac{\mathrm{dX}}{\mathrm{dV}}=\frac{\mathrm{k}(1-\mathrm{X})}{v_{0}}=\frac{\mathrm{k}(1-\mathrm{X})}{v_{\mathrm{A}}\left(1+\Theta_{\mathrm{I}}\right)}
$$







## A.2. Gas Phase


$\mathrm{C}_{\mathrm{TA}}=\frac{\mathrm{r}_{\mathrm{A} 0}}{v_{\mathrm{A}}}=\mathrm{C}_{\mathrm{A} 0}=\frac{\mathrm{r}_{\mathrm{A}}}{\mathrm{RT}_{\mathrm{A}}}$

$\mathrm{C}_{\mathrm{TI}}=\frac{\mathrm{F}_{\mathrm{TI}}}{\mathrm{v}_{\mathrm{I}}}=\frac{\mathrm{F}_{\mathrm{A} 0}+\mathrm{F}_{\mathrm{I} 0}}{\mathrm{v}_{\mathrm{I}}}=\frac{\mathrm{P}_{\mathrm{I}}}{\mathrm{RT}_{\mathrm{I}}}$

Taking the ratio of $\mathrm{C}_{\mathrm{TA}}$ to $\mathrm{C}_{\mathrm{T}}$

$$
\frac{\mathrm{C}_{\mathrm{TI}}}{\mathrm{C}_{\mathrm{TA}}}=\frac{\frac{\mathrm{F}_{\mathrm{TI}}}{v_{\mathrm{I}}}}{\frac{\mathrm{~F}_{\mathrm{TA}}}{v_{\mathrm{A}}}}=\frac{\frac{\mathrm{P}_{\mathrm{I}}}{\mathrm{RT}_{\mathrm{I}}}}{\frac{\mathrm{P}_{\mathrm{A}}}{\mathrm{RT}_{\mathrm{A}}}}
$$

Solving for $\mathrm{v}_{\mathrm{l}}$

$$
v_{\mathrm{I}}=v_{\mathrm{A}} \frac{\mathrm{~F}_{\mathrm{TI}}}{\mathrm{~F}_{\mathrm{TA}}} \frac{\mathrm{P}_{\mathrm{A}}}{\mathrm{P}_{\mathrm{I}}} \frac{\mathrm{~T}_{\mathrm{I}}}{\mathrm{~T}_{\mathrm{A}}}
$$

We want to compare what happens when Inerts and A are fed to the case when only A is fed.

Nomenclature note: Sub I with Inerts I and reactant A fed Sub A with only reactant A fed
$F_{T I}=$ Total inlet molar flow rate of inert, $I$, plus reactant $A, F_{T I}=F_{A 0}+F_{10}$
$F_{T A}=$ Total inlet molar flow rate when no Inerts are fed, i.e., $F_{T A}=F_{A 0}$
$P_{1}, T_{1}=$ Inlet temperature and pressure for the case when both Inerts $(I)$ and $A$ are fed
$P_{A}, T_{A}=$ Inlet temperature and pressure when only $A$ is fed
$C_{A 0}=$ Concentration of $A$ entering when no inerts are presents $C_{A 0}=\frac{F_{A 0}}{v_{A}}$
$\mathrm{C}_{\mathrm{TA}}=$ Total concentration when no inerts are present $=\frac{\mathrm{P}_{\mathrm{A}}}{\mathrm{RT}_{\mathrm{A}}}$
$C_{T I}=$ Total concentration when both $I$ and $A$ are present $=\frac{P_{I}}{R T_{I}}$
$\mathrm{C}_{\mathrm{AOO}}=$ Concentration of $A$ entering when inerts $A$ are entering $=\frac{\mathrm{F}_{\mathrm{A} 0}}{v_{\mathrm{I}}}$
$v_{1}=$ Entering volumetric flow rate with both Inerts $(\mathrm{I})$ and reactant $(\mathrm{A})$

$$
\begin{gathered}
\frac{\mathrm{F}_{\mathrm{TI}}}{\mathrm{~F}_{\mathrm{TA}}}=\frac{\mathrm{F}_{\mathrm{A} 0}+\mathrm{F}_{\mathrm{I} 0}}{\mathrm{~F}_{\mathrm{A} 0}} \equiv\left(1+\Theta_{\mathrm{I}}\right)=\frac{1}{\left(\frac{\mathrm{~F}_{\mathrm{A} 0}}{\mathrm{~F}_{\mathrm{I} 0}+\mathrm{F}_{\mathrm{A} 0}}\right)}=\frac{1}{\mathrm{y}_{\mathrm{A} 0}} \\
\mathrm{y}_{\mathrm{A} 0}=\frac{1}{\left(1+\Theta_{\mathrm{I}}\right)}
\end{gathered}
$$

$$
v_{\mathrm{I}}=v_{\mathrm{A}}\left[\left(1+\Theta_{\mathrm{I}}\right) \frac{\mathrm{P}_{\mathrm{A}}}{\mathrm{P}_{\mathrm{I}}} \frac{\mathrm{~T}_{\mathrm{I}}}{\mathrm{~T}_{\mathrm{A}}}\right]
$$

## A.2.a. Case 1

Maintain constant volumetric flow, $v_{0}$, rate as inerts are added. I.e., $v_{0}=$ $v_{1}=v_{A}$. Not a very reasonable situation, but does represent one extreme. Achieve constant $v_{0}$ varying $P, T$ to adjust conditions so term in brackets, [ ], is one.

$$
\left\lfloor\left(1+\Theta_{\mathrm{I}}\right) \frac{\mathrm{P}_{\mathrm{A}}}{\mathrm{P}_{\mathrm{I}}} \frac{\mathrm{~T}_{\mathrm{I}}}{\mathrm{~T}_{0}}\right\rfloor=1
$$

For example if $\Theta_{l}=2$ then $v_{l}$ will be the same as $v_{A}$, but we need the entering pressures $P_{I}$ and $P_{A}$ to be in the relationship $P_{I}=3 P_{A}$ with $T_{A}=T_{I}$

$$
v_{\mathrm{I}}=\mathrm{v}_{\mathrm{A}}\left\lfloor(1+2) \cdot \frac{\mathrm{P}_{\mathrm{A}}}{3 \mathrm{P}_{\mathrm{A}}} \frac{\mathrm{~T}_{\mathrm{A}}}{\mathrm{~T}_{\mathrm{A}}}\right\rfloor=\mathrm{v}_{\mathrm{A}}\left[3 \bullet \frac{1}{3}\right]=\mathrm{v}_{\mathrm{A}}=\mathrm{v}_{0}
$$

## A.2.a. Case 1

That is the term in brackets, [ ], would be 1 which would keep $v_{0}$ constant with $v_{1}=v_{A}=v_{0}$. Returning to our combined mole balance, rate law and stoichiometry

$$
\frac{\mathrm{dX}}{\mathrm{dV}}=\frac{\mathrm{k}(1-\mathrm{X})}{\mathrm{v}_{0}}
$$




A.2.b. Case 2: Variable $v_{0}$ Constant T, P i.e., $P_{1}=P_{A}, T_{1}=T_{A}$

$$
v_{\mathrm{I}}=v_{\mathrm{A}} \frac{\mathrm{~F}_{\mathrm{TI}}}{\mathrm{~F}_{\mathrm{TA}}}=v_{\mathrm{A}} \frac{\left(\mathrm{~F}_{\mathrm{A} 0}+\mathrm{F}_{\mathrm{I} 0}\right)}{\mathrm{F}_{\mathrm{A} 0}}=v_{\mathrm{A}}\left(1+\Theta_{\mathrm{I}}\right)
$$

$$
v_{I}=v_{A}\left(1+\Theta_{I}\right)
$$

$$
\frac{d X}{d V}=\frac{1}{1,} \frac{k}{1+\Theta}(1-X)
$$




## B. Gas Phase Second Order Reaction

Pure A


$$
\mathrm{C}_{\mathrm{A} 0}=\frac{\mathrm{P}_{\mathrm{A}}}{\mathrm{RT}_{\mathrm{A}}}=\frac{\mathrm{F}_{\mathrm{A} 0}}{\mathrm{v}_{\mathrm{A}}}
$$

Inerts Plus A


$$
\mathrm{C}_{\mathrm{TI}}=\frac{\mathrm{F}_{\mathrm{A} 0}\left(1+\Theta_{\mathrm{I}}\right)}{v_{\mathrm{I}}}
$$

$$
\frac{\mathrm{dX}}{\mathrm{dV}}=\frac{-\mathrm{r}_{\mathrm{A}}}{\mathrm{~F}_{\mathrm{A} 0}}=\frac{\mathrm{kC}_{\mathrm{A} O I}^{2}(1-\mathrm{X})^{2}}{\mathrm{~F}_{\mathrm{A} 0}}
$$

## B. Gas Phase Second Order Reaction

$$
\begin{gathered}
v_{I}=v_{A}\left(1+\Theta_{\mathrm{I}}\right) \frac{\mathrm{P}_{\mathrm{A}}}{\mathrm{P}_{\mathrm{I}}} \frac{\mathrm{~T}_{\mathrm{I}}}{\mathrm{~T}_{\mathrm{A}}} \\
\frac{\mathrm{C}_{\mathrm{A} 0 \mathrm{I}}^{2}}{\mathrm{~F}_{\mathrm{A} 0}}=\frac{\left(\mathrm{F}_{\mathrm{A} 0} / v_{\mathrm{I}}\right)^{2}}{\mathrm{~F}_{\mathrm{A} 0}}=\frac{\mathrm{F}_{\mathrm{A} 0}}{v_{\mathrm{I}}^{2}}=\frac{\mathrm{F}_{\mathrm{A} 0}}{v_{\mathrm{A}} \cdot v_{\mathrm{A}}\left(1+\Theta_{\mathrm{I}}\right)^{2}\left(\frac{\mathrm{P}_{\mathrm{A}}}{\mathrm{P}_{\mathrm{I}}}\right)^{2}\left(\frac{\mathrm{~T}_{\mathrm{I}}}{\mathrm{~T}_{\mathrm{A}}}\right)^{2}} \\
=\frac{\mathrm{C}_{\mathrm{A} 0}}{v_{\mathrm{A}}\left(1+\Theta_{\mathrm{I}}\right)^{2}}\left(\frac{\mathrm{P}_{\mathrm{I}}}{\mathrm{P}_{\mathrm{A}}} \frac{\mathrm{~T}_{\mathrm{A}}}{\mathrm{~T}_{\mathrm{I}}}\right)^{2} \\
\frac{\mathrm{dX}}{\mathrm{dV}}=\frac{\mathrm{k}}{\left(1+\Theta_{\mathrm{I}}\right)^{2}} \frac{\mathrm{C}_{\mathrm{A} 0}}{v_{\mathrm{A}}}\left(\frac{\mathrm{P}_{\mathrm{I}}}{\mathrm{P}_{\mathrm{A}}} \frac{\mathrm{~T}_{\mathrm{A}}}{\mathrm{~T}_{\mathrm{I}}}\right)^{2}(1-\mathrm{X})^{2}
\end{gathered}
$$

## B. Gas Phase Second Order Reaction

For the same temperature and pressures for the cases with and without inerts, i.e., $P_{1}=P_{A}$ and $T_{1}=T_{A}$, then

$$
\frac{\mathrm{dX}}{\mathrm{dV}}=\frac{\mathrm{k}}{\left(1+\Theta_{\mathrm{I}}\right)^{2}} \frac{\mathrm{C}_{\mathrm{A} 0}}{v_{\mathrm{A}}}(1-\mathrm{X})^{2}
$$





## End of Web Lecture 24 Class Lecture 20

