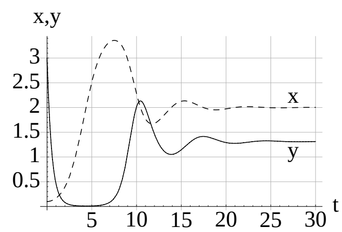
For all problems, *SHOW ALL OF YOUR WORK*. Partial solutions and problems with missing steps will be marked wrong. Continue your work on the back of the page or extra sheet at the end of the exam if you need additional space. **1. Note that there is a table of possibly useful stuff on the back page. 2. You do not need but may use the normal graphing calculator functions of any graphing calculator, but not any differential equations functionality it may have.**

1. Find the following Laplace transforms and inverse transforms:
   a. $\mathcal{L}\{u(t-a)-u(t-b)\}$ $(b>a)$, using the definition of the transform. (4 points)

   b. $\mathcal{L}^{-1}\{\frac{4}{s(s^2+9)}\}$. (4 points)

   c. $\mathcal{L}^{-1}\{\frac{4}{s(s^2+9)}\}$, using a different calculation method than you used in (1b). (4 points)

2. The graph to the right shows the solutions x (dashed curve) and y (solid curve) for a system



$$x' = f(x,y)$$
$$y' = g(x,y).$$

Use these to sketch the corresponding trajectory in the phase plane. The system has three critical points. Based on the given trajectory, can you guess what they are and what their stability is? Explain. Sketch some additional trajectories in the phase plane that reflect your understanding of the system. (12 points)

3. Solve $2x\frac{dy}{dx} = 4y^3(1-3x^2)$. (6 points)

4. Consider the system of differential equations

$$x' = x - xy$$
$$y' = 2y - y^2$$

Determine insofar as you can the behavior of solutions to this system. (16 points)

5. Solve each of the following.
   a. $x'' + 3x' = 0$, with $x(0) = 4$ and $x'(0) = -6$. (6 points)

   b. $x'' + 4x' + 4x = e^{-2t} + 4$. (6 points)

6. Our indomitable mascot Newton the cat is playing with a mass on a spring. The unforced motion of the mass is given by the differential equation $x'' + 4x' + 8x = 0$, where $x(t)$ is the displacement of the mass. Newton introduces a forcing in the system by hitting the mass once, very rapidly, at time $t = 2$. Assume that the mass was not initially moving, and that the force of the impact introduces a sudden change in the mass' acceleration of four acceleration units.
   a. Write down a reasonable differential equation that includes the effect of the forcing and initial conditions. (There may be several possible answers for this.) (6 points)

   b. Solve your differential equation. (8 points)

7. Consider the differential equation $P'(t) = kP(t)(L - P(t))$. What could this model? Why? Find all equilibrium solutions to this differential equation, analyze their stability, and predict the long-term behavior of $P(t)$ for different initial conditions. (10 points)

8. Solve $ty' - y = t^2 \cos(t)$. (6 points)

   a. Use your solution to show that if $y(0) = 0$, there are an infinite number of solutions, whereas if $y(0) = 1$ there are none. (4 points)

   b. Does this conform to or contradict the theorem for the existence and uniqueness of solutions for first-order differential equations? Explain. (6 points)