New *CP*-violation and preferred-frame effects involving polarized electrons

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General approach of the Eöt-Wash group

- Fundamental physics has been very successful & we understand an amazing amount about nature

- But the discovery of dark energy hints that we must be missing something big

- Because we do not have a clue what this is, it is interesting to make sensitive tests of sacred principles which would have profound implications if a principle breaks down.

Equivalence Principle (my first talk)
Gravitational inverse-square law (my second talk)
Lorentz invariance (today's talk)

motivation and history

- in the early 1990's Heckel and I got interested in checking the symmetries of gravity and realized that there were interesting experiments we could do if we had spin-polarized detectors and attractors and that this area was largely unexplored
- the work I'll discuss today is the culmination of three generations of spin experiments and the only one we have published
- we are very grateful to theorists who pointed out interesting implications of an experiment we undertook for naïve reasons
- any additional ideas are welcome!

How good is CPT symmetry?

- Pauli-Luders theorem tells us that any field theory satisfying very general conditions (e.g. Lorentz invariance) must obey CPT

- how can we evaluate sensitivity of CPT tests if we don't have a theory that violates CPT?

Kostelecky's et al.'s preferred-frame approach

- imagine that vector and axial-vector fields were spontaneously generated in the early universe and then inflated to enormous extents

- particles couple to these preferred-frame fields in Lorentz-invariant manners

- this "Standard Model Extension" predicts lots of new observables many of which violate CPT. One observable is $E = \sigma_e \cdot \tilde{b}_e$ where $\tilde{b}_e$ is fixed in inertial space - its benchmark value is $m_e^2 / M_{Planck} \approx 2 \times 10^{-17}$ eV

- Bluhm and Kostelecky suggested that we use our spin pendulum to test if electrons tend to precess about an arbitrary direction in inertial space.

non-commutative geometry

To begin, we must define what we mean by "noncommutative space-time": Noncommutative space-time [4] is a deformation of ordinary space-time in which the space-time coordinates x_μ, representable by Hermitian operators $\widehat{x}_\mu$, do not commute:

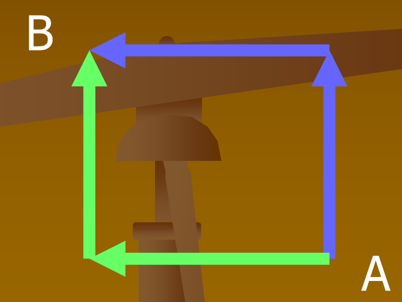
$$[\widehat{x}_\mu, \widehat{x}_\nu] = i\theta_{\mu\nu} \tag{1}$$

Here $\theta_{\mu\nu}$ is the deformation parameter: ordinary space-time is obtained in the $\theta_{\mu\nu} \rightarrow 0$ limit. By convention it is a real tensor [9] antisymmetric under $\mu \leftrightarrow \nu$. Note that $\theta_{\mu\nu}$ has dimensions of length-squared; the physical interpretation of this is that $\theta_{\mu\nu}$ is the smallest patch of area in the $\mu\nu$-plane one may deem "observable," analogous to the role $\hbar$ plays in $[\widehat{x}_i, \widehat{p}_j] = i\hbar\delta_{ij}$, defining the corresponding smallest patch of observable phase space in quantum mechanics. Noncommutative

from Review of the Phenomenology of Noncommutative Geometry
I. Hinchliffe, N Kersting and Y.L. Ma
hep-ph/0205040

effect of non-commutative geometry on spin

non-commutative geometry is equivalent to a “pseudo-magnetic” field and thus couples to spins



$$\mathcal{L}_{eff} = \frac{3}{4} m\Lambda^2 \left(\frac{e^2}{16\pi^2}\right)^2 \theta^{\mu\nu}\overline{\psi}\sigma_{\mu\nu}\psi$$

Anisimov, Dine, Banks and Graesser
Phys Rev D 65, 085032 (2002)
Λ is a cutoff assumed to be 1TeV

N. Deshpande told us about this

A consistent modification of gravity with spontaneous Lorentz-symmetry breaking

Arkani-Hamed et al., JHEP 0405, 074 (2004)

The dynamics of this theory are very interesting.
Nambu-Goldstone bosons from this breaking are "ghosts" that form a kind of gravitational aether

- mimic the cosmological constant
- may behave something like dark matter
- mediate a new force between 2 spins that falls of as $1/r^2$

$$V(r) = \frac{-M^2}{F^2}\frac{1}{8\pi r}\Big(A(\alpha,\theta_v,\gamma)(\vec{S}_1 \diamond \vec{S}_2) + 2B(\alpha,\theta_v,\gamma)(\vec{S}_1\cdot\hat{r})(\vec{S}_2\cdot\hat{r})$$

$$+ C(\alpha,\theta_v,\gamma)(\vec{S}_1 \diamond \hat{v})(\vec{S}_2 \diamond \hat{v}) + D(\alpha,\theta_v,\gamma)\Big((\vec{S}_1 \diamond \hat{v})(\vec{S}_2\cdot\hat{r}) + (\vec{S}_1\cdot\hat{r})(\vec{S}_2 \diamond \hat{v})\Big)\Big)$$

where $\cos\theta_v = \hat{r}\cdot\hat{v}$.

where $\alpha = Mrv$, $\gamma = MRv$, and

$$\vec{V} \diamond \vec{W} \equiv \vec{V}\cdot\vec{W} - (\vec{V}\cdot\hat{r})(\vec{W}\cdot\hat{r}),$$

Spin-dependent Potential + Ether Drift

$$V_\pi(r) = \frac{-1}{8\pi r}\left(\frac{M}{F}\right)^2 (A(r, cos\theta_{rv})(\vec{S}_1 \diamond \vec{S}_2) + 2B(r, cos\theta_{rv})(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r})$$

$$+ C(r, cos\theta_{rv})(\vec{S}_1 \diamond \hat{v})(\vec{S}_2 \diamond \hat{v}) + D(r, cos\theta_{rv})[(\vec{S}_1 \diamond \hat{v})(\vec{S}_2 \cdot \hat{r}) + (\vec{S}_2 \diamond \hat{v})(\vec{S}_1 \cdot \hat{r})])$$

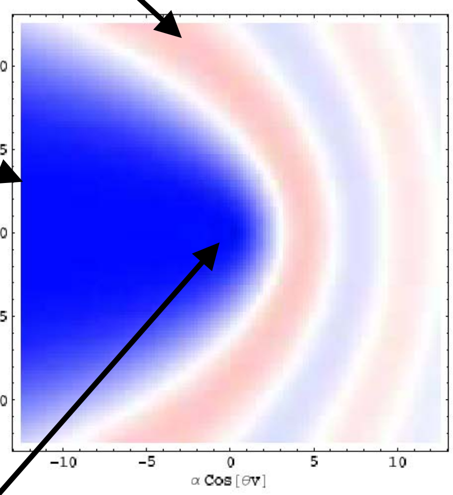
where $\vec{S}_1 \diamond \vec{S}_2 = \vec{S}_1 \cdot \vec{S}_2 - (\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r})$

gravitational aether or Goldstone wakes

Assuming point sources,

negative

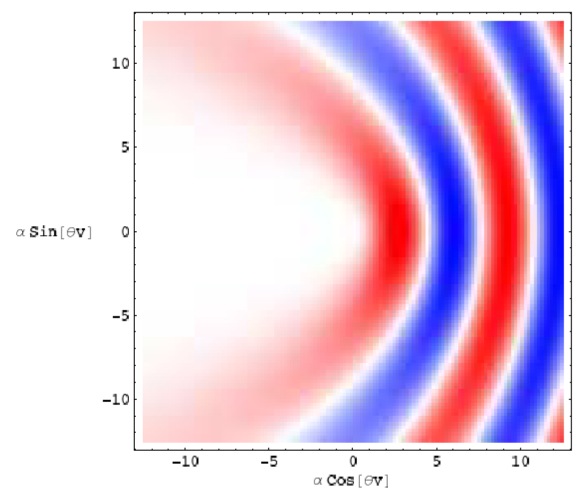
positive



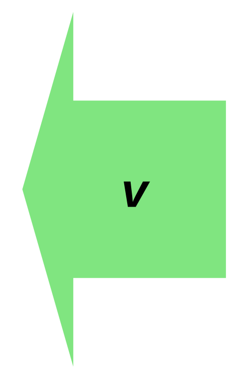
source

$A(r, \cos\theta_{rv})$

C and D look like this



$B(r, \cos\theta_{rv})$



Exotic 1-boson exchange forces

Macroscopic CP violation from exchange of axion-like particles
(Wilczek and Moody, PRD 30, 130 (1964))

$$V_{eA}(r) = g_{\mathrm{P}}^{e} g_{\mathrm{S}}^{A} \frac{\hbar}{8\pi m_e c} \boldsymbol{\sigma}_e \cdot \left[\hat{\boldsymbol{r}} \left(\frac{1}{r\lambda} + \frac{1}{r^2} \right) e^{-r/\lambda} \right],$$

Velocity-dependent forces (Dobrescu and Mocioiu hep-ph/0605342)

$$V_{eN}(r) = \boldsymbol{\sigma}_e \cdot \left[A_{\perp} \frac{\hbar}{c} \frac{(\tilde{\boldsymbol{v}} \times \hat{\boldsymbol{r}})}{m_e} \left(\frac{1}{r\lambda} + \frac{1}{r^2} \right) + A_v \frac{\tilde{\boldsymbol{v}}}{r} \right] e^{-r/\lambda},$$

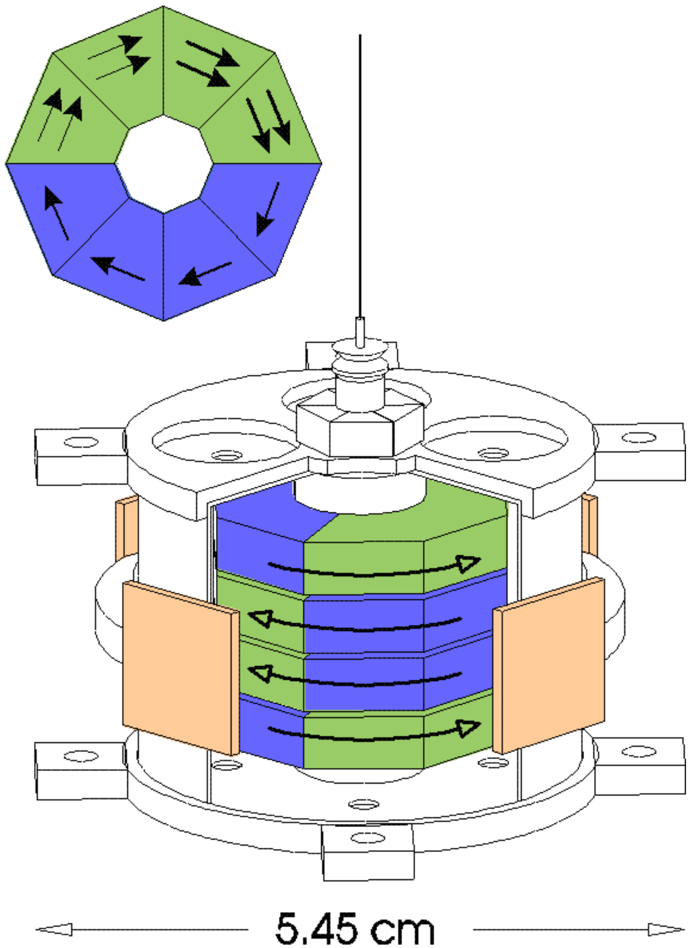
Scalar or vector boson exchange

Vector bosons with V,A couplings

The Eöt-Wash spin pendulum experiment

Claire Cramer Blayne Heckel

the Eöt-Wash spin pendulum



- 9.6 x 10^{22} polarized electrons
- negligible mass asymmetry
- negligible composition asymmetry
- flux of B confined within octagons
- negligible external B field

- Alnico: all B comes from electron spin: spins point opposite to B

- $SmCo_5$: Sm 3^+ ion has spin pointing along total B and its spin B field is nearly canceled by its orbital B field--so B of $SmCo_5$ comes almost entirely from the Co's electron spins

- therefore the spins of Alnico and Co cancel and pendulum's net spin comes from the Sm and J = – S

Why is the magnetic moment of Sm^{3+} ion so small?

Sm^{3+} ion has $(4f)^5\,(6s)^2$ electronic configuration

Hund's rule#2: orbital state has largest possible antisymmetric value L=5 (M=3+2+1+0 –1)

Hund's rule #1: electron spins symmetric with maximum value S=5/2

Hund's rule #3: in < ½ filled shell J has minimum value J=5/2

Expect μ = - 5/7μ_B , μ_L = - 30/7μ_B & μ_S = 25/7μ_B

Estimating N_p, the number of polarized electrons in the pendulum

When the Alnico (a relatively soft ferromagnet) is magnetized to the same degree as the SmCo5 (a hard ferromagnet), the Alnico and Co spins cancel
and the net spin is due essentially entirely from the Sm

$$N_{\mathrm{p}} = \frac{B_0|R|}{\mu_0\mu_B} V\eta \tag{10}$$

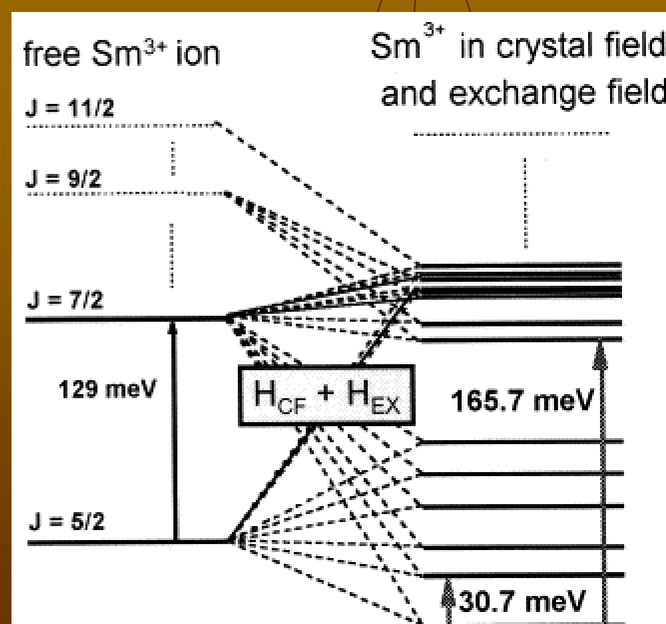
where B_0 is the magnetic field inside a puck, R is the ratio of Sm to Co spin moments in room-temperature $\mathrm{Sm\,Co_5}$ (a negative quantity because the Sm and Co spins are directed oppositely), $\eta = 0.65$ accounts for the puck's octagonal shapes and $V = 4.12\ \mathrm{cm}^3$ is the total volume of $\mathrm{Sm\,Co_5}$.

We have the made simplifying assumption that B
in Alnico and Co arises entirely from spin moments
We will relax this to obtain our final estimate for N_p

get experimental info on spin content of Sm Co_5 from circularly polarized photon scattering and polarized neutron scattering

- Polarized neutron scattering shows effects of exchange and crystalline fields and finds that the room temp Sm magnetic moment is very small: $-0.04\mu_B$ vs $-8.97\mu_B$ for the 5 Co's; therefore we neglect Sm contribution to B

P. Tils et al., J. Alloys and Compounds 289, 28 (1999)



≈ kT

Compton scattering of circularly polarized synchrotron radiation by A. Koizumi et al., J. Phys. Soc. Japan 66, 318 (1986) shows that

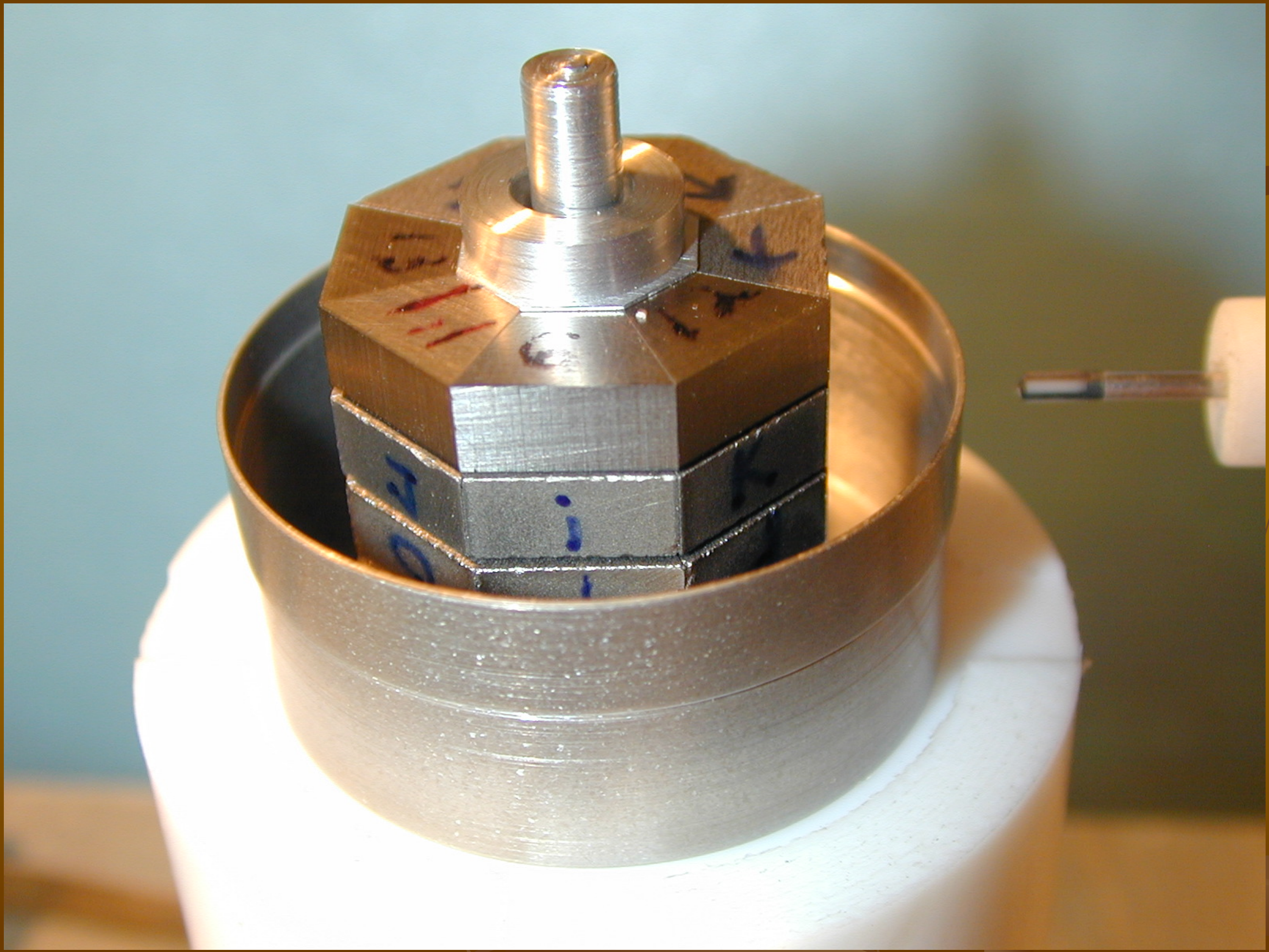
- the Sm ion in room-temp Sm Co_5 has $\mu_S = -1.79 \pm 0.31\ \mu_B$ which is smaller than the bare ion's expectation of $-25/7\mu_B$, and essentially consistent with the n scattering results
- the ratio of Sm to Co spin moments in room-temp Sm Co_5 is R=–0.23 ±0.04

Neutron scattering by Givord etal., Appl. Phys. 50, 2008 (1979) yields

- the spin contribution to the room-temp Sm3+ magnetic moment $\mu_S(Sm)=+3.56\ \mu_B$
- from much other work we deduce $\mu_S(Co_5)=-7.25\ \mu_B$
- So that R=–0.49

We take the mean and equivalent Gaussian spread to be R=–0.36±0.08

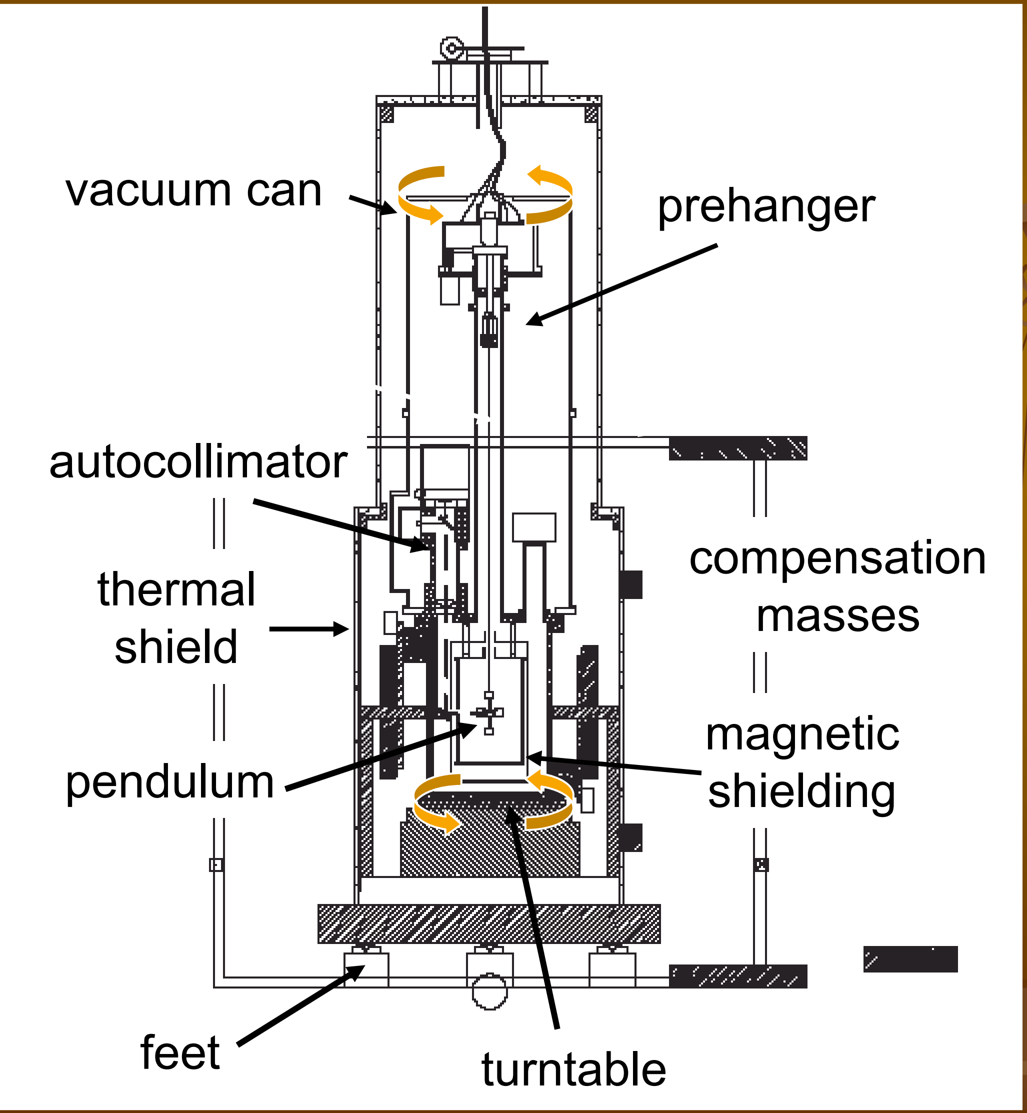
measuring the stray magnetic field of the spin pendulum



B inside = 9.5 kG

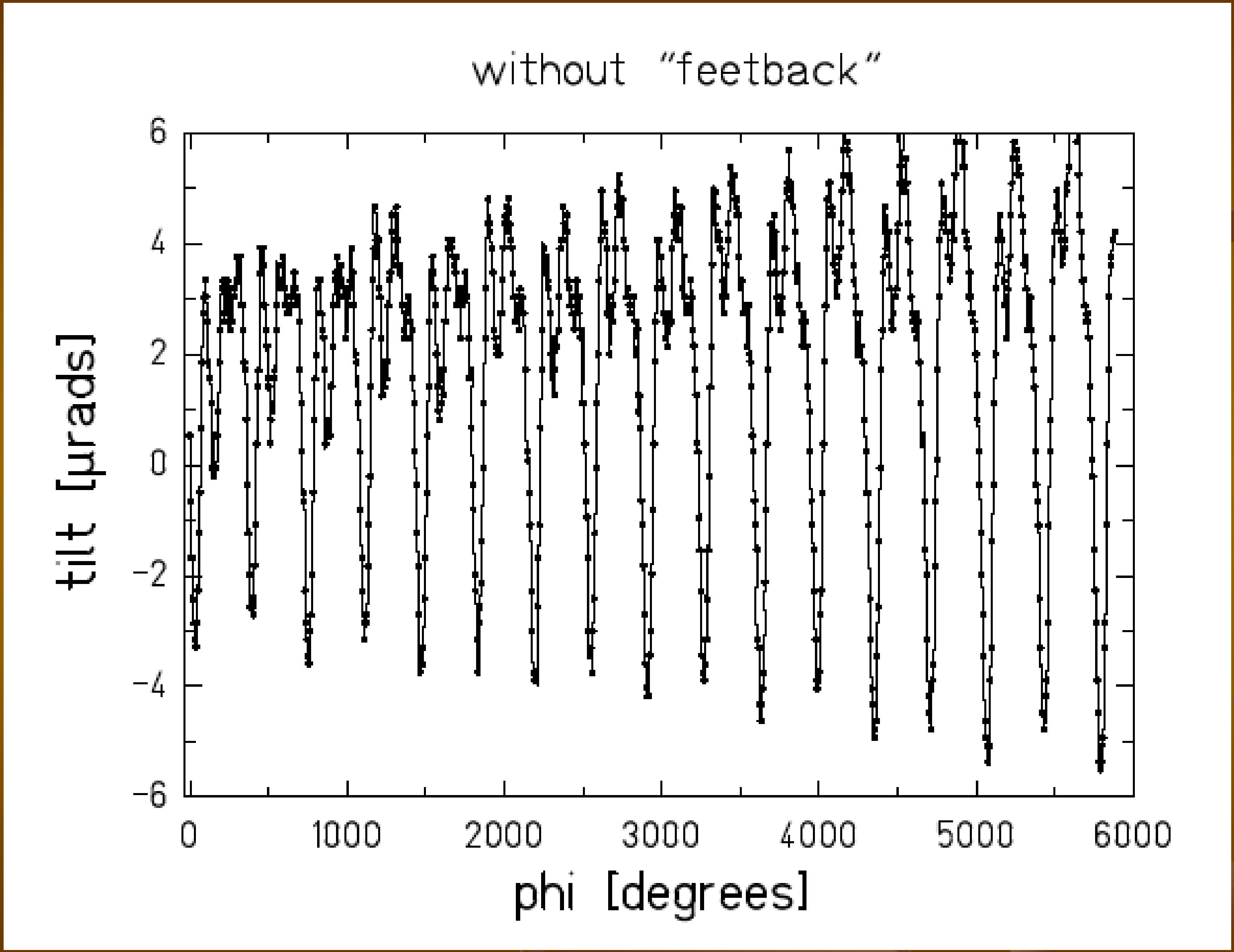
B outside ≈ few mG

The EW rotating torsion balance

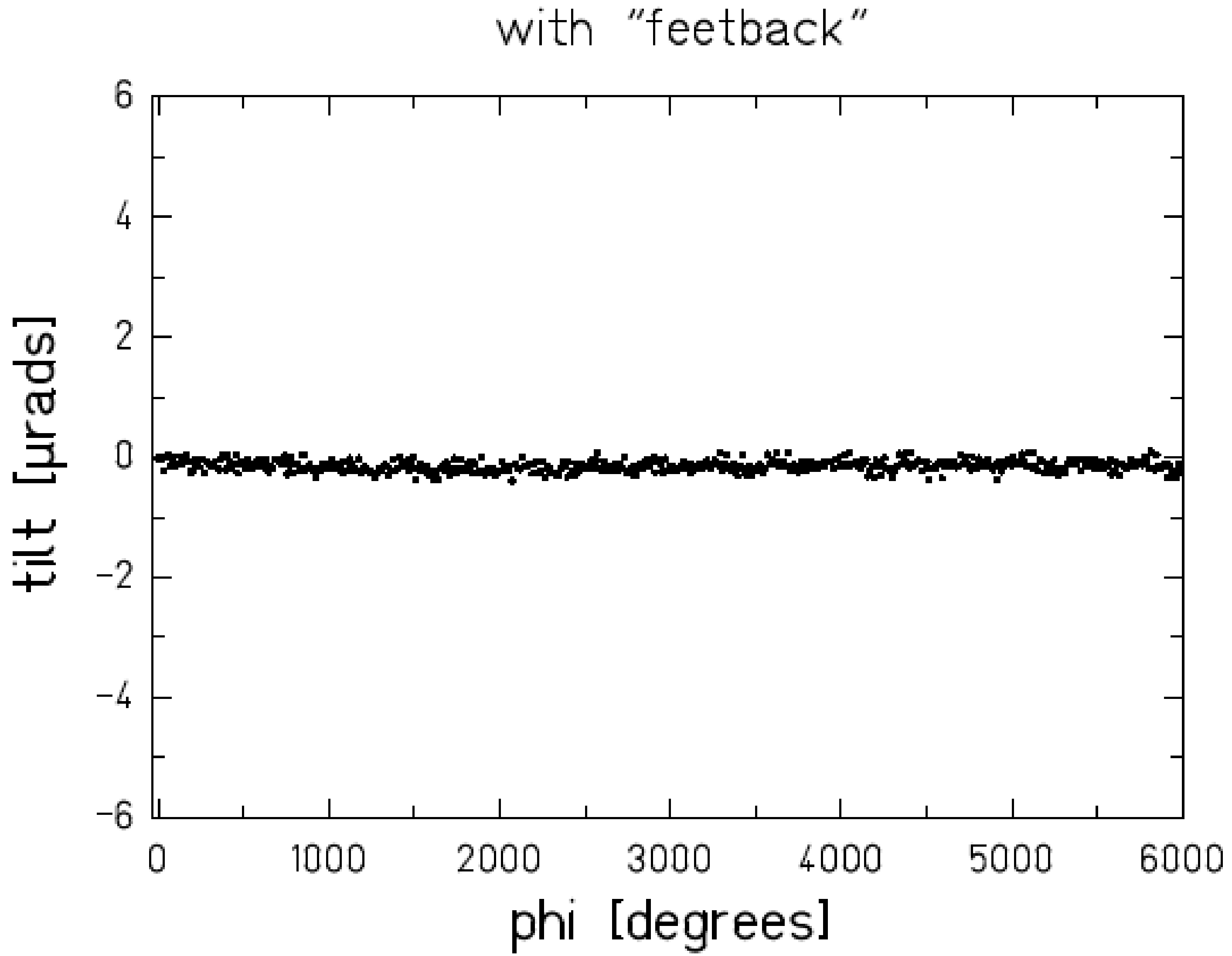


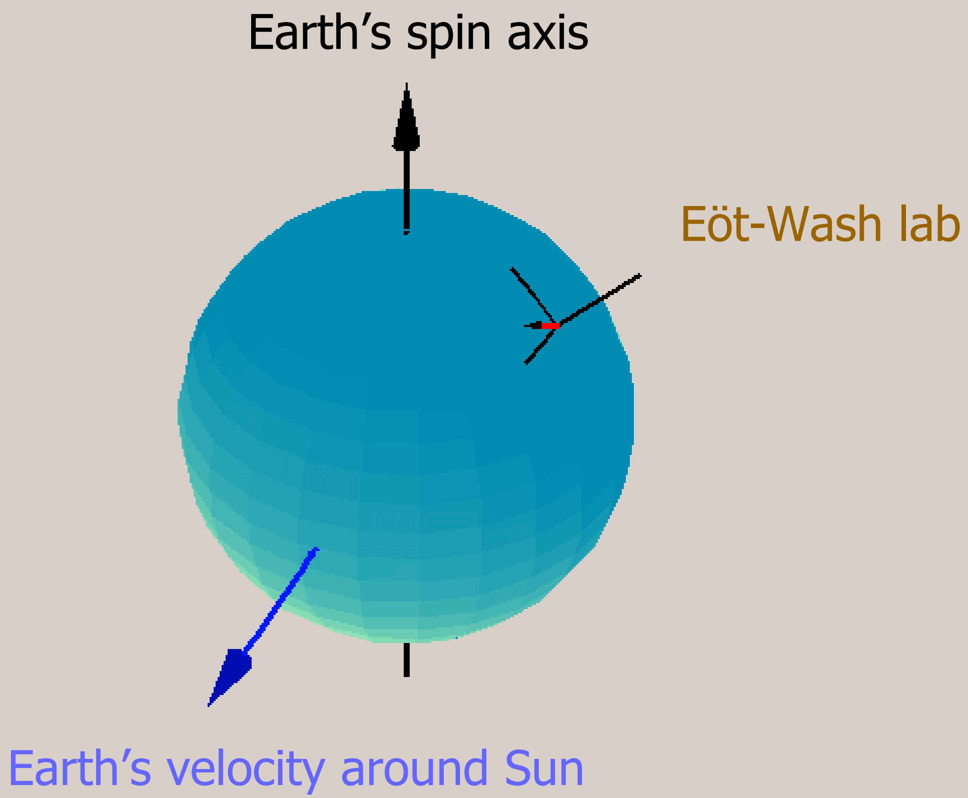
the "feetback" leveling system

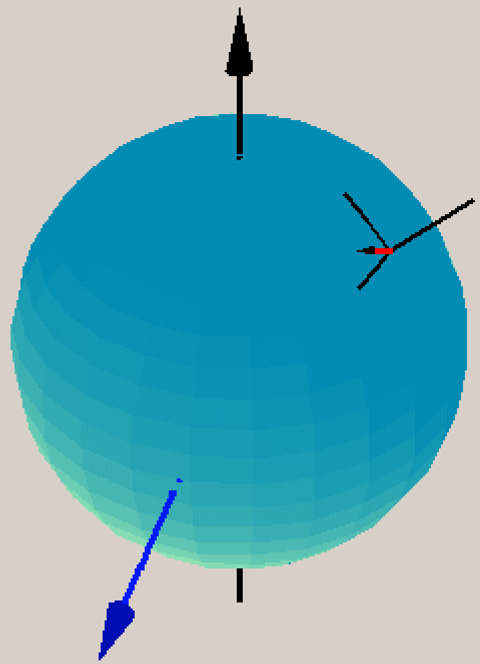
- orthogonal rotating electronic tilt sensors continuously measure the tilt of the rotating instrument, correcting for varying tilt of the lab floor and imperfections in the turntable itself
- this information is fed to Peltier elements controlling the temperature of the feet and causes them to expand or shrink by a few μm
- developed by Ulrich Schmidt



without "feetback"
tilt [µrads]
phi [degrees]
6
4
2
0
−2
−4
−6
0
1000
2000
3000
4000
5000
6000


with "feetback"
6
4
2
0
-2
-4
-6
tilt [µrads]
0
1000
2000
3000
4000
5000
6000
phi [degrees]


Earth’s spin axis
Eöt-Wash lab
Earth’s velocity around Sun



Data analysis procedure

We define a quantity beta such that the pendulum's energy has the form

$$E = -N_{\mathrm{p}}\, \boldsymbol{\sigma}_{\mathrm{p}} \cdot \boldsymbol{\beta}\ ,$$

If so, the pendulum will experience a torque, tau, given by

$$\boldsymbol{\tau} = N_{\mathrm{p}}\, \boldsymbol{\sigma}_{\mathrm{p}} \times \boldsymbol{\beta}$$

and we use the observed torques on the pendulum to infer the two horizontal components of beta at any point in time. These β_N & β_E measurements can then be analyzed for:

- effects fixed in inertial space
- effects associated with the sun
- etc.

We test for preferred-frame effects, assuming the preferred frame to be the one in which the CMB is essentially isotropic (the dipole term vanishes)

The first effect violates rotational invariance

$$V = -\boldsymbol{\sigma} \cdot \boldsymbol{A} \ .$$

sidereal modulation

The second effect violates parity and boost invariance as well

$$V = -B\boldsymbol{\sigma} \cdot \boldsymbol{v}/c \ ,$$

sidereal and annual modulations

The third effect violates rotational and boost invariance

$$V = -\sum_{i,j} \sigma_i \frac{v_j}{c} C_{ij} \ .$$

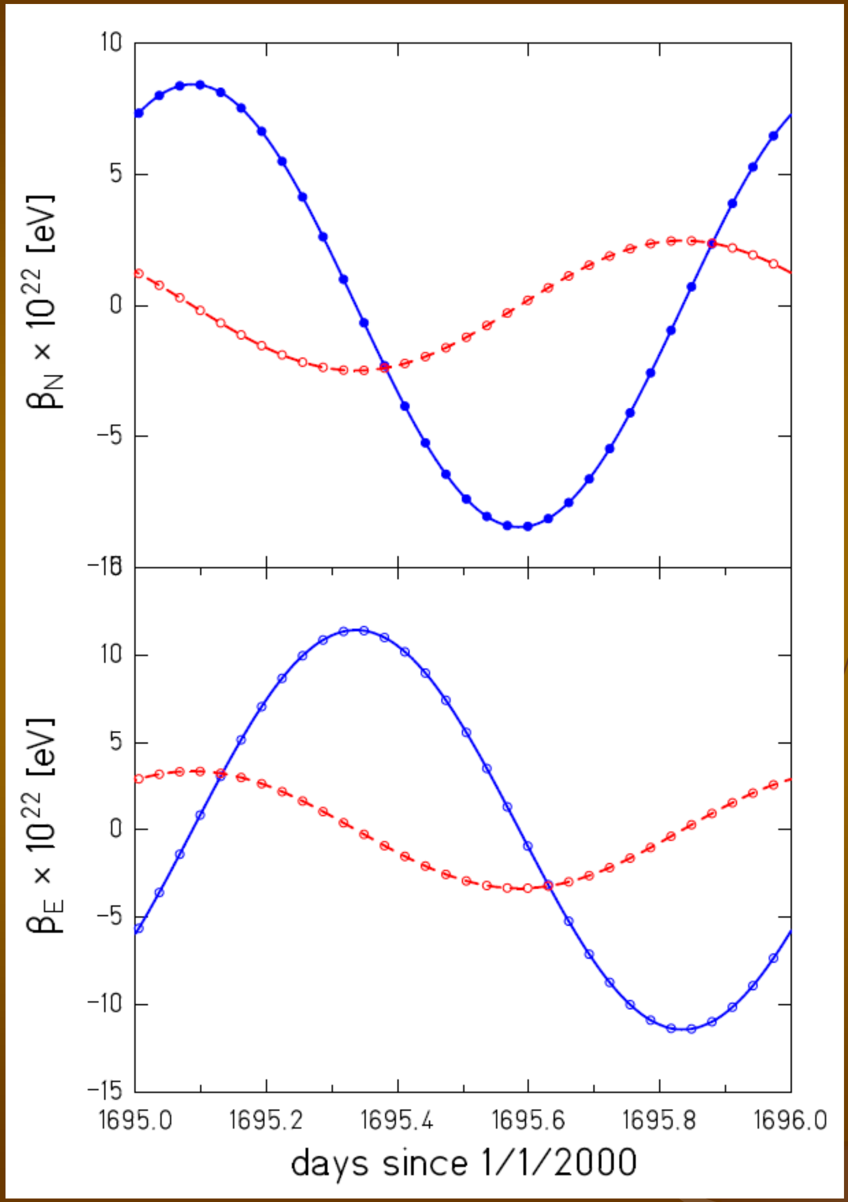
sidereal and annual modulations

Signature of rotational symmetry violating A

Use coord system where
X points toward vernal equinox
Z points along earth's spin vector
Y=Z x X

All torques are modulated at the turntable rotation frequency. In addition
signals of A_X and A_Y have sidereal modulations
but Az signal is fixed in the lab frame

Signature of rotational and boost symmetry breaking C



signal is modulated at the turntable rotation frequency, and has additional sidereal and annual modulations

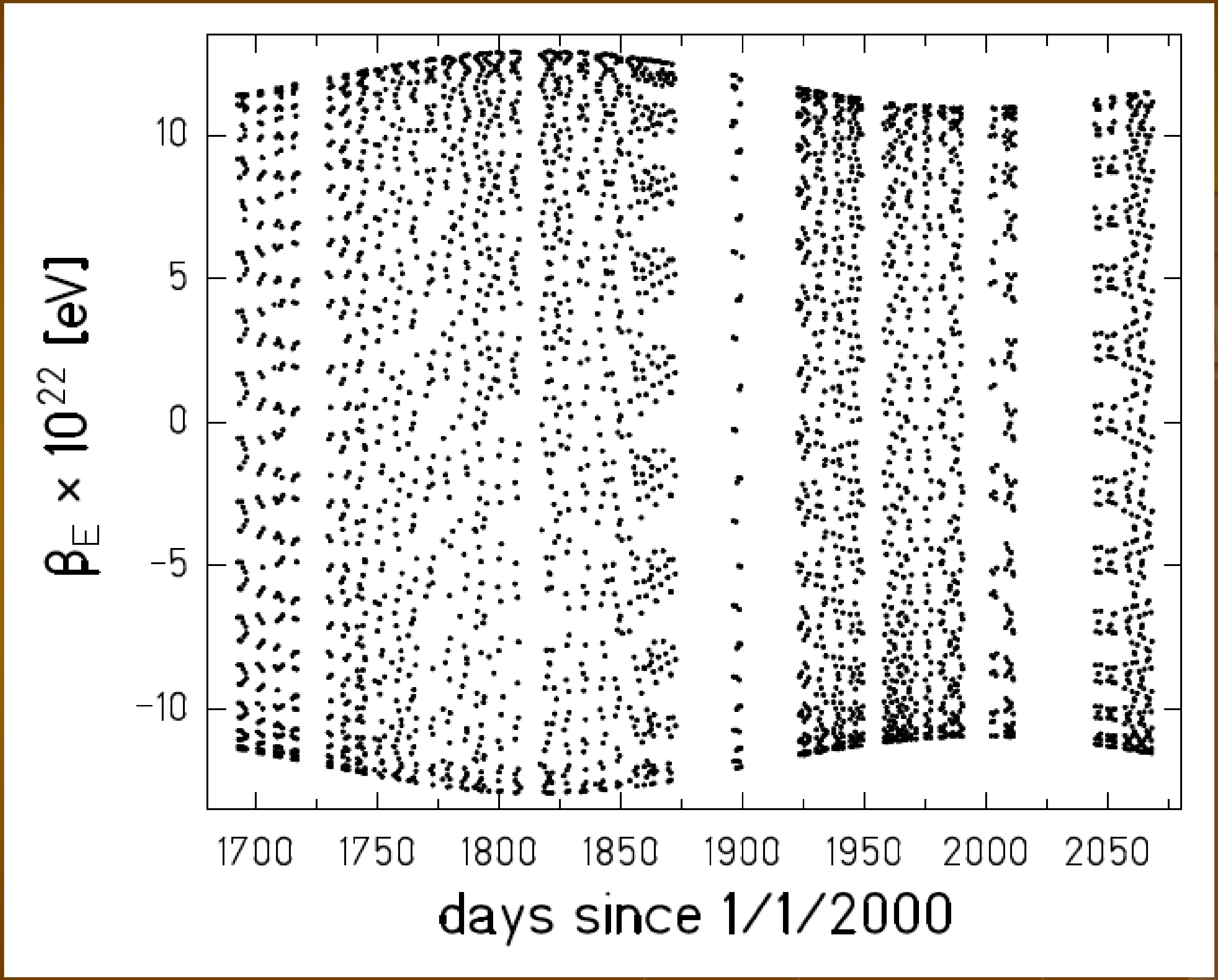
blue: expected $c_{XX}=10^{-18}$ ev signal

red: expected $c_{YY}=10^{-18}$ ev signal

Effects are reduced because $v/c \approx 10^{-3}$

each is point placed at the time of one of our measurements

expected $c_{XX}=10^{-18}$ ev signal



each is point placed at the time of one of our measurements-almost 4000 points extending to 2825 on the horizontal axis are omitted for clarity

Calibration of the torque scale

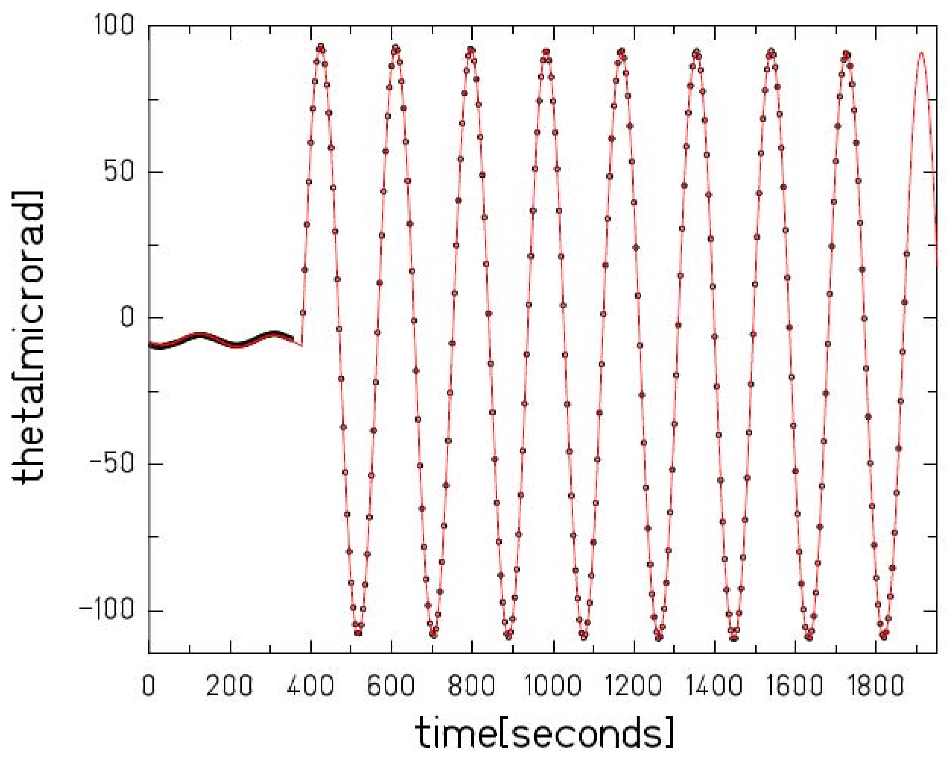
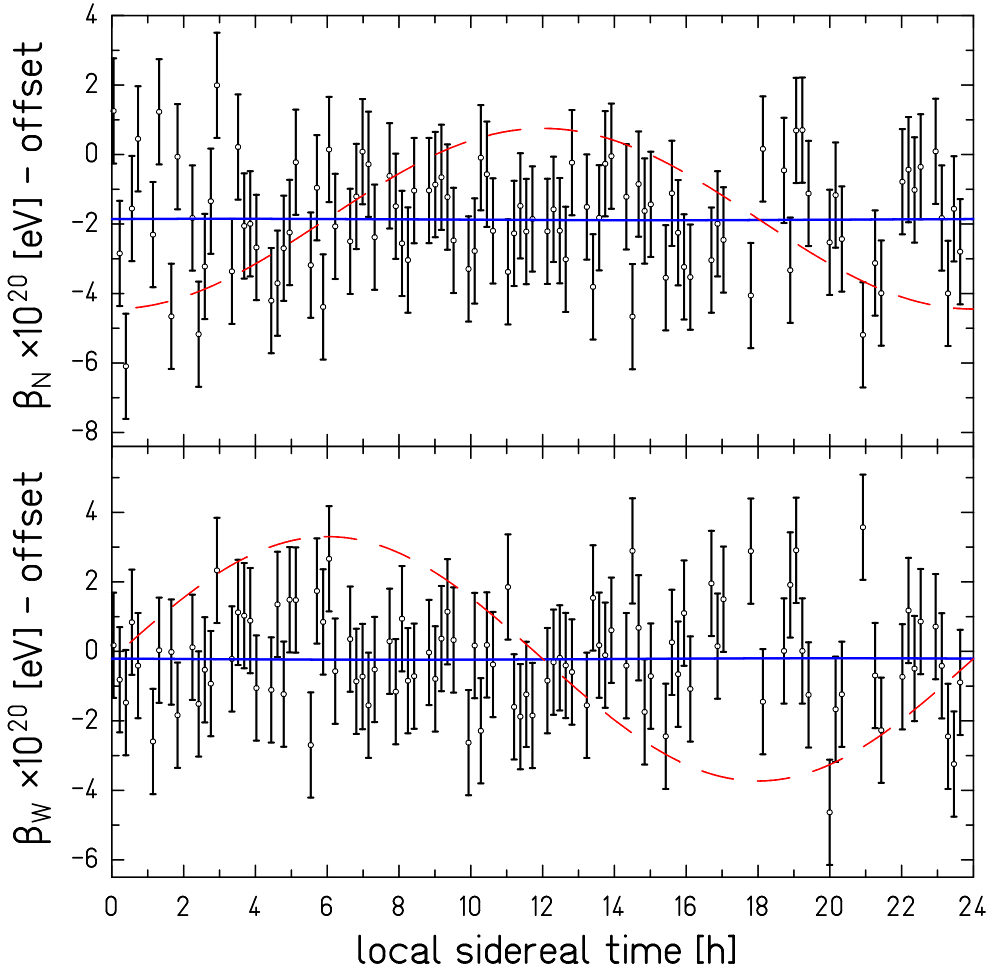


FIG. 4: Dynamical calibration of the torque scale, showing the pendulum twist as a function of time. At $t = 377.7$ s the turntable rotation frequency was abruptly changed from 0.55556 mHz to 0.55611 mHz. The resulting change in the pendulum's oscillation amplitude and phase calibrated the angular deflection scale and determined the pendulum's free oscillation frequency f_0. The smooth curve is the best fit.

These data plus the calculated pendulum moment of inertia fix the torque scale

astronomically modulated data spans a period of 3 years: a 118 hour stretch is shown below



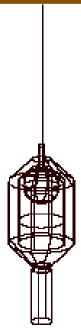
Definition of β:

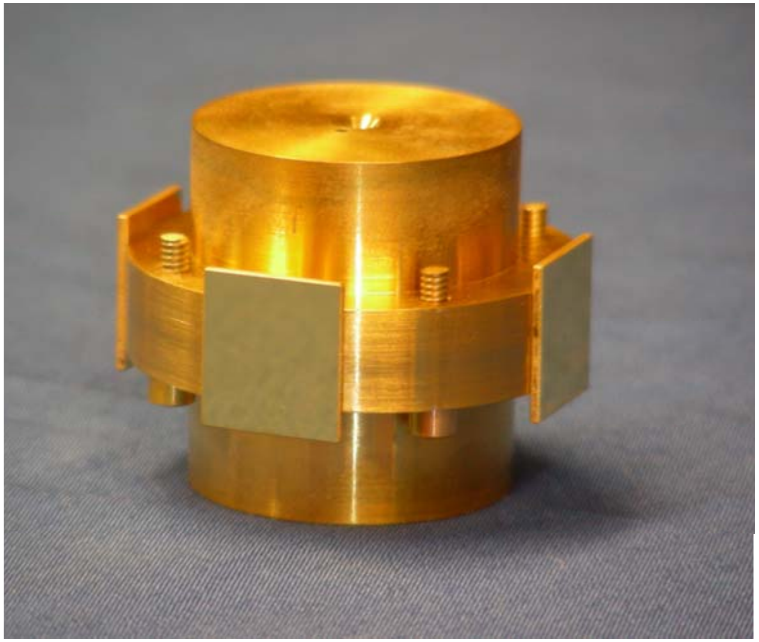
$E_{pend} = -N_p\ \beta \cdot \sigma$

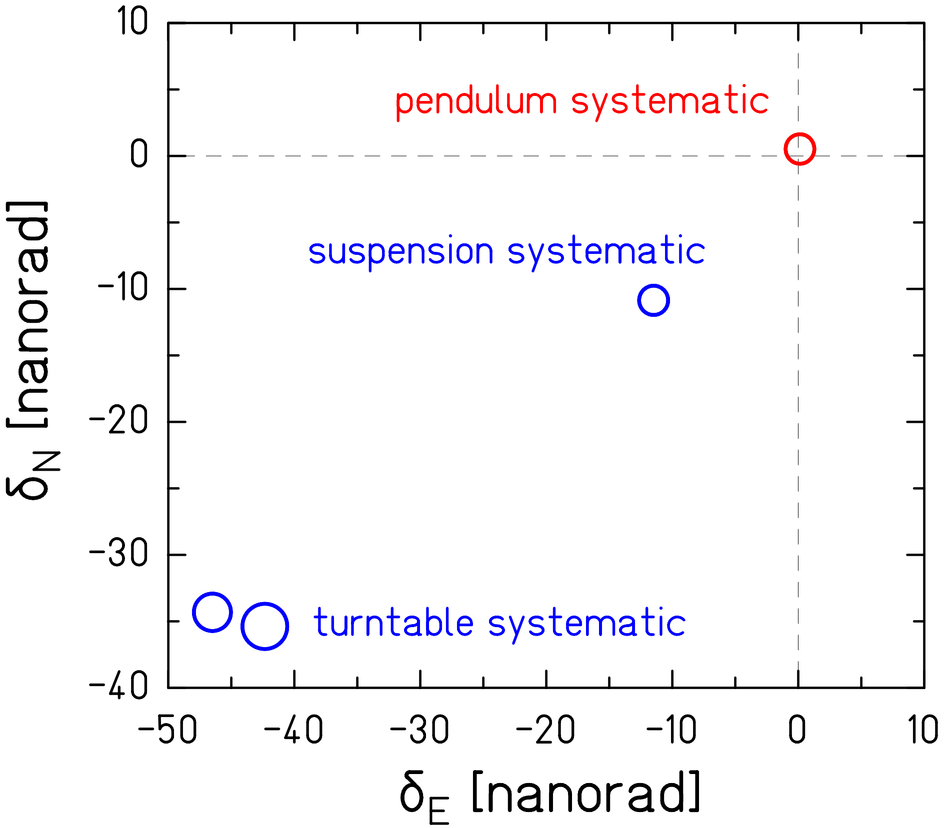
simulated signal from assumed $A_x = 5 \times 10^{-20}$ eV

best fit out-of-phase sine waves corresponding to arbitrary preferred-frame signal

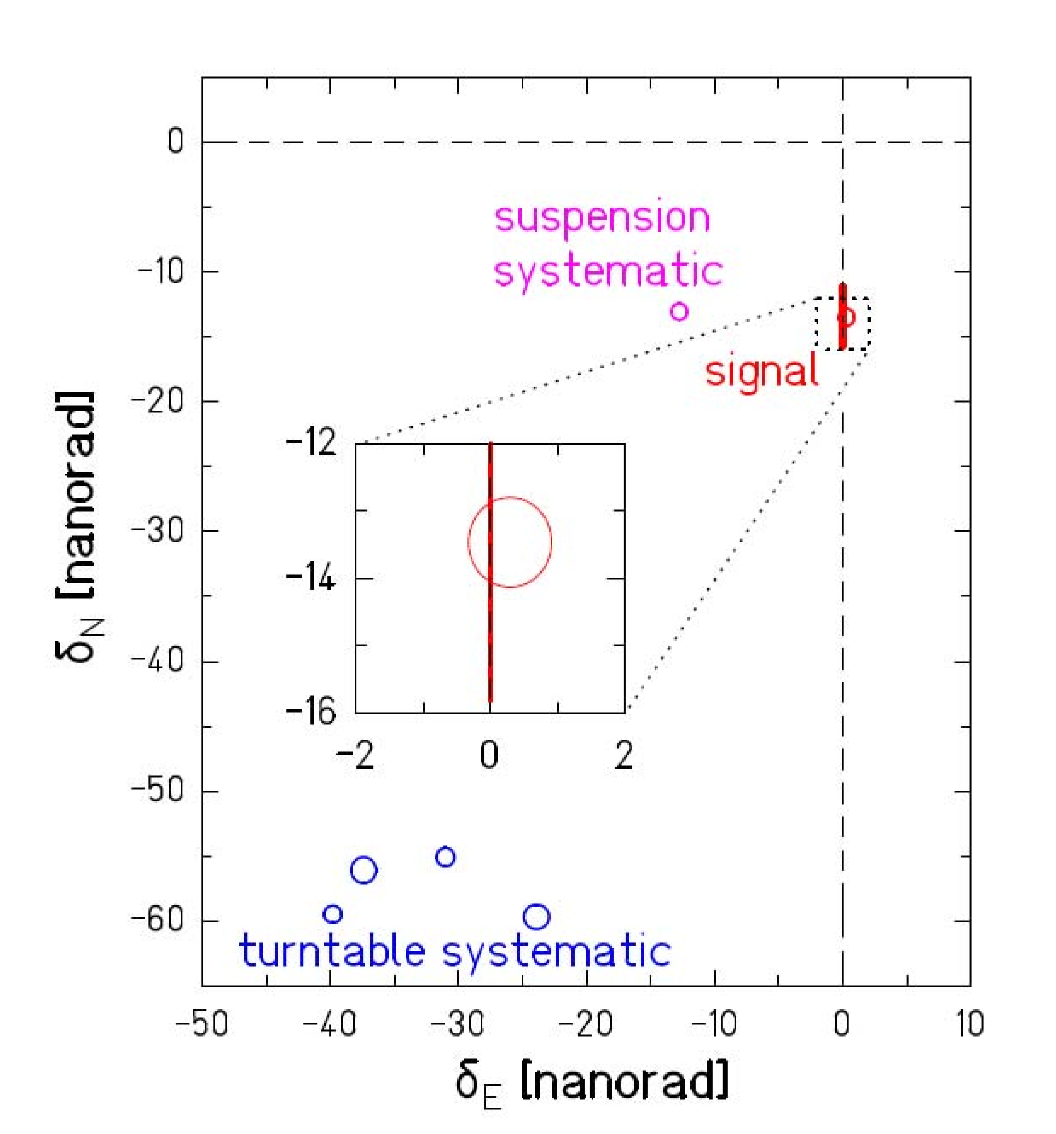
systematics study with a zero-moment pendulum and the ball-cone thing







lab-fixed spin pendulum signal taken with the "ball-cone thing"



we see a signal that points exactly South The red vertical line will be explained in the next slide.

extraction of lab-fixed signals

TABLE V: Three separate measurements of lab-fixed signals. Units of torque, $\kappa\,\delta$ and $\beta = \kappa\delta/N_{\rm p}$ are 10^{-16} N m and 10^{-20} eV, respectively. Dates refer to the mean time of the measurements. Errors in β include the systematic and random errors given in Table IV but not the scale-factor uncertainty.

signal	15/10/06 value	15/6/07 value	8/3/08 value
$\kappa\,\delta_{\rm N}$	-2.49 ± 0.11	-2.57 ± 0.08	-2.52 ± 0.12
$\beta_{\rm N}$	-1.62 ± 0.07	-1.67 ± 0.05	-1.64 ± 0.08
$\beta^{\rm gyro}$	-1.62	-1.62	-1.62
$\beta_{\rm N} - \beta^{\rm gyro}$	$+0.00 \pm 0.07$	-0.05 ± 0.05	-0.02 ± 0.08
$\kappa\,\delta_{\rm E}$	$+0.04 \pm 0.11$	-0.06 ± 0.08	$+0.08 \pm 0.12$
$\beta_{\rm E}$	$+0.03 \pm 0.07$	-0.04 ± 0.05	$+0.05 \pm 0.08$

TABLE I: Average daily components the systematic effects

effect	daily variation
ambient magnetic field	2.0 ± 0.4 mG
tilt	4.7 ± 2.1 nrad
Q_{22} gravity gradient	$(1.10 \pm 0.27) \times 10^{-3}$ g/cm^3
Q_{21} gravity gradient	$(5.0 \pm 3.4) \times 10^{-4}$ g/cm^3
temperature drift	0.34 ± 0.21 mK
1ω temperature	2.2 ± 1.7 μk

TABLE IV: Error budget for lab-fixed signals showing uncertainties in $|\beta| \times 10^{22}$. A scale-factor uncertainty equaling 18% of the central value must be folded into the quadratic sum of the random and statistical errors.

date	10/2006 error [eV]	6/2007 error [eV]	3/2008 error [eV]
systematic effect			
tilt	0.52	0.88	0.36
gravity gradient	0.77	0.56	0.65
temp drift	0.14	0.14	0.20
1ω temp	0.14	0.14	0.20
turntable speed	0.11	0.11	0.11
magnetic	1.06	1.06	0.51
total systematic error	1.43	1.50	0.95
random error	7.5	5.6	7.7

The only correction we applied for for diurnal variations in the lab's magnetic field

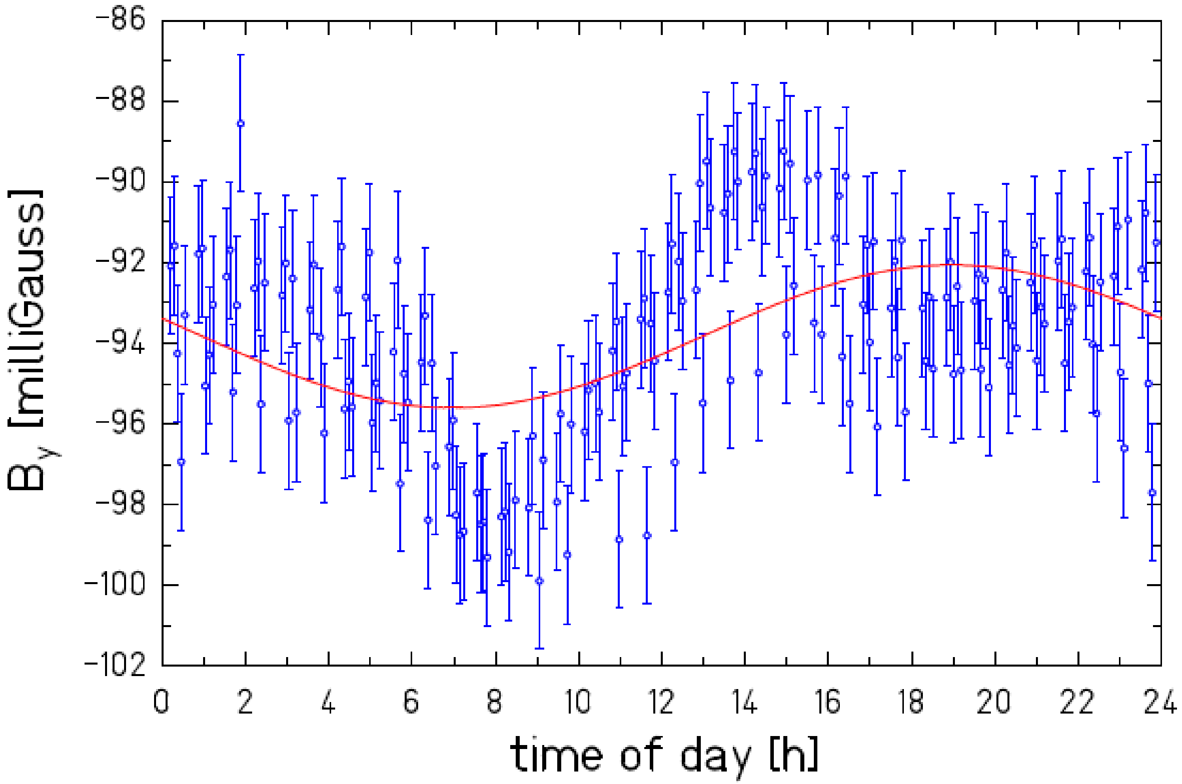


FIG. 12: Daily variation of the ambient magnetic field measured with a flux-gate magnetometer.

The gyrocompass

Anschütz's gyrocompass.

Anschuetz-Kaempfe and Sperry separately patented gyrocompasses in UK and US. In 1915 Einstein ruled that Anschütz's patent was valid.

Our gyrocompass.

Earth's rotation Ω acting on J of pendulum produces a steady torque along suspension fiber

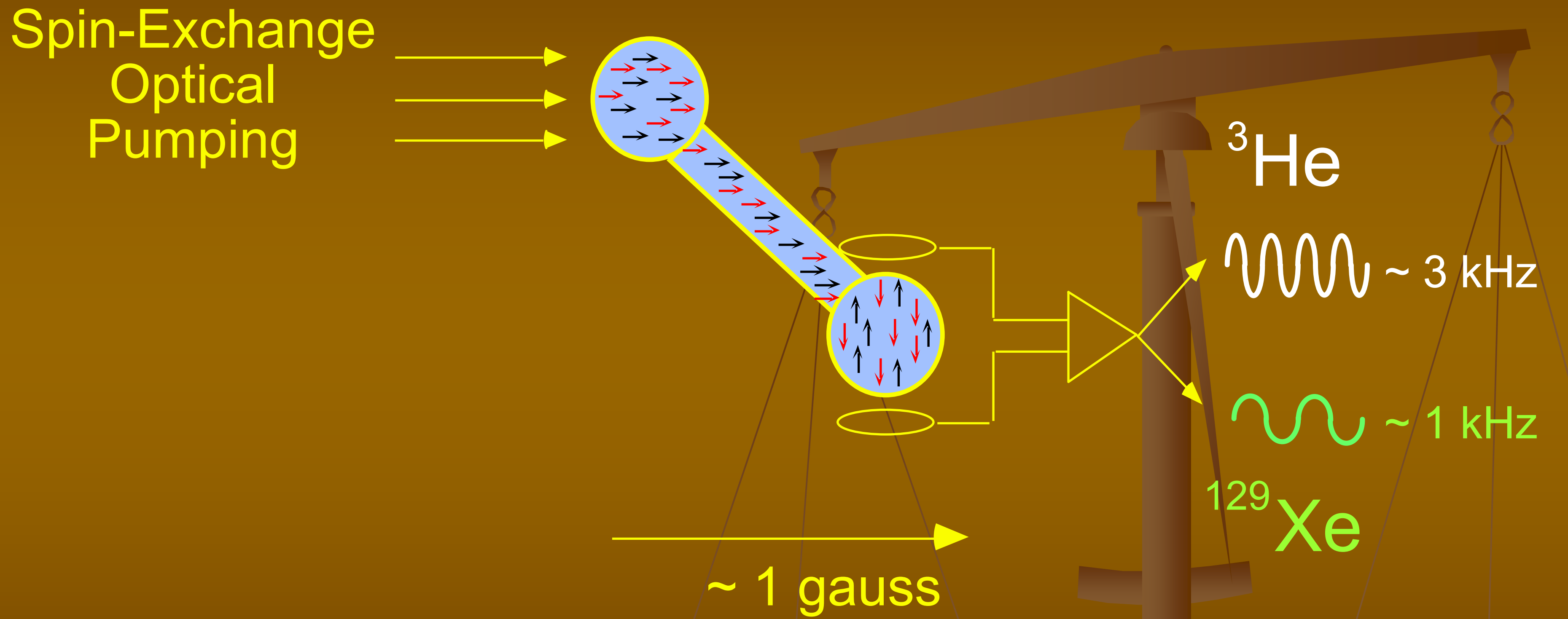
$| \Omega \times J \cdot n |$ where n is unit vector along local vertical. Because $S=-J$ this is equivalent to $\beta_N = -1.61 \times 10^{-20}$ eV

constraints on Lorentz violation

TABLE VI: 1σ constraints from our work and from Hou et al.[22] on the Lorentz-violating rotation parameters defined in Eq. 1. Units are 10^{-22} eV. The errors in A_Z are larger than those in A_X and A_Y because of the greater systematic uncertainty in lab-fixed signals. It is assumed that C terms can be neglected.

parameter	this work	Hou et al.[22]
A_X	-0.67 ± 1.31	-108 ± 112
A_Y	-0.18 ± 1.32	-5 ± 156
A_Z	-4 ± 44	107 ± 2610

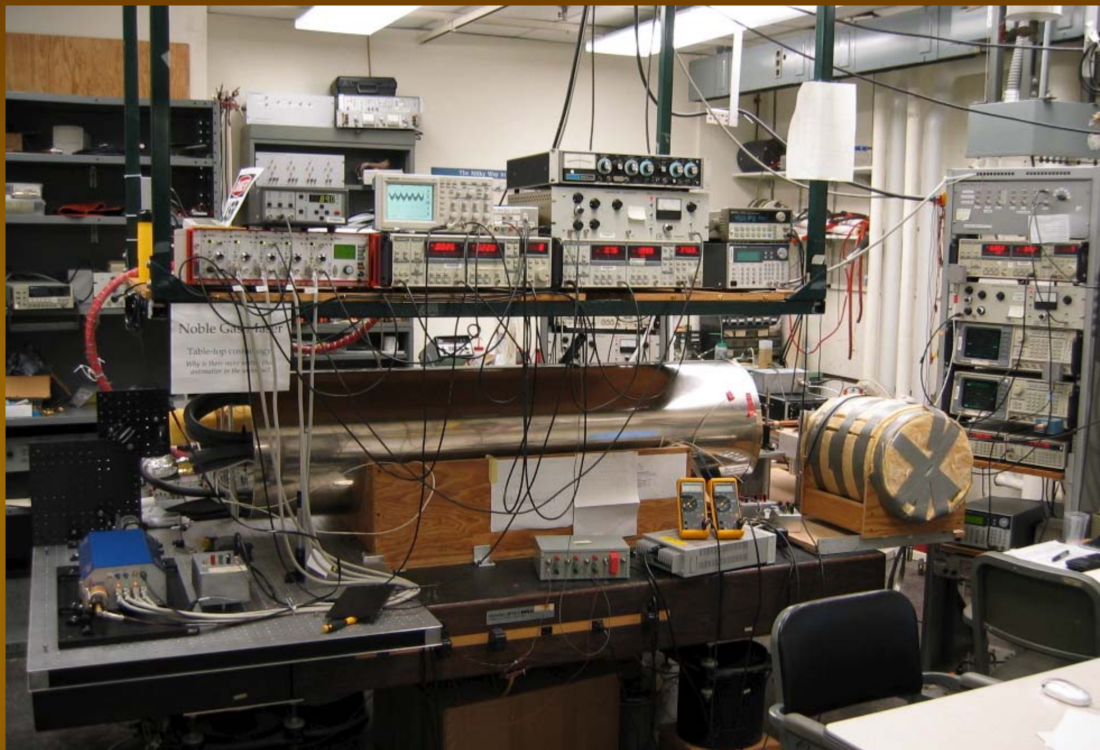
Two-Species Noble Gas Maser



One species serves as a magnetometer, the other free-runs

~ 100 nanohertz frequency sensitivity on timescale of hours

Walsworth Group: ^{3}He/^{129}Xe & Hydrogen masers



$$\left|\tilde{b}^{n}_{X,Y}\right| \leq (6 \pm 5) \times 10^{-23}\ \mathrm{eV}$$

PRL **85**, 5038 (2000)

$$\left|b^{n}_{T}\right| \leq (1.5 \pm 0.9) \times 10^{-18}\ \mathrm{eV}$$

PRL **93**, 230801 (2004)

$$\left|\tilde{b}^{e^-}_{XY} + \tilde{b}^{p}_{XY}\right| \leq \left(3 \pm 2\right) \times 10^{-18}\ \mathrm{eV}$$

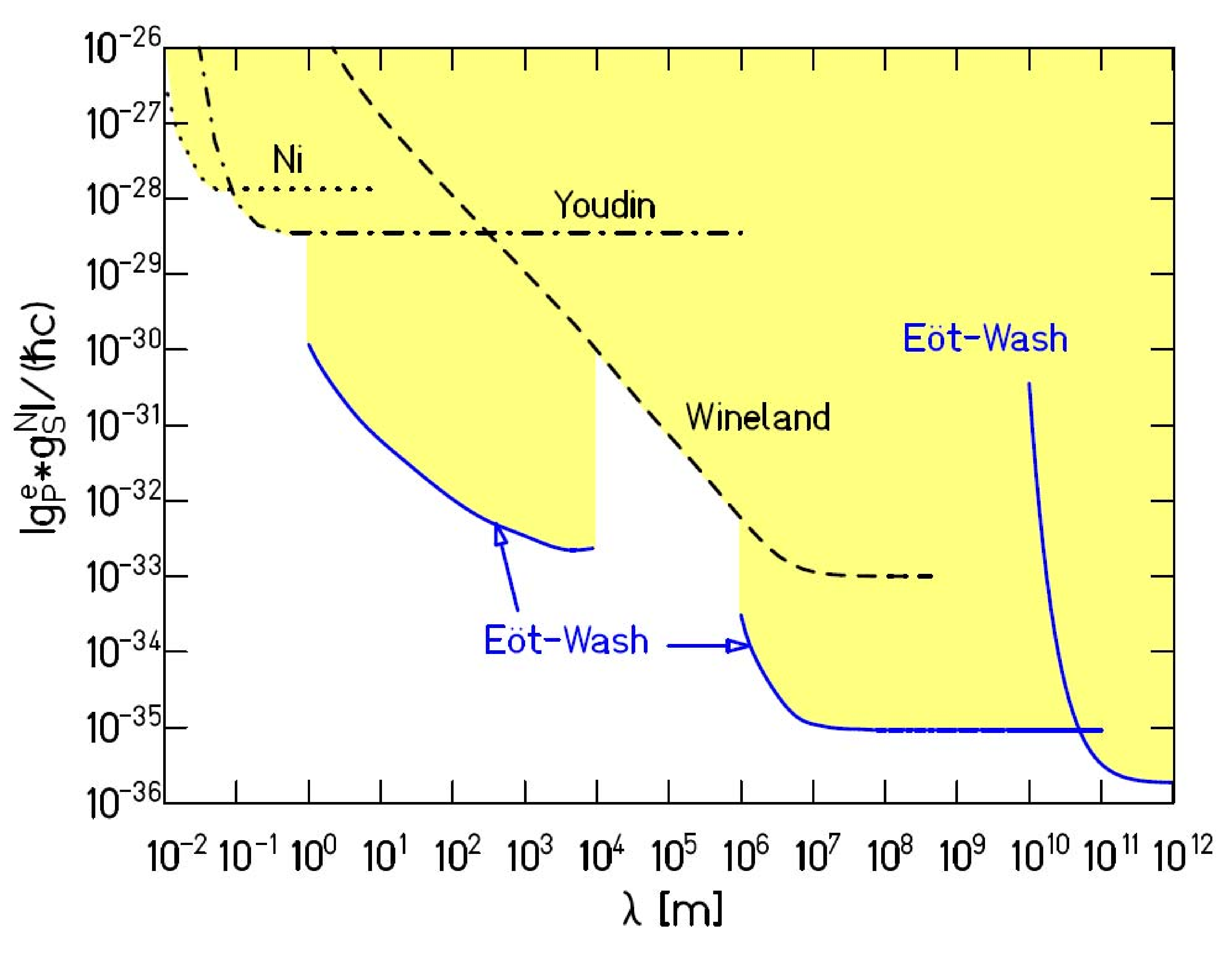
PRD **63**, 111101 (2001): PRA **68**, 63807 (2003)

constraints on exotic boson couplings

TABLE X: 1σ boson-exchange constraints from interactions with the Sun and Moon. Note that $f_v = 2g_A^e g_V^N$. The solar and lunar constraints assume $\lambda >> 1.5 \times 10^{11}$ m and $\lambda >> 4 \times 10^8$ m, respectively.

parameter	solar constraint	lunar constraint
$g_P^e g_S^N/(\hbar c)$	$(-3.5 \pm 8.5) \times 10^{-37}$	$(+0.2 \pm 1.6) \times 10^{-34}$
$f_\perp/(\hbar c)$	$(-0.1 \pm 2.1) \times 10^{-32}$	$(-1.1 \pm 8.6) \times 10^{-29}$
$g_A^e g_V^N/(\hbar c)$	$(+0.2 \pm 1.2) \times 10^{-56}$	$(-3.1 \pm 2.4) \times 10^{-50}$

2σ upper limits on $g_P^e\ g_S^N$ for axion-like particles

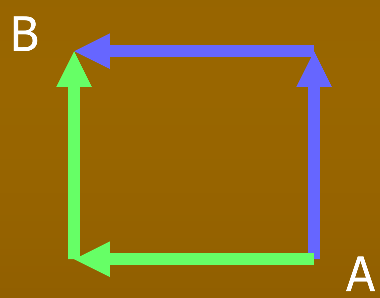


an amusing number

- our "spin pendulum" upper limit on the energy required to invert an electron spin about an arbitrary axis fixed in inertial space is $\sim 10^{-22}$ eV
- this is comparable to the electrostatic energy of two electrons separated by ~ 90 astronomical units

effect of non-commutative geometry on spin

$$\mathcal{L}_{eff} = \frac{3}{4} m\Lambda^2 \left(\frac{e^2}{16\pi^2}\right)^2 \theta^{\mu\nu}\overline{\psi}\sigma_{\mu\nu}\psi$$



Λ is a cutoff assumed to be 1TeV
Anisimov, Dine, Banks and Graesser
hep-ph/2010039

minimum observable patch of area implied by our results

$$|\theta^{\mu\nu}| = 5 \times 10^{-59}\ \mathrm{m}^2$$

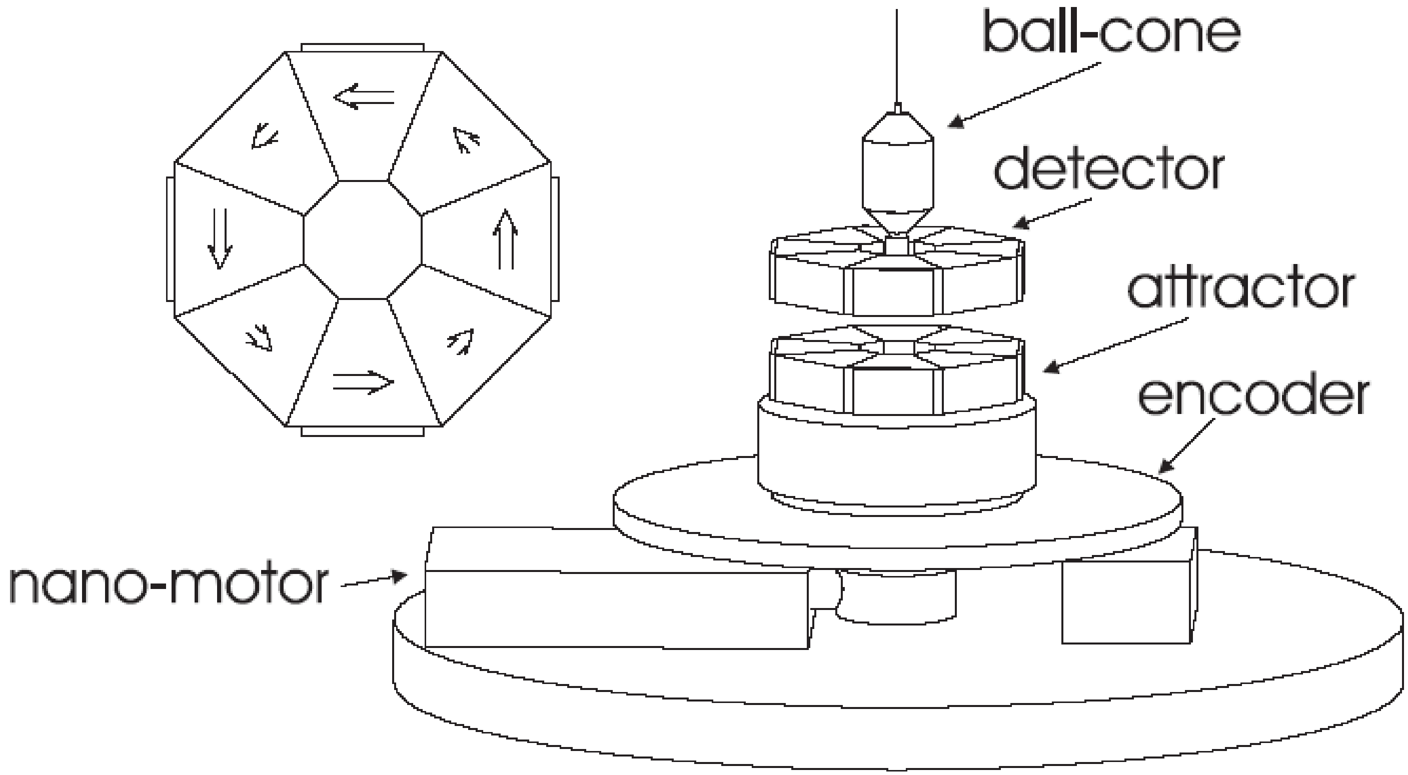
5×10^{-59} m^2 seems very small
and indeed it is

but in another sense it is also quite large
5×10^{-59} m^2 ~ $(3 \times 10^{13}$ GeV$)^2$ ~ $(10^6 L_P)^2$
where L_P is the Planck Length
$\sqrt{(\hbar G/c^3)} = 1.6 \times 10^{-35}$ m

or ~ $(330 L_U)^2$
where L_U is the Grand Unification length
$L_U = \hbar c / 10^{16}$ GeV

but that is still pretty good, especially compared to the alternatives

Apparatus for studying the interaction between 2 spins PhD project of William Terrano



This is an old figure-Will's pendulum and attractor have 20 instead of 8 segments—a mu-metal shield between attractor and detector is not shown

conclusions

I've been talking this week about testing powerful, deep and beautiful principles of physics that one learns to appreciate as an undergraduate

- isotropy of space
- Einstein's Equivalence Principle
- Newton's inverse square law

Why bother?

we understand a great deal about the physical world, but there is a broad consensus that we must be missing something very big. Of course, neither I nor anyone else knows what this is

But testing our most cherished principles with really high sensitivity is important and very challenging (i.e. fun).

And the potential payoffs could be huge.

$$V_{eA}(r) = g_{\mathrm{P}}^{e} g_{\mathrm{S}}^{A} \frac{\hbar}{8\pi m_e c} \boldsymbol{\sigma}_e \cdot \left[\hat{\boldsymbol{r}} \left(\frac{1}{r\lambda} + \frac{1}{r^2} \right) e^{-r/\lambda} \right],$$

$$V_{eN}(r) = \boldsymbol{\sigma}_e \cdot \left[A_{\perp} \frac{\hbar}{c} \frac{(\tilde{\boldsymbol{v}} \times \hat{\boldsymbol{r}})}{m_e} \left(\frac{1}{r\lambda} + \frac{1}{r^2} \right) + A_v \frac{\tilde{\boldsymbol{v}}}{r} \right] e^{-r/\lambda},$$

TABLE I. 1σ constraints on the Kostelecký $\tilde{\boldsymbol{b}}^e$ parameters from our work and from Hou *et al.* [5]. Units are 10^{-22} eV.

Parameter	This work	Hou *et al.*
$\tilde{b}_{\mathrm{x}}^{e}$	$+0.1 \pm 2.4$	-108 ± 112
$\tilde{b}_{\mathrm{y}}^{e}$	-1.7 ± 2.5	-5 ± 156
$\tilde{b}_{\mathrm{z}}^{e}$	-29 ± 39	107 ± 2610

TABLE II. 1σ constraints from interactions with the Sun. These values assume $\lambda > 1$ AU.

Parameter	Constraint
$g_{\mathrm{P}}^{e} g_{\mathrm{S}}^{N}/(\hbar c)$	$(-0.4 \pm 1.6) \times 10^{-36}$
$A_{\perp}/(\hbar c)$	$(-2.4 \pm 6.4) \times 10^{-34}$
$A_v/(\hbar c) = g_{\mathrm{A}}^{e} g_{\mathrm{V}}^{N}/(4\pi\hbar c)$	$(+3.0 \pm 1.7) \times 10^{-57}$

general one-boson exchange spin-spin interaction assuming only rotational and translational invariance

$$\mathcal{V}_2 = \frac{1}{4\pi r}\,\vec{\sigma}\cdot\vec{\sigma}\,'\,e^{-m_0 r} \quad ,$$

$$\mathcal{V}_3 = \frac{1}{4\pi m_e^2\, r^3}\left[\vec{\sigma}\cdot\vec{\sigma}\,'(1+m_0 r) - \left(\vec{\sigma}\cdot\hat{\vec{r}}\right)\left(\vec{\sigma}\,'\cdot\hat{\vec{r}}\right)(3+3m_0 r+m_0^2 r^2)\right] e^{-m_0 r} \quad ,$$

$$\mathcal{V}_{11} = -\frac{1}{4\pi m_e\, r^2}\,(\vec{\sigma}\times\vec{\sigma}\,')\cdot\hat{\vec{r}}\,(1+m_0 r)\,e^{-m_0 r} \quad ,$$

$$\mathcal{V}_{6,7} = -\frac{1}{8\pi m_e\, r^2}\left[(\vec{\sigma}\cdot\vec{v})\left(\vec{\sigma}\,'\cdot\hat{\vec{r}}\right) \pm \left(\vec{\sigma}\cdot\hat{\vec{r}}\right)(\vec{\sigma}\,'\cdot\vec{v})\right](1+m_0 r)\,e^{-m_0 r} \quad ,$$

$$\mathcal{V}_8 = \frac{1}{4\pi r}\,(\vec{\sigma}\cdot\vec{v})\left(\vec{\sigma}\,'\cdot\vec{v}\right) e^{-m_0 r} \quad ,$$

$$\mathcal{V}_{14} = \frac{1}{4\pi r}\,(\vec{\sigma}\times\vec{\sigma}\,')\cdot\vec{v}\,e^{-m_0 r} \quad ,$$

$$\mathcal{V}_{15} = -\frac{1}{8\pi m_e^2\, r^3}\left\{\left[\vec{\sigma}\cdot\left(\vec{v}\times\hat{\vec{r}}\right)\right]\left(\vec{\sigma}\,'\cdot\hat{\vec{r}}\right) + \left(\vec{\sigma}\cdot\hat{\vec{r}}\right)\left[\vec{\sigma}\,'\cdot\left(\vec{v}\times\hat{\vec{r}}\right)\right]\right\} \times \left(3+3m_0 r+m_0^2 r^2\right) e^{-m_0 r} \quad ,$$

$$\mathcal{V}_{16} = -\frac{1}{8\pi m_e\, r^2}\left\{\left[\vec{\sigma}\cdot\left(\vec{v}\times\hat{\vec{r}}\right)\right](\vec{\sigma}\,'\cdot\vec{v}) + (\vec{\sigma}\cdot\vec{v})\left[\vec{\sigma}\,'\cdot\left(\vec{v}\times\hat{\vec{r}}\right)\right]\right\}(1+m_0 r)\,e^{-m_0 r}$$