For all problems, *SHOW ALL OF YOUR WORK*. While partial credit will be given, partial solutions that could be obtained directly from a calculator or a guess are worth no points. Continue your work on the back of the page or extra sheet at the end of the exam if you need additional space. *You do not need but may use the normal graphing calculator functions of any graphing calculator, but NOT any differential equations functionality it may have.* If you need to borrow a graphing calculator, ask me.

1. For each of the following, set up the appropriate form of the particular solution y_p but *do not* determine the values of the coefficients in your solution. Be sure it is clear why your expression for y_p has the form it does.

   a. $y'' + 3y' + 2y = 4e^{-2x} - \pi e^{2x}$. (8 points)

   b. $y'' + 4y = x(4 + 2\sin(2x))$. (8 points)

2. Use the eigenvalue method to solve the initial value problem $\vec{x}' = \begin{pmatrix} 3 & 8 \\ 2 & -3 \end{pmatrix} \vec{x}$, $\vec{x}(0) = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$. (16 points)

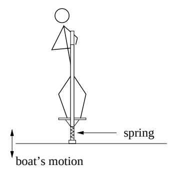
3. What is an eigenvalue problem? What condition do we impose to find the eigenvalues of a matrix? (4 points)

   a. How does the condition that you indicated in (3) follow from your statement of what an eigenvalue problem is? (4 points)

   b. Why do we impose the condition that you indicated in (3) (that is, what does imposing the condition guarantee us)? (4 points)

4. Consider the system $\vec{x}\,' = \begin{pmatrix} 2 & -5 \\ 4 & 6 \end{pmatrix} \vec{x}$. The eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 2 & -5 \\ 4 & 6 \end{pmatrix}$ are $\lambda = 4 \pm 4i$ and $\vec{v} = \begin{pmatrix} -1 \pm 2i \\ 2 \end{pmatrix}$. Find the general (real-valued) solution to this problem. (12 points)

5. A clown stands on a pogo stick (note: a pogo stick is essentially a spring; see figure to the right) on the deck of a boat. The up-and-down motion of the boat is periodic with a frequency ω. The clown's vertical displacement is then governed by the initial value problem



$$y'' = -cy' - 20y + 17\sin(\omega t), \quad y(0) = 0, \quad y'(0) = 0.$$

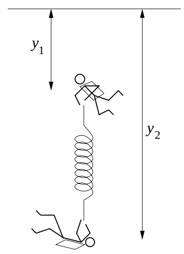
a. Is it possible that this equation will allow the the clown's displacement to exhibit beats? Explain any conditions that would have to be true for this to happen. (4 points)

b. Suppose, regardless of what you said in (5a), that the clown's motion exhibits beats. Sketch a graph illustrating this. Explain in a couple of sentences what it says about the clown's displacement (that is, what can you say about how the displacement changes with time? Is the displacement large or small? Growing of decaying?, etc.). (6 points)

c. Now suppose that $c = 4$ and that the boat's motion continues with $\omega = 4$ for a long time. Solve the initial value problem (insofar as you need to) and find an expression describing the long-term behavior of the clown's displacement. (12 points)

6. Write down a differential equation for which you could use Variation of Parameters but *could not* use the Method of Undetermined Coefficients. Explain what characteristics of your equation make it a correct answer to this question. Be sure to include in your explanation what you use Variation of Parameters to find. *Note that you do not need to solve the problem that you write down.* (6 points)

7. Two skydivers, after leaping one-after-the-other from a plane, are playing with a long spring (figure to the right). The spring's equilibrium length is L. A system modeling this is



$$
\begin{aligned}
m_1\, y_1'' &= m_1 g - c y_1' + k(y_2 - y_1 - L) \\
m_2\, y_2'' &= m_2 g - c y_2' - k(y_2 - y_1 - L).
\end{aligned}
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   a. Explain what each of the terms in the system represent, and therefore why it is a good model for this situation. (6 points)

   b. Rewrite this as a *first-order* system of differential equations. (6 points)

   c. Rewrite your system as a matrix equation. (4 points)