For all problems, *SHOW ALL OF YOUR WORK*. While partial credit will be given, partial solutions that could be obtained directly from a calculator or a guess are worth no points. Continue your work on the back of the page or extra sheet at the end of the exam if you need additional space. *You do not need but may use the normal graphing calculator functions of any graphing calculator, but NOT any differential equations functionality it may have.*

1. Find the real-valued solutions to the following, as indicated.

   a. Find the general solution to $x'' + 4x' + 8x = 4t$. The complementary homogeneous solution is $x_c = c_1 e^{-2t}\cos(2t) + c_2 e^{-2t}\sin(2t)$. (6 points)

   b. Find the general solution to $\frac{d^3x}{dt^3} + 8x = 3\sin(2t)$. The complementary homogeneous solution is $x_c = c_1 e^{-2t} + c_2 e^{t/2}\cos(\sqrt{3}\,t/2) + c_3 e^{t/2}\sin(\sqrt{3}\,t/2)$. (8 points)

   c. Find the solution to $x'' + 3x' + 2x = 3e^{-2t}$, $x(0) = 2$, $x'(0) = 0$. The complementary homogeneous solution is $x_c = c_1 e^{-2t} + c_2 e^{-t}$. (10 points)

2. Find the real-valued solutions to the following, as indicated.

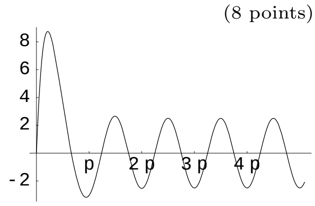
   a. Find the general solution to $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. (10 points)

   b. If $\mathbf{y} = (x_1\ x_2\ x_3)^{\mathrm{T}}$, use the eigenvalue method to find the general solution to $\mathbf{y}' = \mathbf{A}\mathbf{y}$ if $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & -4 \\ 0 & 1 & 0 \end{pmatrix}$. *(For 10 points, you may solve instead for* $\mathbf{A} = \begin{pmatrix} 0 & -4 \\ 1 & 0 \end{pmatrix}$*).* (15 points)

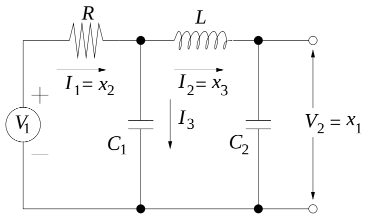
3. For the following,
   i. write down a differential equation that could give the behavior described *(Note that there might be a number of reasonable answers to this!)*, and
   ii. **without** solving the equation, write down what functions you expect to see in the solution to the problem (e.g., e^{-t^2}, $12\ln(\arctan(t))$, or t^{762}).

   a. Resonance, with the resonant solution having frequency $\omega = 7$. (8 points)

   b. The behavior shown to the right. (8 points)



4. A circuit that is commonly found in electronic equipment is a "two-loop low-pass filter," which regulates an input voltage by filtering out high frequency oscillations in that voltage. For the interested, the circuit is shown in the figure to the right. A model for the circuit is the following. Write it in the matrix form $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}$. (All capital letters and ω in the equations are constants, and primes denote derivatives with respect to t.)



$$x_1' = \frac{1}{C_2} x_3$$

$$x_2' = -\frac{1}{RC_1} x_2 + \frac{1}{RC_1} x_3 - \frac{\omega}{R}\cos(\omega t)$$

$$x_3' = -\frac{1}{L} x_1 - \frac{R}{L} x_2 + \frac{1}{L}\cos(\omega t)$$

5. Let $\mathbf{M} = \begin{pmatrix} -1/2 & -2 \\ 1/3 & 4/3 \end{pmatrix}$ and $\mathbf{P} = \begin{pmatrix} -6 & 12 \\ 2 & -3 \end{pmatrix}$.

   a. Is $\mathbf{P}$ the inverse matrix for $\mathbf{M}$? (Why or why not?) (5 points)

   b. If $\mathbf{x} = (x_1\ x_2)^T$ and $\mathbf{b} = (1\ 3)^T$, how many solutions does $\mathbf{M}\,\mathbf{x} = \mathbf{0}$ have? How many solutions are there to $\mathbf{M}\,\mathbf{x} = \mathbf{b}$? Be sure to indicate why you give the answers you do. *Note that you do not have to solve these problems to answer this question!* (6 points)

6. Consider the problem $\mathbf{y}' = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \mathbf{y}$, where $\mathbf{y} = (x_1\ x_2)^T$.

   a. Verify that the vector functions $\mathbf{y}_1 = \begin{pmatrix} e^t \\ 0 \end{pmatrix}$ and $\mathbf{y}_2 = \begin{pmatrix} te^t \\ \frac{1}{2}e^t \end{pmatrix}$ are solutions to this problem. (6 points)

   b. Write a general solution to the problem. What is true that allows you to construct the general solution in the manner you do? (6 points)

   c. Use your general solution to find the particular solution to this problem satisfying the initial condition $\mathbf{y}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. (6 points)