Michigan Math and Models of Inquiry Based Learning in Entry-Level Mathematics

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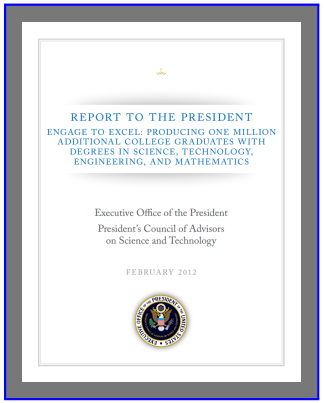
Where we are now

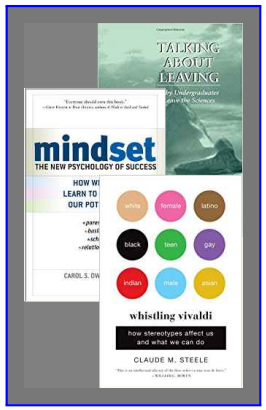
"May you live in interesting times." –(not a) Chinese curse

- Our times:
  - [2011] Each year only about 50% of students earn a grade of A, B, or C in college algebra. *[Saxe & Braddy, 2015]*
  - [2013] only 43% of undergraduate math majors are awarded to women; Between 1990 and 2011, the number of math majors increased by 25%, while the those awarded to blacks didn't change (and the number to women increased only about 10%). *[Bressoud, 2013]*
  - [2012] . . . college mathematics teaching and curricula [should be] developed and taught by faculty from mathematics-intensive disciplines other than mathematics. . . " *[PCAST, 2012]*
- Thus: there is significant pressure to teach well.



What we know about teaching, learning, and retention

- We understand learning better (than ever?) now:
  - Active learning improves student understanding and disposition, *[Laursen et al., 2014]*
  - Increases student performance (0.47σ, more on concept inventories) and decreases DWF rates 45%, *[Freeman, et al., 2014]*
  - Especially for underrepresented groups. *[Laursen, et al., 2011]*
- And Active Learning can address retention and inclusion issues:
  - Mindset and math disposition, stereotype threat, and inclusion of underrepresented groups. *[Dweck, 2007; Steele, 2010; Seymour & Hewitt, 1997]*

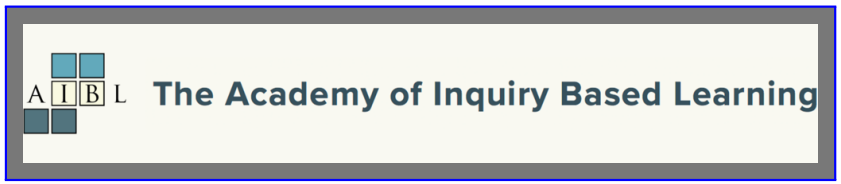


- Our community is clear on this: "A wealth of research has provided clear evidence that active learning results in better student performance and retention. . . we call on [faculty and policy makers]. . . to ensure that effective active learning is incorporated into post-secondary mathematics classrooms." *[CBMS Statement, 2016]*

A Rose by any other Name

- A note about "Inquiry Based Learning." Different terms we see:
  - Inquiry Based Learning (IBL),
  - Active Learning (though, what is "passive learning"?),
  - the Moore Method, or Modified Moore Method, and more.
- The *Academy of Inquiry Based Learning* (AIBL) defines IBL as:
  - *a form of active learning in which students are given a carefully scaffolded sequence of mathematical tasks and are asked to solve and make sense of them, working individually or in groups.*

    [inquirybasedlearning.org]
  - And it has core principles:
    - deep engagement in rich mathematical activities, and
    - opportunities to collaborate with peers.

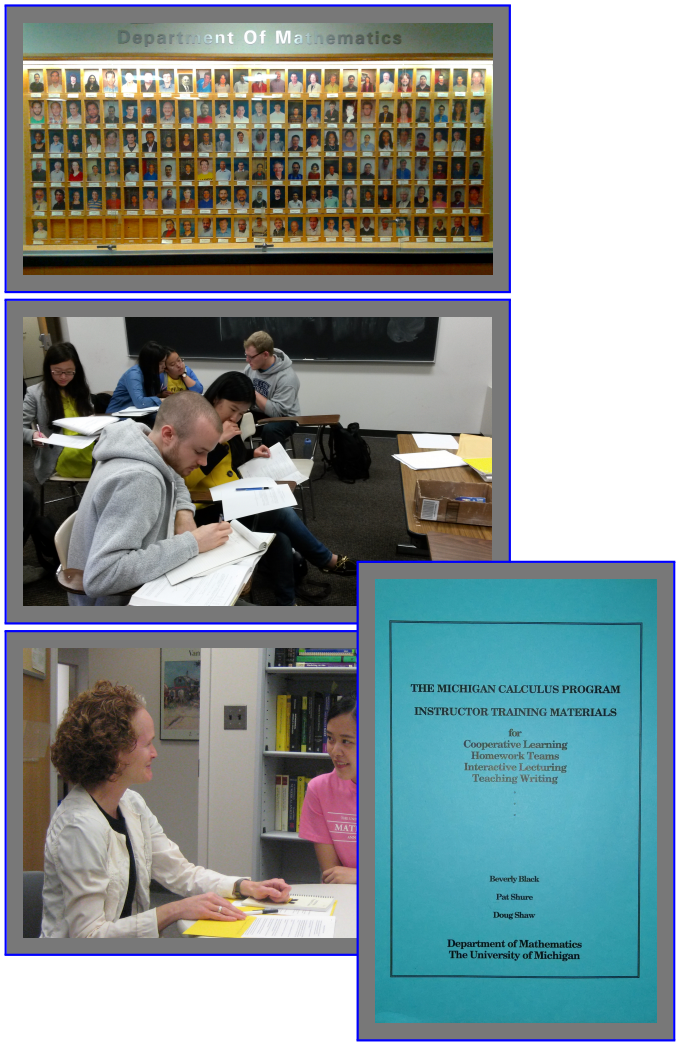


Overview and Outline

- Context
  - What matters, and why.
- History and Michigan Math
  - What "Michigan Math" is.
  - Its implementation and continuation.
  - Evidence for our success.
- Other Models
  - University of Nebraska, Lincoln.
  - San Diego State University.

Department Context

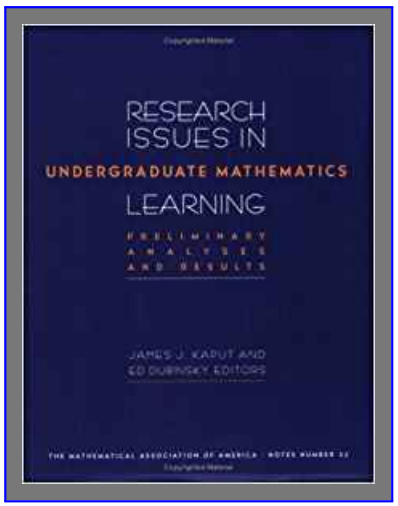
- It's a big department: 128 faculty, 135 graduate students, and over 600 undergraduate majors.
- The undergraduate program is multifaceted: 5 two-year introductory sequences (one standard, four honors), an IBL center, clubs,. . .
- And has broad support structures: placement, advising, tutoring, math lab, computer labs and testing, DHSP.
- And significant instructor support: training, lesson plans, class visits, coordinated courses, on-line homework.



Long, long ago in a Department far, far away

- Local and national challenges with calculus: failure to generate proficiency and promote retention in STEM fields, especially Math, and college-level mathematics serving as a bottleneck not a pump. *[Steen, 1987; PCAST, 2012]*
- 1990: Dept chair called by dean and regent
  - *Graduating seniors "universally" named calculus as their worst experience at UM.*
    –Don Lewis
- Calculus Reform: NSF funded over 350 projects between 1987 and 1995, incl. at Michigan.
  - Learning context: *"This is a time for establishing, no matter how tentatively, [frameworks]. . . that can help. . . guide further work."* *[Kaput & Dubinsky, 1994]*

Don Lewis, 1926–2015



New Wave Calculus

- Michigan's Calculus Reform: 1991–95 implementation ("Calculus and calculators"?)
  - Reduced class sizes from 35–45 to 24.
  - Adopted a reformed textbook.
  - Instituted use of graphing calculators.
  - Implemented cooperative learning both in and out of the classroom.
  - Extensively revised and extended the new instructor training program.
- By 2000, program continued (but class sizes were up to 32).

calc class, 90s

Michigan Calculus (Today)

- Michigan Calculus is the Michigan Introductory Program:
  - Math 105, Data, Functions, & Graphs,
  - Math 115 & 116, Calculus I & II.
- Defining characteristics.
  - Active learning in classes of 32 (until 2015) or 18 (since).
  - Conceptual focus and assessment.
    - And Mastery/Gateway assessment.
  - Structure and Instructor support: common lesson plans, online homework, exams, grading, course scale, instructor support.
  - Student Support.
    - Placement, and mid-course drop-back course (math 115 $\rightarrow$ 110).
    - Math Lab.

ma105 class, 10s

Active Learning

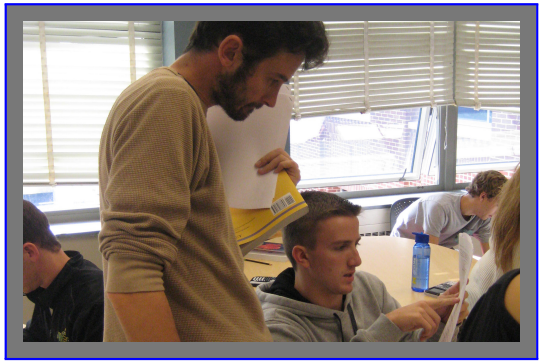
Sample class:

10:10–10:15am	Group work on introductory problem
10:15–10:20am	Announcements
10:20–10:30am	Summary of group work solutions
10:30–10:40am	Mini-lecture on new material
10:40–11:10am	Group work on new material
11:10–11:20am	Discussion of solution that group wrote on board
11:20–11:25am	Group discussion
11:25–11:30am	Summary of remaining group work

Total:

group work: $\sim$40 min

lecture: $\sim$40 min



Paul Kessenich, math 115(?)

Some Observations

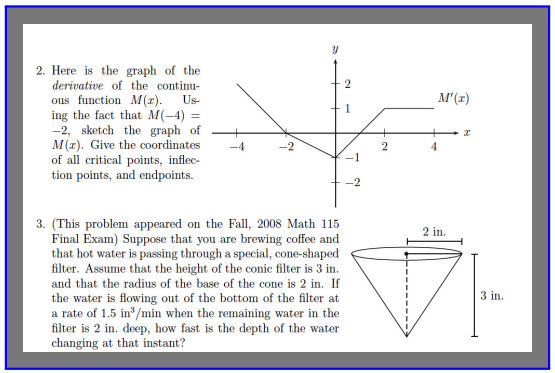
- There are some lucky(?) characteristics of our environment.
  - Classroom furniture and layout have improved**(!)** with time.
  - 80 minutes is a good class length.
  - Class sizes of 18 are a decrease**(!)** from earlier.

. . . but 32 works too.

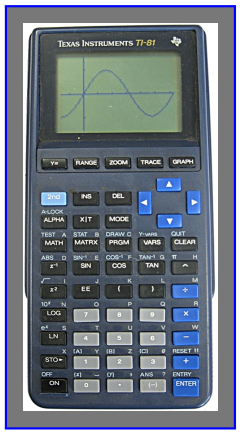
Intro Program classroom

Course Focus

- Exams and most homework are highly conceptual
- Calculators and notecards are allowed (in general) on all exams and homework.
- Skills (e.g., derivatives, integrals) are tested through gateway tests.


2. Here is the graph of the *derivative* of the continuous function $M(x)$. Using the fact that $M(-4) = -2$, sketch the graph of $M(x)$. Give the coordinates of all critical points, inflection points, and endpoints.

3. (This problem appeared on the Fall, 2008 Math 115 Final Exam) Suppose that you are brewing coffee and that hot water is passing through a special, cone-shaped filter. Assume that the height of the conic filter is 3 in. and that the radius of the base of the cone is 2 in. If the water is flowing out of the bottom of the filter at a rate of 1.5 in^3/min when the remaining water in the filter is 2 in. deep, how fast is the depth of the water changing at that instant?



What is Conceptual?

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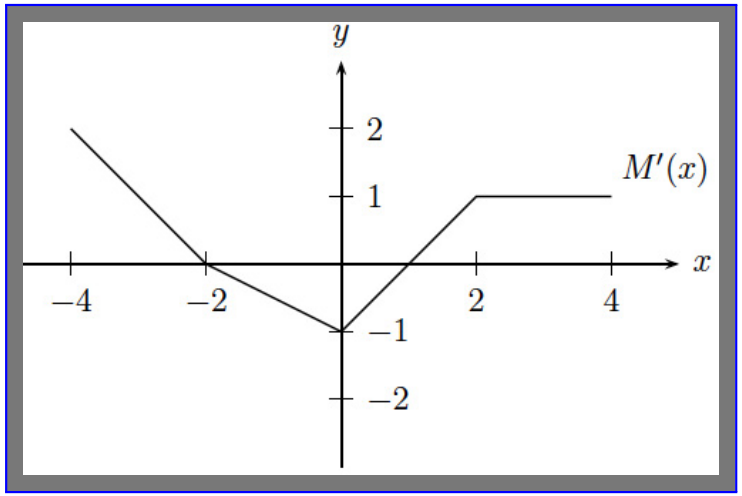

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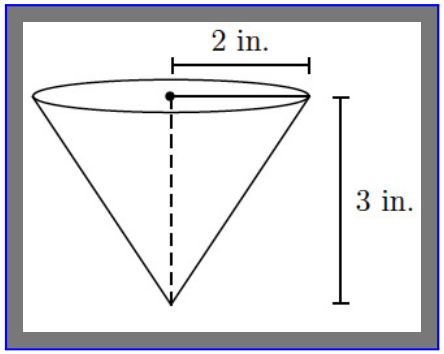
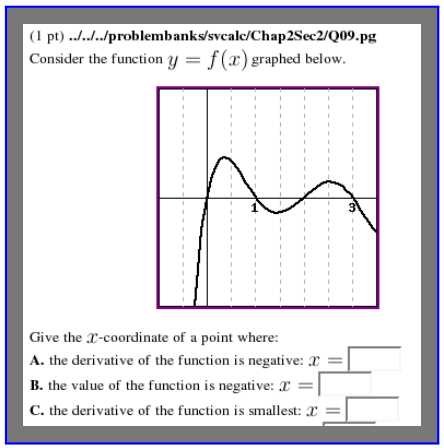


Assessment

- Uniform Exams (3)
  - highly conceptual,
  - allow calculators and notecards.
- Team Homework (5–8)
  - teams of four students,
  - with designated roles: manager, reporter, scribe, clarifier;
  - require solutions written in full sentences;
  - (also) highly conceptual.
- Web Homework ($\approx$ daily)
  - largely drawn from the textbook
- Gateway testing (1-2/course)
  - Test basic or essential skills,
  - Allow unlimited practice,
  - Taken for credit in a proctored lab.

Exam: A wind turbine, spinning counterclockwise at a constant rate… At exactly 1pm, a blade is pointing straight toward the ground. Find a formula for the height of the blade above the ground.

Team HW: Coulomb's law describes electrical force, $F(d)$ between two electrically charged objects. What is a reasonable domain for $F(d)$? Why should it be invertible? Fit a formula to data.

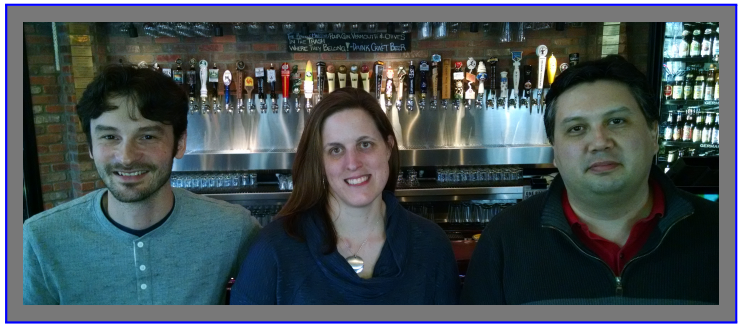


web homework

Gateway: Find the derivative of $S(r) = r\cos(r^2 + 1) - \pi$.

Structure

- Courses are very Uniform
  - Syllabus
  - Daily Schedule
    - 4–6 weekly class schedules.
  - Assessment
    - all but quizzes, which are by section.
    - And exam grading: no instructor grades their section, teams grade each problem.
    - Final grading scale: is set by the coordinator, with some leeway for instructors to change grades.
  - Pedagogy
    - strong emphasis on active, group learning.
- All of these are managed by a Course Coordinator for each course.
  - One course teaching release.



Paul Kessenich, Angela Kubena, Fernando Carreon

. . . or two, or three

Support

- Course Coordinators, Introductory Program Directors, Department Administration *(and me)*
- Training Week
  - All new graduate students and post-docs.
  - Week-long training program.
- Lesson Plans
  - For all of math 105, first third of math 115.
- Weekly Course Meetings
- Class visits
  - For all new instructors, At least once in first semester.
- Midterm Evaluations

Scissors congruence training session

Lesson Plans!

Math 115 - Lesson 3: *Section 1.3 - New Functions From Old*

Notices		**Assignments**	
		READ:	Section 1.4
REMINDERS:	Team HW due date and time (beginning of class)	DO:	WeBWorK 1.3
		DUE:	
ANNOUNCE:	Date for upcoming quiz over course material		

Suggested Lesson Plan: [Time is shown as number of minutes after the hour or 1/2-hour]

[10 - 25] Give a short quiz on the Student Guide and/or the reading for today's class, if you have indicated you will do so. This need not be long or difficult, just enough to determine if they have actually read the guide and are doing the section reading before class. Announce the date and sections to be covered for an upcoming in-class quiz over the course material (an actual math quiz!).

[25 - 35] Discuss the quiz immediately after it is collected. Make certain that students understand the course grading policy (and that YOU do, too–ask if you are unsure), the fact that this course will require a minimum of 8 hours of outside of class work, etc.
Take a couple of minutes to make sure students are dong the web homework and meeting with their teams. Clear up any lingering "course administrivia" questions.

Note: In this lesson, there are several good opportunities for students to work together at the blackboards. And colored chalk is extremely useful for this section.

[35 - 55] It's very important for students to recognize basic "manipulations" (transformations) of the functions in their library. They should be comfortable with the following facts and know how to use them:

- $f(x) + k$ and $f(x + k)$ represent vertical and horizontal shifts of $f(x)$, respectively. (They should know the direction and magnitude of the shifts as well.)
- $-f(x)$ and $f(-x)$ represent vertical and horizontal "flips".
- $kf(x)$ and $f(kx)$ represent vertical and horizontal scalings, either "stretches" or "shrinks" (aka

Training Week

University of Michigan Mathematics Department
Professional Development Program, Fall 2018

	Monday, Aug 27	Tuesday, Aug 28	Wednesday, Aug 29	Thursday, Aug 30	Friday, Aug 31	
8:30	ALL NEW graduate students Math Dept Orientation 8:30-10:45 am EH B844					8:30
9:00		**Video Lecturing (Groups A-G)** *(7 minutes each)* MH		**Running an Interactive Classroom Session I** *(10 minutes each)* MH	**Running an Interactive Classroom Session II** *(10 minutes each)* MH	**9:00**
9:30	**Faculty** Introductions to the Week 10:30-10:45 am EH B860	**Asking Questions (Groups AA-GG)** NUB 1528	**Extended Individual Practice Lecturing** *(12 minutes each graduate students only)* MH [Note: Starts at 9:30 am]			9:30
10:00						**10:00**
10:30		**Video Lecturing (Groups AA-GG)** *(7 minutes each)* MH				10:30
11:00	**CRLT Players** Power Center 11:00 am-12:15 pm	**Asking Questions (Groups A-G)** NUB 1528		**Where is the Line?** WEIS 296	**Course Administrivia & Course Meetings** NUB 1518, 1528	**11:00**
11:30						11:30
12:00		Lunch (on own)	Lunch *(on own)*	**Team HW in Action** WEIS 296	Chair's Lunch *(for all)* EH Atrium	**12:00**
12:30				Lunch (on own)		12:30
1:00	**CRLT Workshop for Math** EH B860	**Michigan Math In Action** *(Course features, student profiles)* NUB 1528	**Most Things You Worry About Never Happen** MH 1449	**Understanding Student Understanding** NUB 1528	**Getting Ready for Your First Class** MH 2325 1:30-3:30 pm	**1:00**
1:30						1:30
2:00			*Refreshments*			**2:00**
2:30		*Refreshments*	**A Day in the Life** *(The Interactive Classroom)* MH	*Refreshments*		2:30
3:00		**The Groupwork Fractal** *(The Cooperative Classroom)* MH		**Math 105/115 Course Meetings** or **Course Specific Q&A** NUB 1512, 1518 and 1528	**Capsule Research Talks** *(20 minutes each)* 1:30-5 pm	**3:00**
3:30						3:30
4:00	***Dominicks!!???!*** 812 Monroe St *(must be at least 21 years old, bring ID)*	**Professional Responsibilities** (Chair) EH 1360	**Breakout Sessions** EH labs	**Fake It 'till You Make It** *(grad students only)* NUB 1509		**4:00**
4:30						4:30
5:00		**GEO meeting** *(grad students only) EH 1068*				5:00

Note: All shaded sessions are required.

The Gateway Lab (EH B069) is available Monday 8/27 and Tuesday 8/28 (hours to be announced on Monday).

Implementation

- Implementation
  - *"We got everything in place so that it was a done deal."* –Pat Shure
  - Was **not** entirely smooth.
  - *"After a few complaints. . . any unsympathetic department chair or dean might have quickly squelched the new program without a fair trial. Fortunately, we had full support from both."* –Mort Brown
  - *"It is perhaps a mistake to mention [the] supportive climate [in the department] as the last item. . . [identifying] the reasons [for the program's success]. . . This is the school that has drawn national attention for what they have been able to do **as a department**."* –Wayne Roberts

Pat Shure

. . . and Continuation

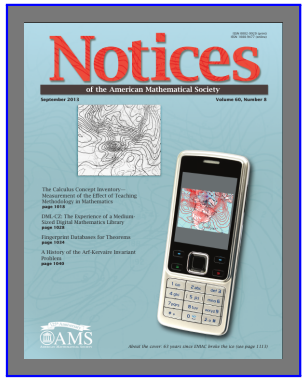
- Internal Support
  - *". . . the level of internal skepticism and outright opposition that I expected to find, while present, is much less than I expected."*
    –Wayne Roberts
  - Support from Department Administration:
    Don Lewis, Al Taylor, Mel Hochster
  - Internal Champions:
    Pat Shure, Mort Brown, Karen Rhea, Stephen DeBacker
- Evidence of Success

Mort Brown

Evidence of Success

"Never mistake activity for achievement." –John Wooden

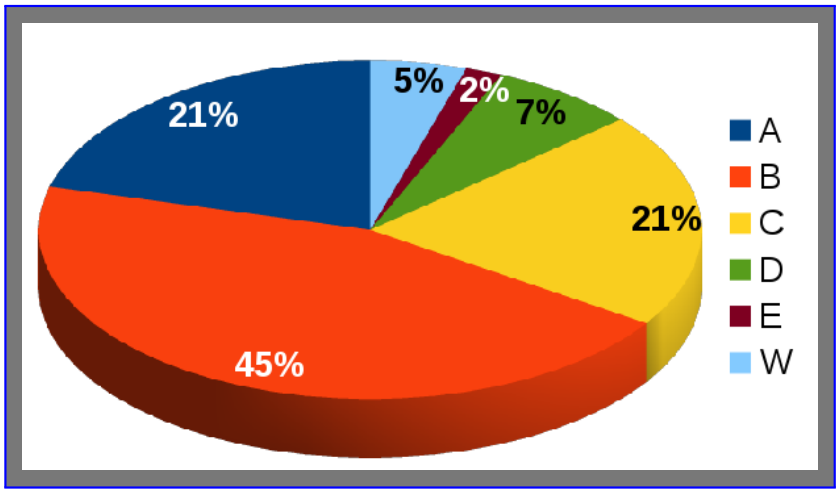
- Assessment of a(ny) program is essential.
- Assessment of Michigan Math:
  - 1990s: Site visits.
    - 1995: Wayne Roberts, Sharon Ross, Jeff Eiseman
      "The positive things I had heard are in fact true; indeed, the depth of activity. . . go[es] well beyond what I knew about it."



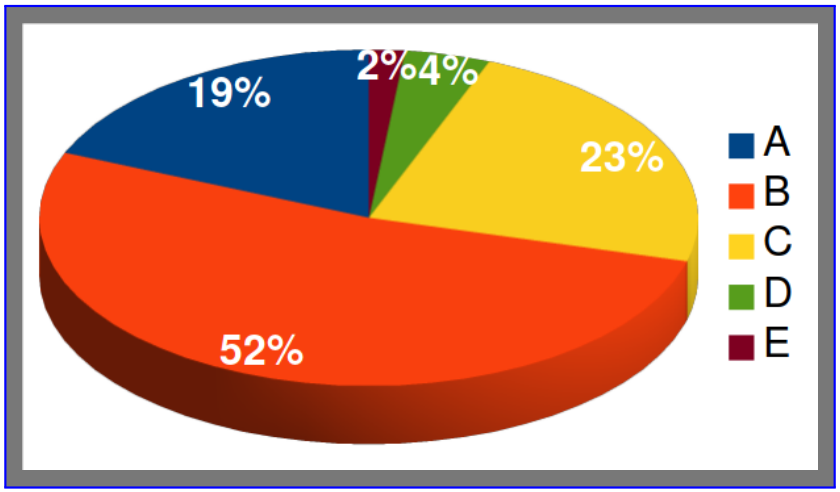
Notices, 2013-08

  - 2000s: Calculus Concept Inventory.
    - 2008: Pre-/Post- test of calculus concepts, used at many institutions
      Average normed gain over all sections was as good as the best seen before. And two standard deviations above the existing average.
  - 2010s: The Calculus Study
    - 2010: We are the obvious large midwestern university.
  - 2018: DFW Rate

DF(W) Rate, Calculus I, Fall 2014 and 2018



calculus I, F14 (incl. W)



calculus I, F18 (excl. W)

Caveats:

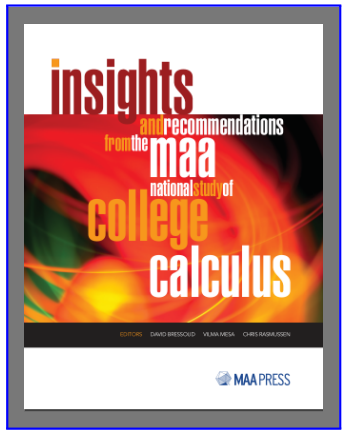
- We are not "any" university.
- Grades are, of course, scaled.

"Never mistake activity for achievement." *–John Wooden*

Characteristics of Successful Programs in Calculus

"Our survey revealed that Calculus I, as taught in our colleges and universities, is extremely efficient at lowering student confidence, enjoyment of mathematics, and desire to continue in a field that requires further mathematics. The institutions we selected bucked this trend."

–Bressoud & Rasmussen



[Bressoud, Mesa, Rasmussen; 2015]

- Local Data. Regular collection and use of local data to guide program modifications as part of continual improvement efforts.
- Placement. Effective procedures for placing students appropriately into their first Precalculus to Calculus II (P2C2) course (both initial placement and re-placing students after the term begins).
- Coordination System. A coordination system for instruction that (i) makes use of a uniform textbook and assessments and (ii) goes beyond uniform curricular elements to include regular P2C2 instructor meetings in development of de facto communities of practice.

Characteristics, cont.

- Course Content. Course content that challenges and engages students with mathematics.
- Active Pedagogy. The use and support of student-centered pedagogies, including active learning strategies.
- GTA Preparation and Development. Robust teaching development programs for teaching assistants.
- Student Support Service. Proactive student support services (e.g., tutoring centers, services for first-generation students) that foster students' academic and social integration.

U(M) Math Learning Center

Other Models

- University of Nebraska
- San Diego State University

Nebraska

- Trajectory
  - Started (2012) in courses before calculus, where scales corresponded to available resources.
  - Then moved “up,” to calculus.
- Changes
  - Course Coordination
  - Administrative Structure
  - New Support

Nebraska: Course Changes

- Class time changes: 3×50 min to 3×75 min, or 5×50 min to $3\times50 + 2\times75$ min.
- Workbooks and lesson plans to make active learning easier.
- Common exams and grading
- Online homework
- Better classrooms
- Calculus readiness "mastery" test

UNL math classroom

San Diego State University

- Changed calculus and precalculus.
- And changed “everything.”, following closely the seven characteristics of successful programs from the MAA insights document.

Michael O’Sullivan, SDSU math chair

- Effort was spearheaded by the department chair, with support from the administration.

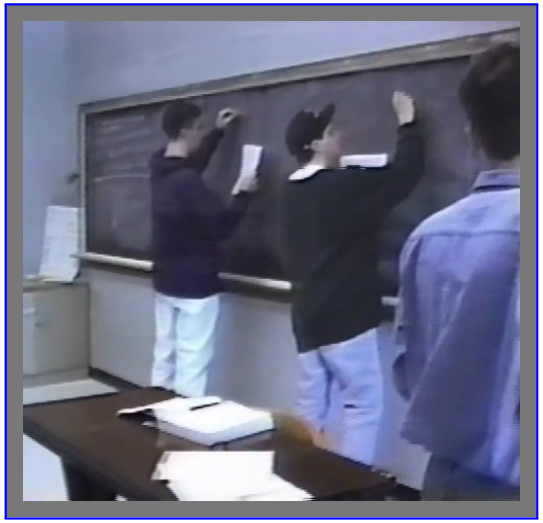
SDSU Changes

- New placement procedures
- Coordination of P2C2 courses
- Addition of an active learning lab
  - Extra contact hour.
  - Now, two breakout/recitations, one a traditional problem session, one active learning.
  - And breakout sessions reduced from 40 to 30 students.
- TA Professional Development
- New Math Learning Center

Calculus breakout session

Reflections on Successful Change

- There may now be a sense of what some of the characteristics of successful programs are.
  - Dynamic, data driven; Placement; Coordination; Challenging content; Active learning; Instructor support; Student support.



calculus class, '90s

- And there are some common themes in the changes we've described.
  - Strong departmental leadership and support.
  - Strong institutional support, including resources.
  - Passionate internal champions
    - *. . . who move to adopt structures that conform to the "known" characteristics.*