Quantum Critical Behavior in Strongly Interacting Rydberg Gases

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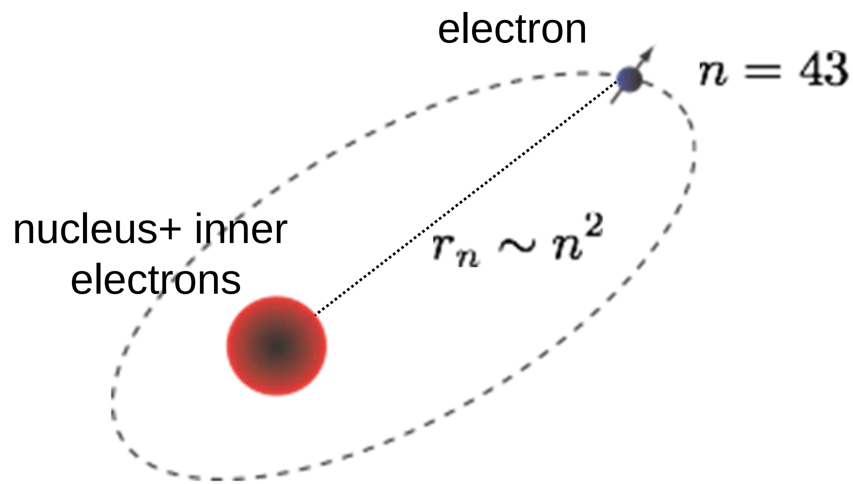
Rydberg atoms

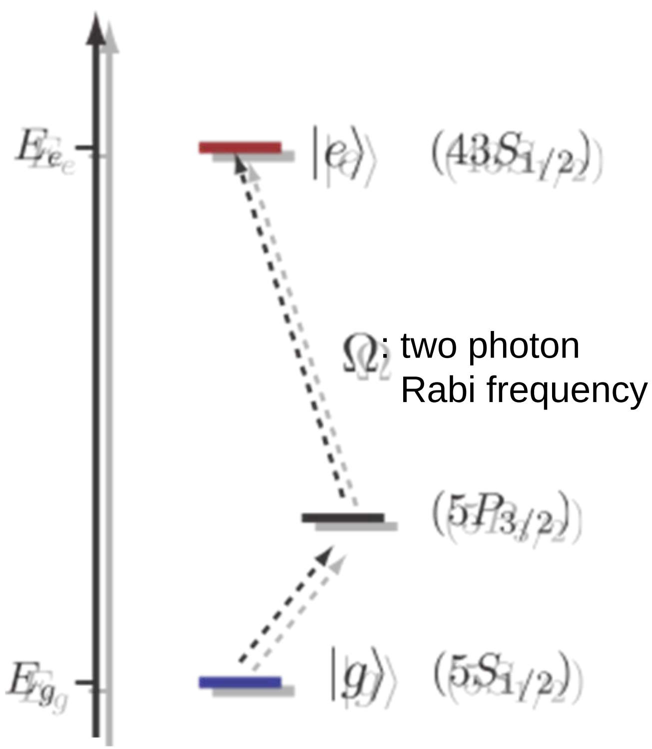
Rydberg atom

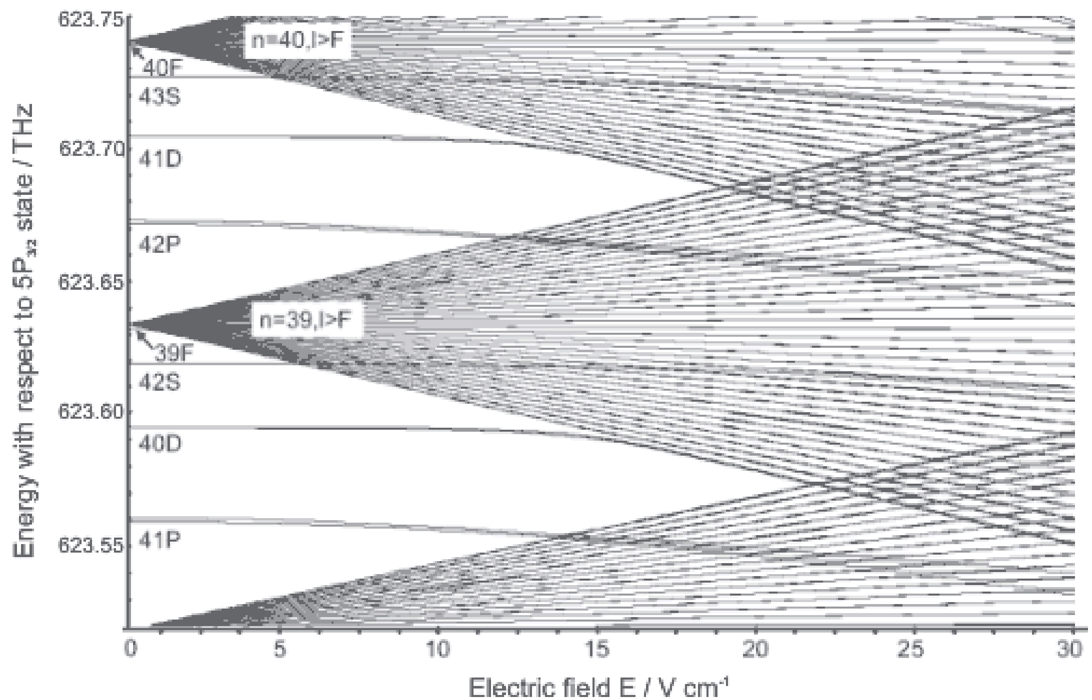
- atom with an electron in an
  highly excited shell

- large dipole moment

$$d \sim n^2$$







Blockade

Rydberg-Rydberg interaction

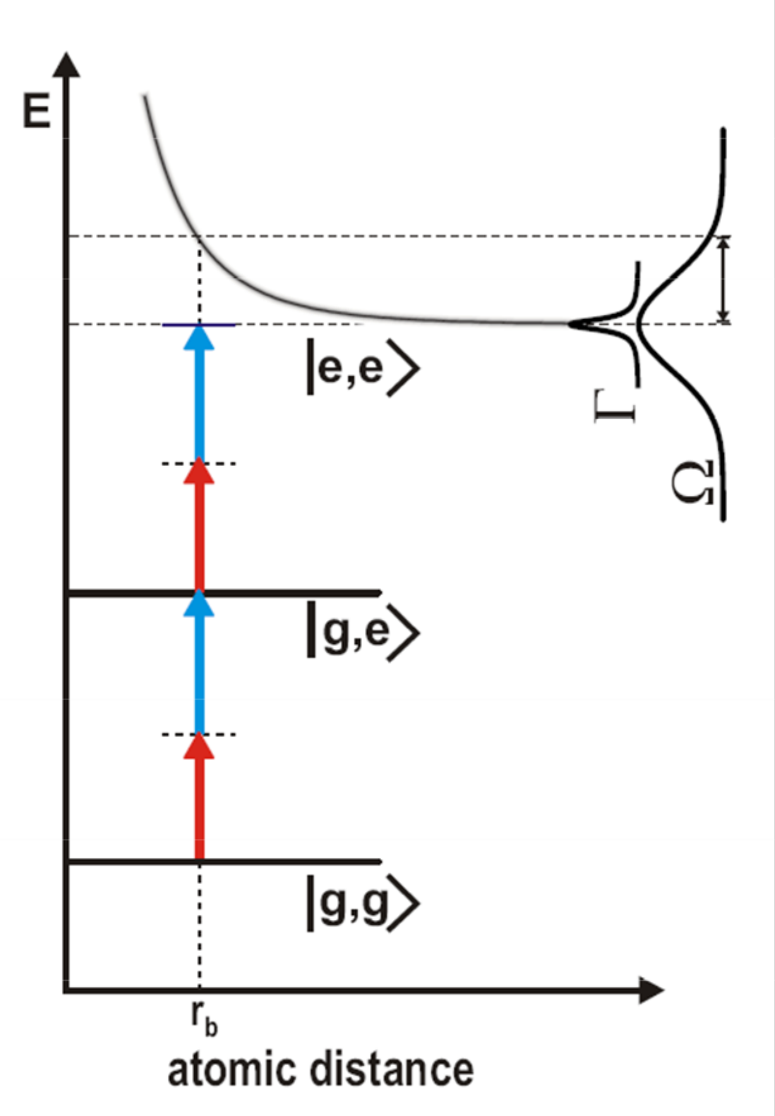
- dipole-dipole interactions

- strong van der Waals repulsion

$$V(r) = \frac{C_6}{r^6} \qquad C_6 \sim n^{11}$$

- strong blockade regime:
    - blockade radius

$$r_b = \sqrt[6]{C_6/\hbar\Omega} \sim 5\ \mu\mathrm{m}$$



Quantum Information

- implementation of quantum gates

Jaksch, Cirac, Zoller, Rolston, Côté, Lukin, Zoller
PRL 2000

Blockade

Small Samples

- blockade larger than system size $r_B < L$
- only one Rydberg excitation in the system

$|g, g, g, g, \ldots\rangle$: no excitation

$\Rightarrow$ $|g, g, \ldots, g, e, g, \ldots\rangle$

Ω : Rabi frequency

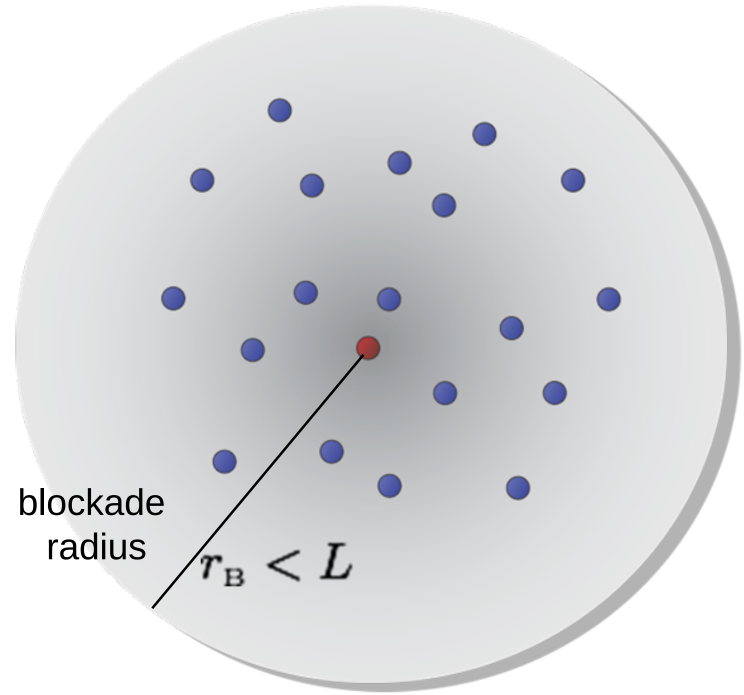
i^{th} single excitation

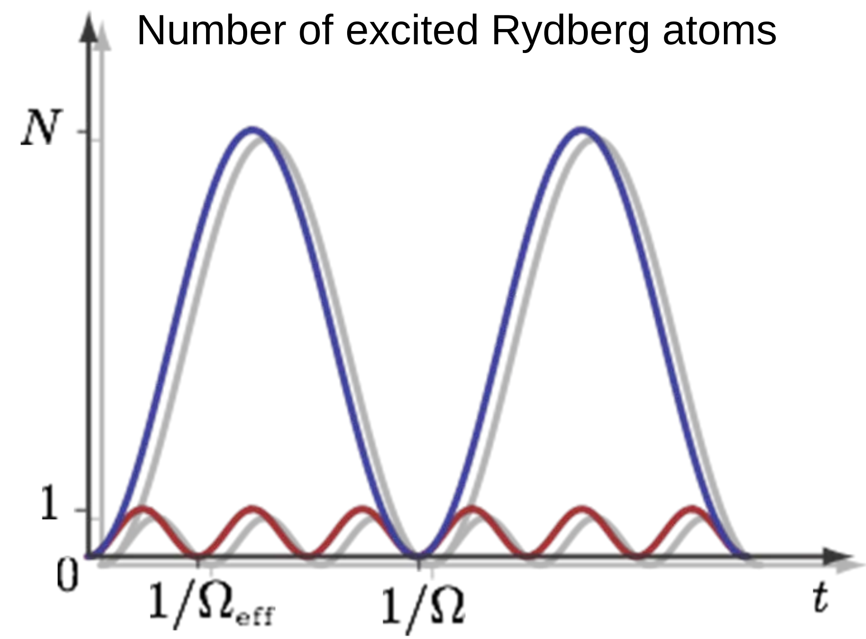
- excited state is coherent superposition

$$\sum_{i=1}^{N} |g, g, \ldots, g, e, g, \ldots\rangle / \sqrt{N}$$

- system exhibits coherent oscillations

$\Rightarrow$ $\Omega_{\text{eff}} = \sqrt{N}\Omega$





Blockade

Large Samples

- blockade radius smaller than system size $r_B < L$

- correlated many-body system

- strong blockade regime:

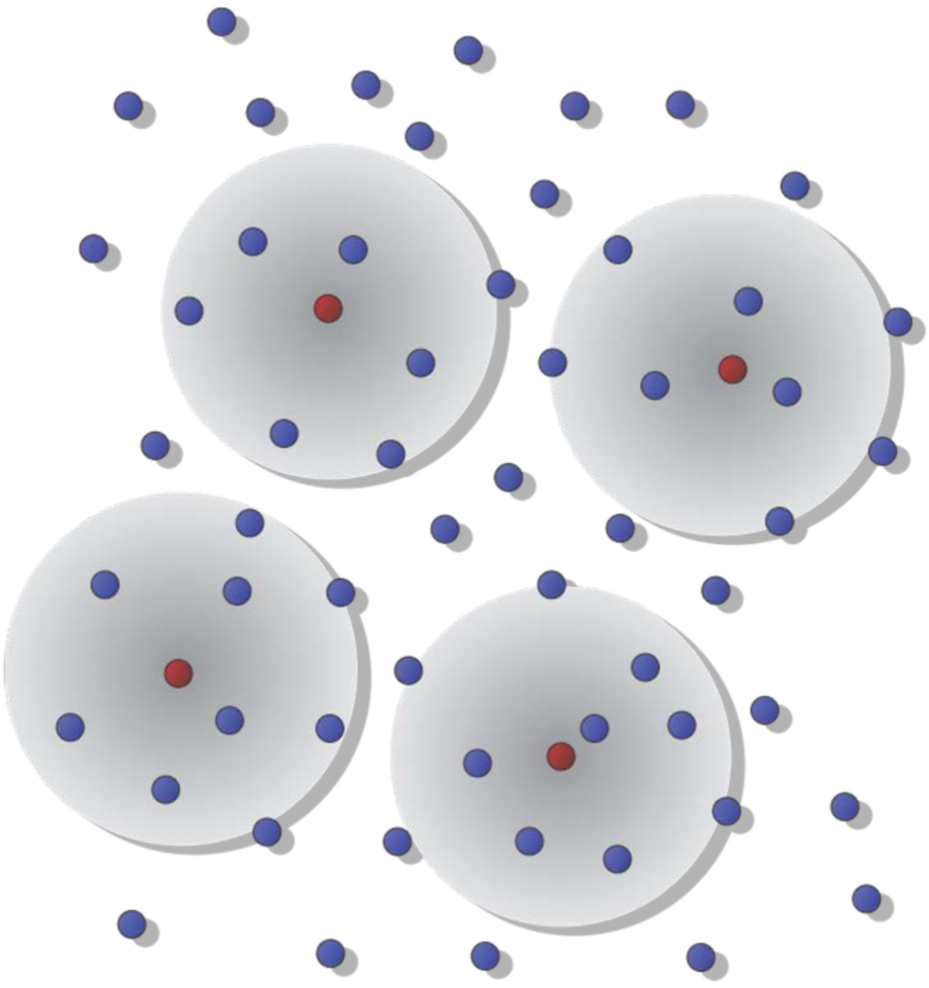
  - blockade radius

$$r_b = \sqrt[6]{C_6/\hbar\Omega} \sim 5\ \mu\text{m}$$

  - number of particles within blockade radius

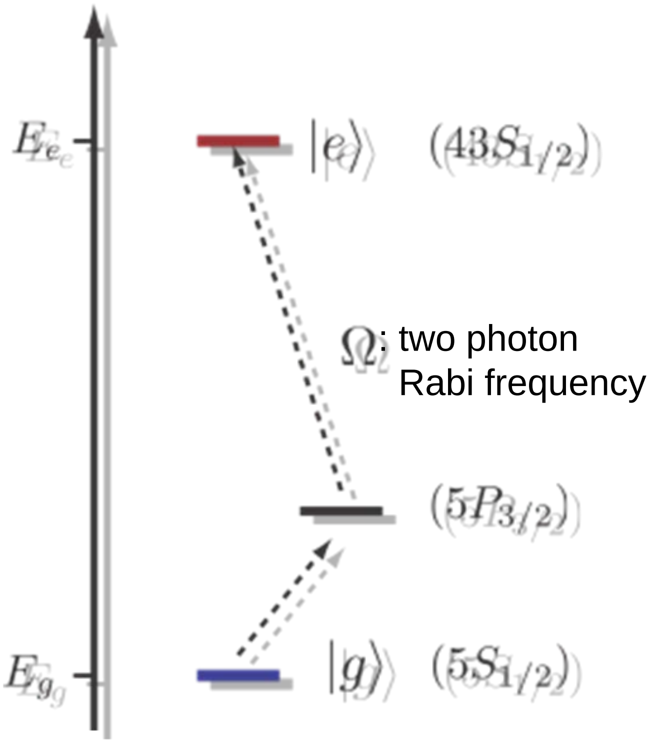
$$N_b = \sqrt{C_6 n^2/\hbar\Omega} \sim 1000$$

(Experiments: T. Pfau, Stuttgart)



What are the properties of this strongly correlated state

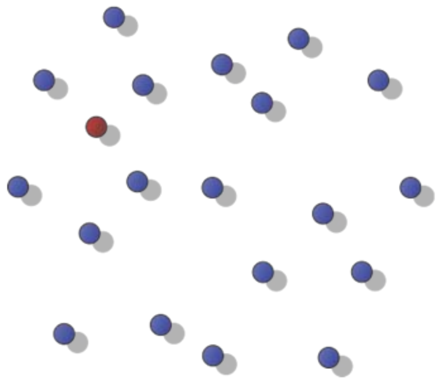
Setup



Rydberg excitation

- resonant excitation of Rydberg states

- frozen motion of the atoms during Rydberg excitation



Hamiltonian

Effective spin system

- rotating wave approximation (rotating frame)

$$| \uparrow\rangle_i = |e\rangle_i$$
$$| \downarrow\rangle_i = |g\rangle_i$$

- mapping to spin-1/2 system

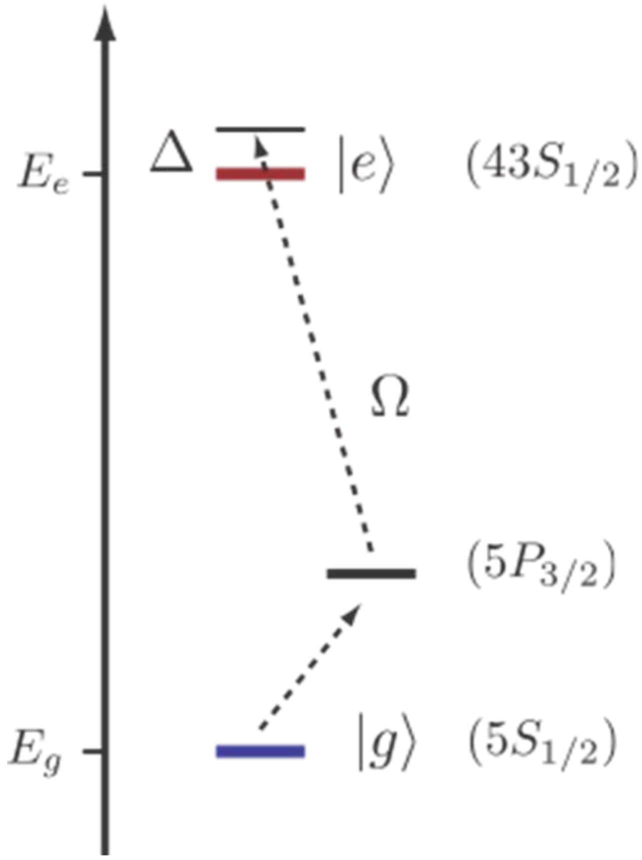
$$\sigma_i^z = |e\rangle\langle e|_i - |g\rangle\langle g|_i$$
$$\sigma_i^x = |e\rangle\langle g|_i + |g\rangle\langle e|_i$$

- number of excited Rydberg atoms

$$P_i^e = (\sigma_i^z + 1)/2$$

$$n_i^e = \langle P_i^e \rangle$$

$$N_e = \sum_i n_i^e$$



Hamiltonian

$$H = \frac{C_6}{2}\sum_{i\neq j}\frac{P_i^e P_j^e}{|\mathbf{r}_i - \mathbf{r}_j|^6} + \frac{\hbar\Omega}{2}\sum_i \sigma_i^x - \frac{\hbar\Delta}{2}\sum_i \sigma_i^z$$

van der Waals repulsion — Rabi frequency — detuning

$\mathbf{r}_i$: particle position

n : averaged particle density

d : dimension of the system

- dimensionless parameter $\alpha = \frac{\hbar\Omega}{C_6 n^{6/d}}$

- characteristic energy scale $E_c = C_6 n^{6/d}$

interparticle distance

$$\frac{H}{E_c} = \frac{1}{2}\sum_{i\neq j} P_i^e P_j^e \frac{n^{6/d}}{|\mathbf{r}_i - \mathbf{r}_j|^6} + \frac{\alpha}{2}\sum_i \sigma_i^x - \frac{\tilde{\Delta}}{2}\sum_i \sigma_i^z$$

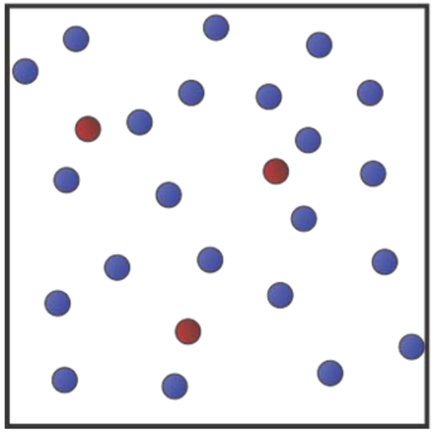
Contains the Hamiltonian the relevant details to describe the experiments?

- coherent dynamics
- neglects ionization
- no accidential resonances due to interactions
- “frozen” motion of atoms

Numerical integration

Small system size

- randomly places atoms in a box with periodic boundary conditions



$N = 50 - 100$

$\alpha \approx 0.01$

50 random initial conditions

- Hamiltonian

$$H = \frac{1}{2} \sum_{i \neq j} P_i^e P_j^e \frac{n^{6/d}}{|\mathbf{r}_i - \mathbf{r}_j|^6} + \frac{\alpha}{2} \sum_i \sigma_i^x$$

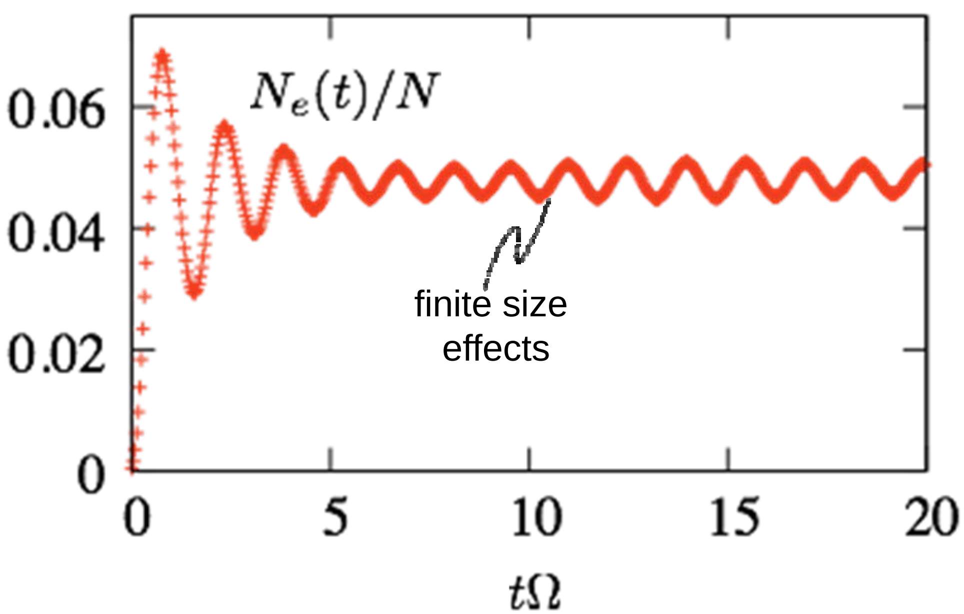
- reduction of Hilbertspace due to van der Waals interaction

Initial state

- all atoms are prepared into the ground state $|G\rangle = \Pi_i |g\rangle_i$

- coherent evolution of the system with $U = \exp(-\frac{iHt}{\hbar})$

- number of excited Rydberg state $\langle N_e(t) \rangle$



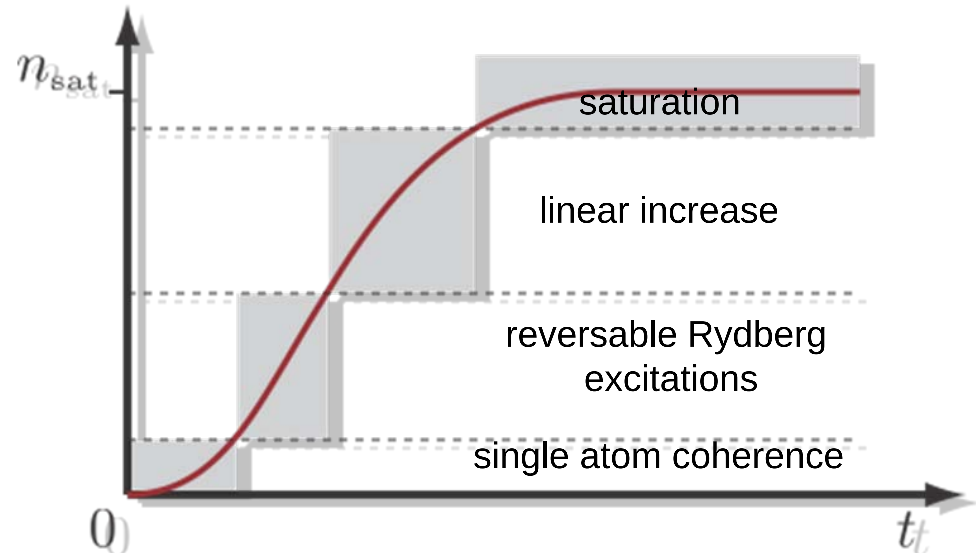
Saturation

Characteristic time evolution

- initial condition:
  all atoms in ground state

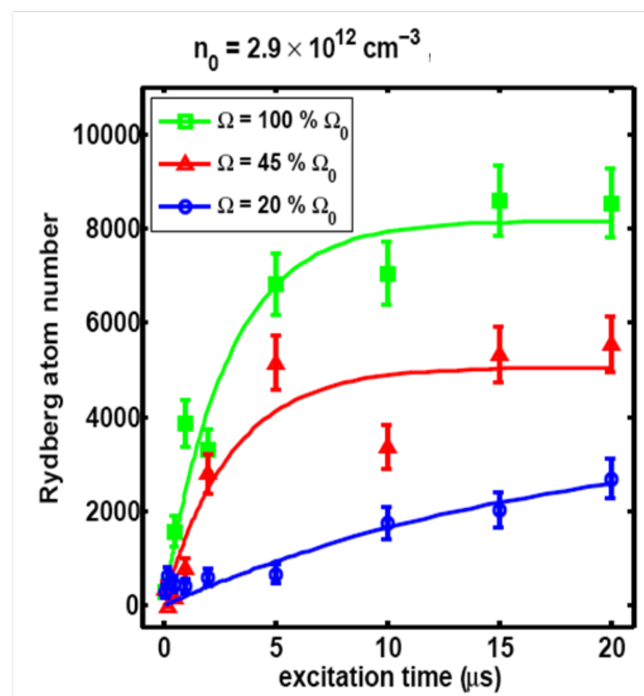
- switching on of laser:

    - single atom coherence
      on short time scales
    - intermediate regime with
      Blockade effects
    - saturation in a steady state



Equilibrium state on long time scales

- relation to ground state of
  the Hamiltonian?
- “thermal” equilibrium state?



Phase Diagram

Ground state $\Omega = 0$

- classical Hamiltonian without quantum fluctuations

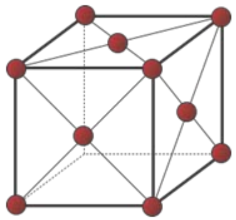
$$H = \frac{C_6}{2}\sum_{i\neq j}\frac{P_i^e P_j^e}{|\mathbf{r}_i - \mathbf{r}_j|^6} + \frac{\hbar\Omega}{2}\sum_i \sigma_i^x - \frac{\hbar\Delta}{2}\sum_i \sigma_i^z$$

Crystalline phase

$\Delta > 0, \Omega = 0$

- finite number of excitation: $\langle n_e \rangle > 0$

- crystalline structure: closed sphere packing



Second order quantum phase transition

$$\langle n_e \rangle \sim \Delta^{d/6}$$

Paramagnet, “Vacuum”

$\Delta < 0, \Omega = 0$

- all particles in the ground state: $\langle n_e \rangle = 0$

- initial state of the experiment

Why a crystalline phase

Hamiltonian

- Hamiltonian for excited Rydberg atoms

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \frac{1}{2} \sum_{i \neq j} V_{vdW}(\mathbf{r}_i - \mathbf{r}_j)$$

$$V_{vdW}(\mathbf{r}) = \frac{C_6}{r^6}$$

strong van der Waals repulsion

- dimensionless parameter

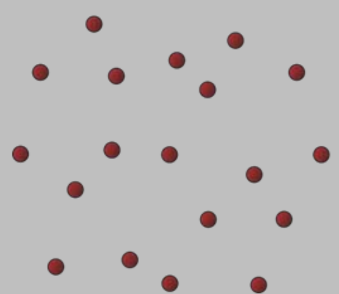
$$r_s = \frac{E_{\rm int}}{E_{\rm kin}} = \frac{C_6 m}{\hbar^2 a^4}$$

interparticle distance

Exp:
- Wigner crystal (Wigner, '34)
- 2D crystals with polar molecules

Liquid phase

- no broken symmetry

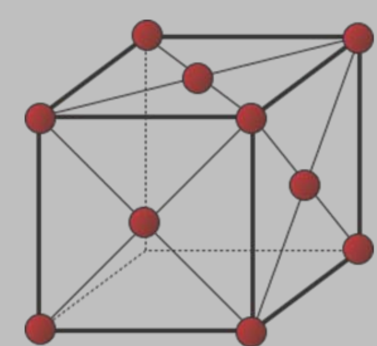


Quantum phase transition

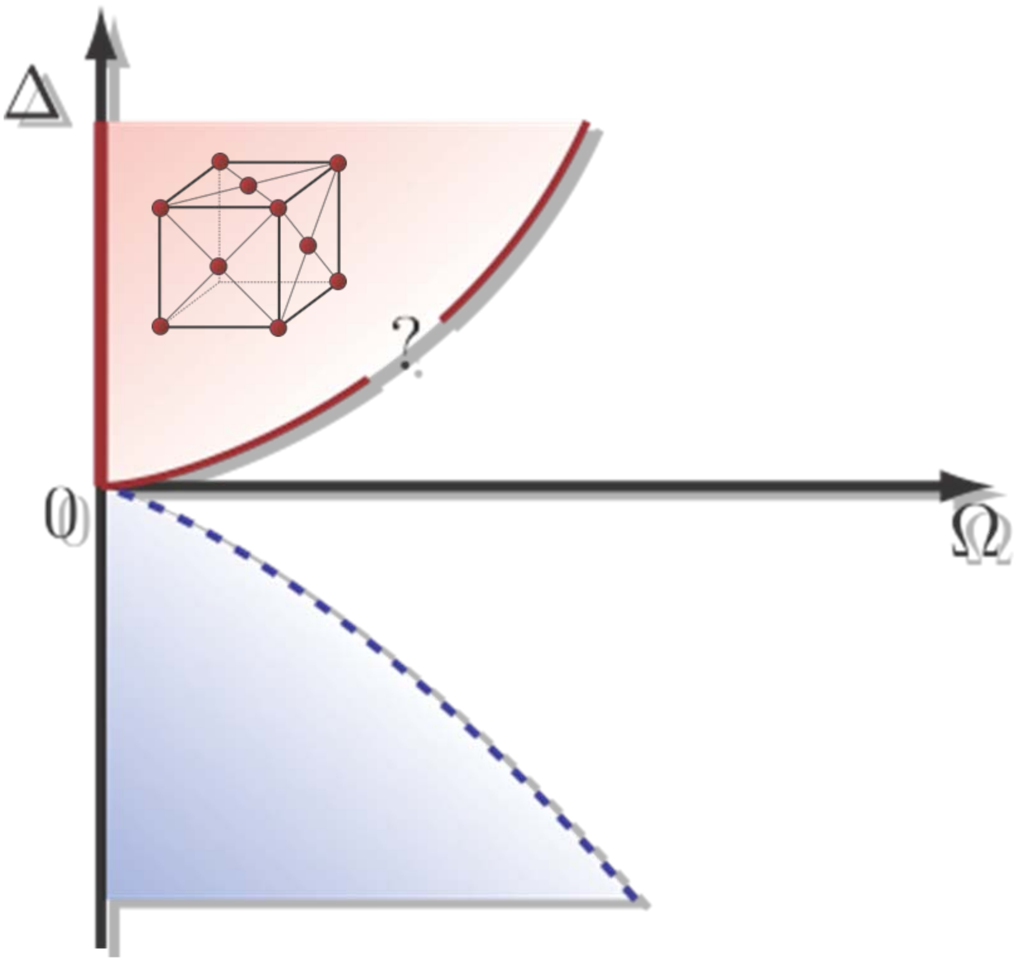
$r_s \approx 10$

Crystalline phase

- breaks translation invariance
- phonon modes
- FCC crystal? (cubic closed packing)



Phase Diagram



Crystalline phase

$$\Delta > 0, \Omega \ll \Delta$$

- regime dominated by a crystalline structure

Quantum critical region

$$\Delta \approx 0, \Omega \gg \Delta$$

- diverging length scale

$$\alpha = \frac{\hbar\Omega}{C_6 n^{6/d}}$$

Paramagnet, “Vacuum”

$$\Delta > 0, \Omega \ll \Delta$$

- independent Rabi oscillations: large detuning

$$\langle n_e \rangle \sim n\,\Omega^2/\Delta$$

Critical theory

Critical region $\Delta = 0$

- diverging length scale:
  blockade radius $r_{\mathrm{B}} = (C_6/\hbar\Omega)^{1/6}$

- dimensionless parameter: $$\alpha = \frac{\hbar\Omega}{C_6 n^{6/d}} = \frac{1}{r_{\mathrm{B}}^6 n^{6/d}}$$

Universality $\alpha \rightarrow 0$

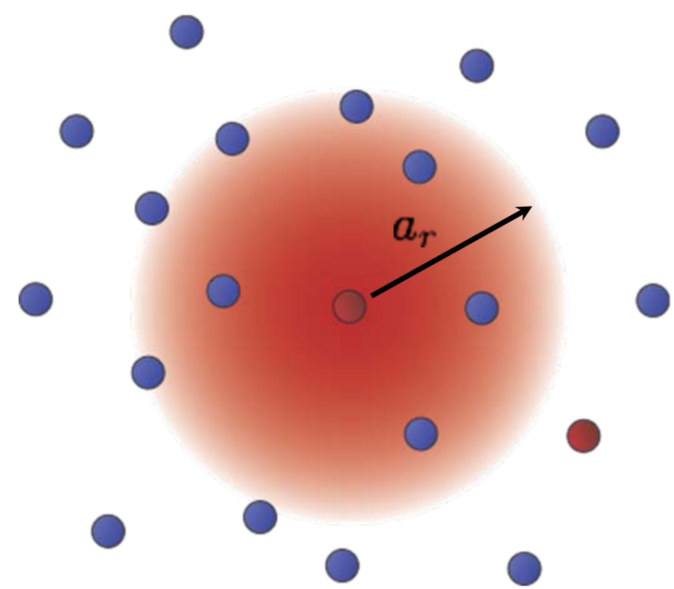
- scaling exponents for all observables:

$$f_e = N_e/N = c\alpha^{\nu}$$

universal exponent (ν)

non-universal prefactor (c)

- independent on the microscopic realization
  - atom distribution:
    lattice vs. random
  - atomic species
  - short-range interactions



Mean field theory

Approximation

- select a single atom
- surrounded by a bath of atoms
- interaction produces an effective potential

$$h_z = \frac{1}{2}\sum_j \langle P_j^e \rangle \frac{n^{6/d}}{|\mathbf{r}_i - \mathbf{r}_j|}$$

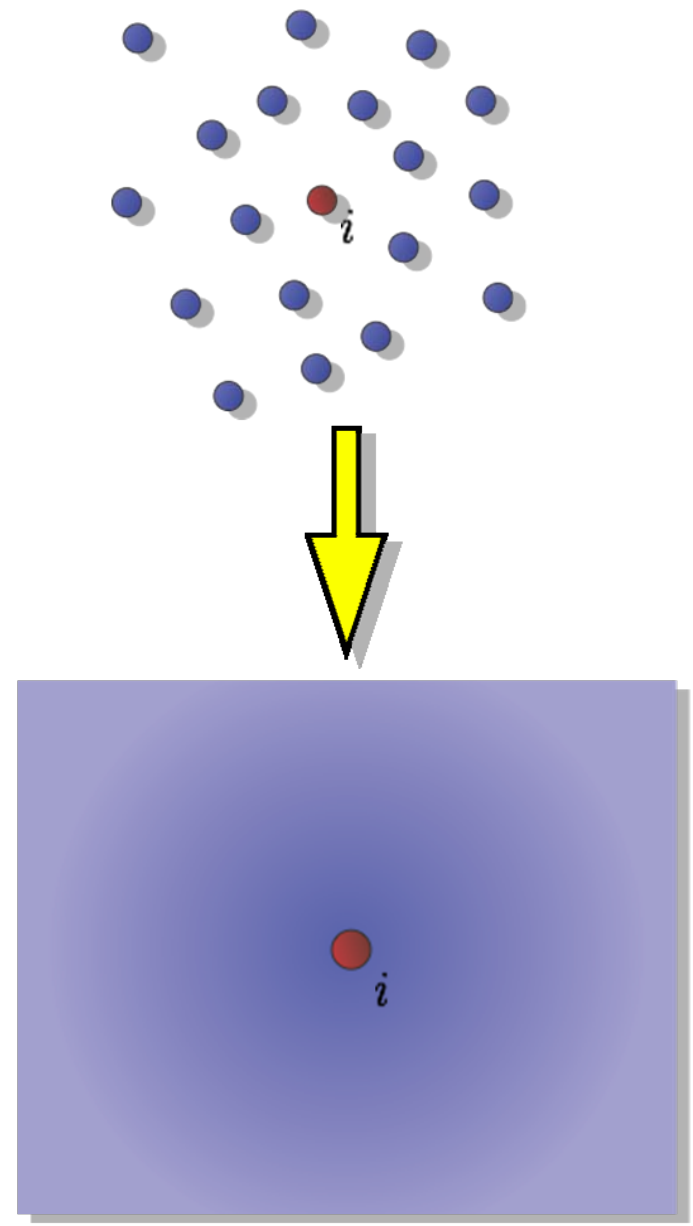
- local Hamiltonian

$$H_i = \frac{\alpha}{2}\sigma_i^z + P_i^e h_z = \frac{\alpha}{2}\sigma_i^x + \frac{h_z}{2}\sigma_i^z + \frac{h_z}{2}$$

- simple (inappropriate) ansatz:

$$\langle P_j \rangle = \langle P_i \rangle = f_e$$

Ignores correlations
due to Blockade



Density-density correlations

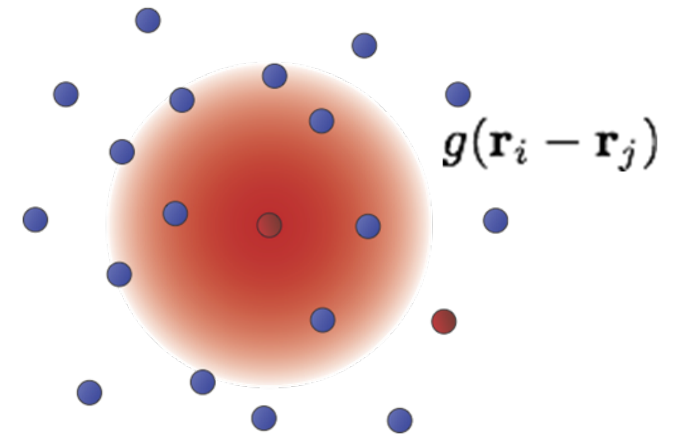
Rydberg Blockade

- suppression of Rydberg due to van der Waals repulsion

$$\langle P_i^e P_j^e \rangle = g(\mathbf{r}_i, \mathbf{r}_j)\langle P_i^e \rangle \langle P_i^e \rangle$$
$$= g(\mathbf{r}_i - \mathbf{r}_j) f_e^2$$

translation invariance

$$f_e = \langle P_i^e \rangle = \langle P_j^e \rangle$$: mean Rydberg excitations

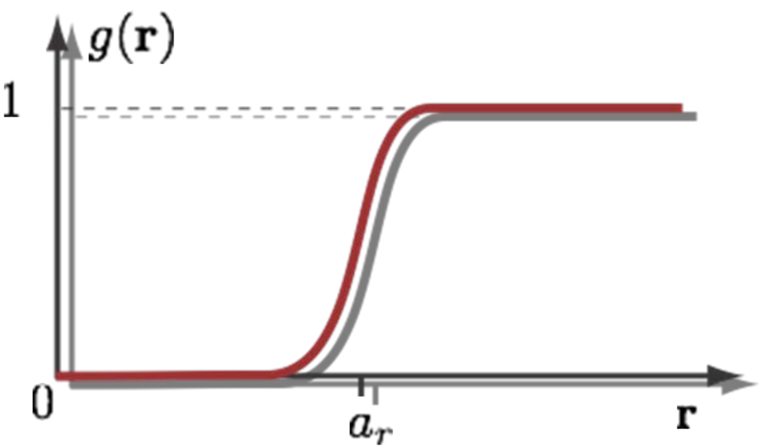


- properties of density-density correlation function

$$f_e n \int d\mathbf{r}\,[1 - g(\mathbf{r})] = 1$$

- characteristic length scale

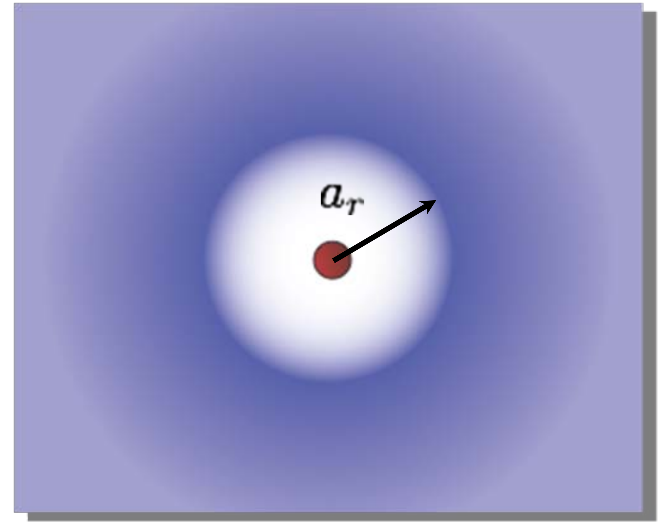
$$a_r = 1/(f_e n)^{1/d}$$



Mean-field theory

- effective potential

$$h_z = \sum_j g(\mathbf{r}_i, \mathbf{r}_j) \langle P_j \rangle \frac{n^{6/d}}{|\mathbf{r}_i - \mathbf{r}_j|^6}$$
$$= f_e \int d\mathbf{r}\, g(\mathbf{r}/a_r) \frac{n^{6/d}}{|\mathbf{r}|}$$



- local single particle Hamiltonian

$$H_i = \frac{\alpha}{2}\sigma_i^z + P_i^e h_z = \frac{\alpha}{2}\sigma_i^x + \frac{h_z}{2}\sigma_i^z + \frac{h_z}{2}$$

- self-consistency condition

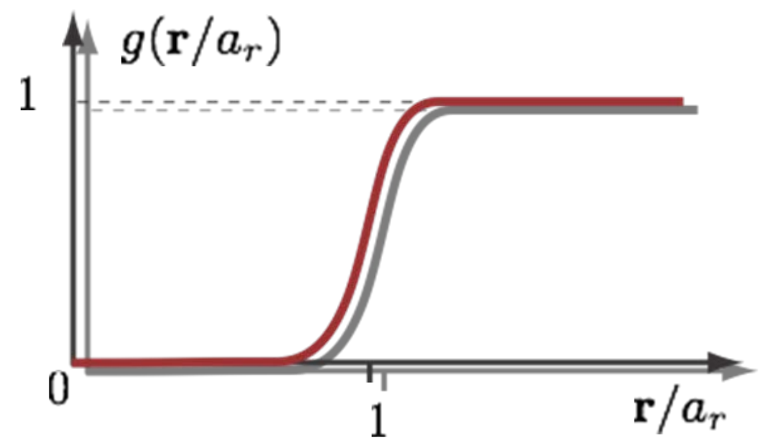
$$f_e = \langle P_i^e \rangle = \langle P_j^e \rangle$$
$$a_r = 1/(n f_e)^{1/d}$$

- energy conservation
  - sudden switching of the laser: $\langle H \rangle = 0$
  - determines equilibrium state

Ansatz for density-denisty correlations:

dimensional factor

$$g(x) = \Theta\left[x - 4(\pi/3)^{1/3}\right]$$



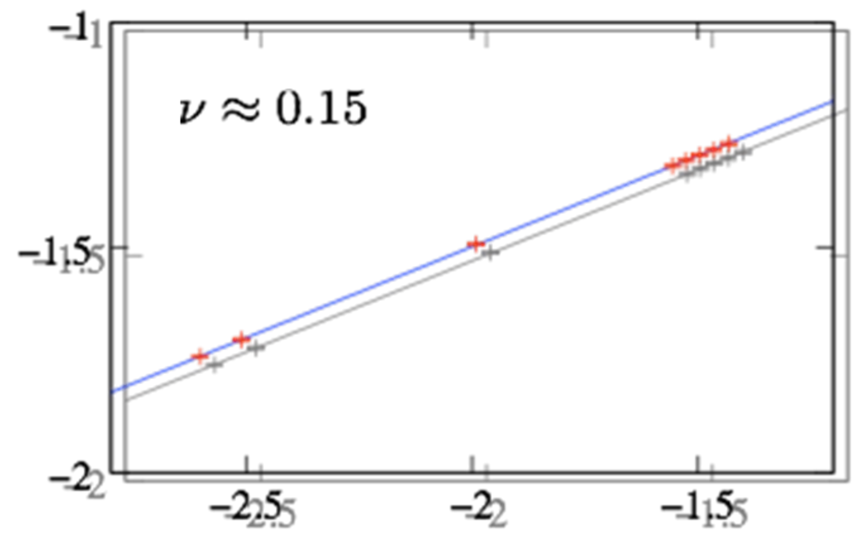
Mean-field theory

Solution

- independent on equilibrium state

$$f_e \sim \alpha^\nu = \left(\frac{\hbar\Omega}{C_6 n^{6/d}}\right)^\nu \qquad \nu = \frac{2d}{12+d}$$

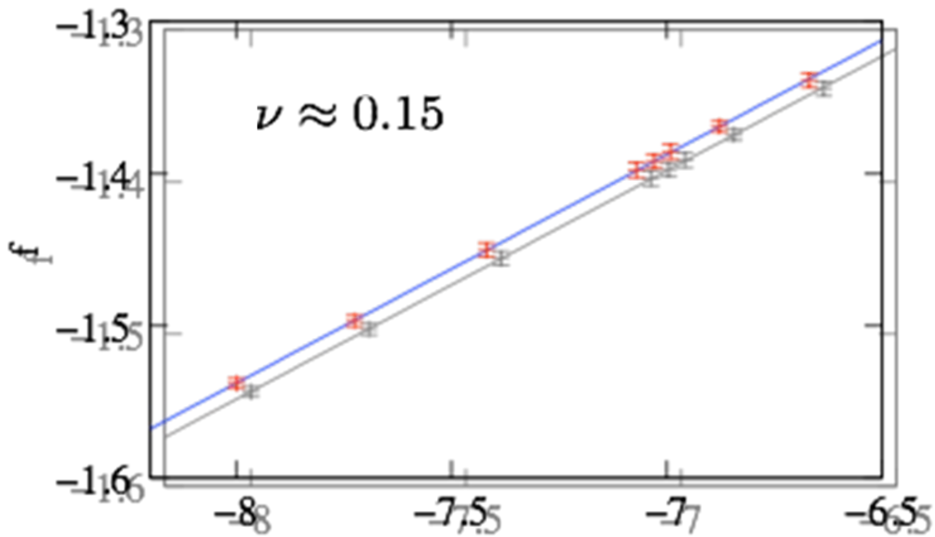
3D numerical analysis



Comparison mean-field vs. numerics (coherent evolution)

	3D system	1D system
Numerics:	$\nu \approx 0.15$	$\nu \approx 0.15$
Mean-field:	$\nu = 2/5 = 0.4$	$\nu = 2/13 \approx 0.154$

1D numerical analysis



Upper critical dimension

Hamiltonian

- corrections to mean-field approximation

$$H = \sum_i H_i + \frac{1}{2} \sum_{i \neq j} g(\mathbf{r}_i, \mathbf{r}_j) \left(P_i^e - f_e\right) \left(P_j^e - f_e\right) \frac{n^{6/d}}{|\mathbf{r}_i - \mathbf{r}_j|^6}$$

mean-field hamiltonian

fluctuations around the mean-field

Upper critical dimension

- above critical dimension, mean-field provides correct scaling exponents

$$f_e = N_e/N = c\alpha^\nu$$

universal exponent

non-universal prefactor

- van der Waals interactions: many nearest neighbor

- for $\Omega = 0$ mean-field is exact

upper critical dimension?

$$d_c = 1$$

- new universality class?

Local density approximation

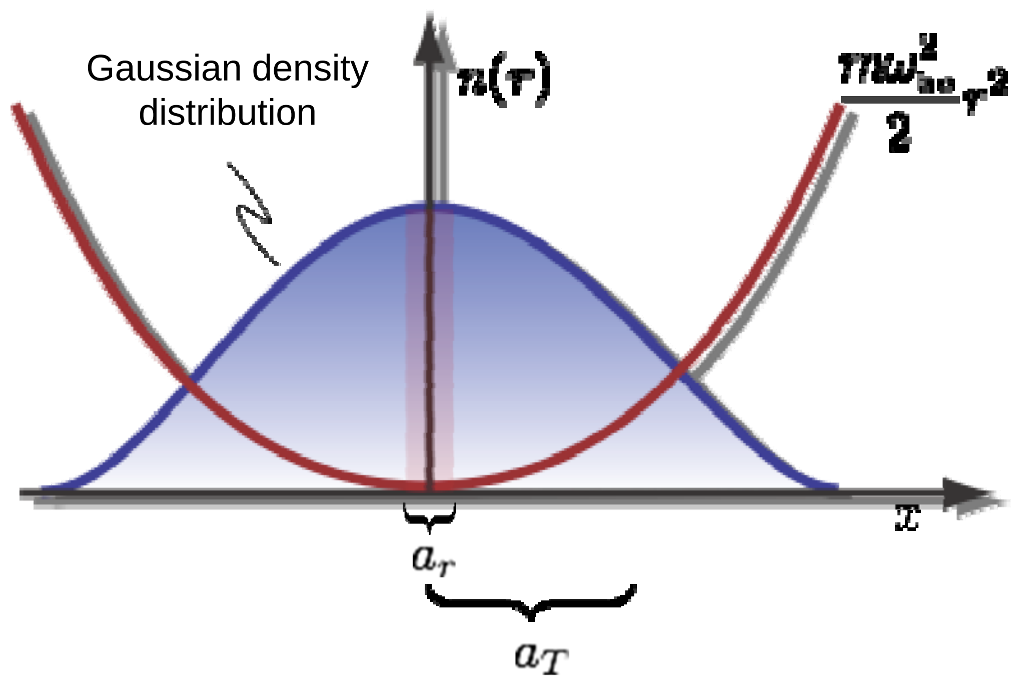
Local density

- harmonic trapping potential
- thermal gas with
  density distribution

$$n(r) \sim \exp\left(-\frac{m\omega_{\rm ho}^2}{2T} r^2\right)$$

- smoothly varying trap

$$a_T = \sqrt{T/m\omega_{\rm ho}^2} \gg a_r = 1/(n f_e)^{1/d}$$



local density approximation

$$N_e = \int d\mathbf{r}\, n(\mathbf{r}) f_r(\alpha) \sim \int d\mathbf{r}\, n(\mathbf{r}) \left[\frac{\hbar\Omega}{C_6 (n(\mathbf{r}))^{6/d}}\right]^\nu$$

$$\frac{N_e}{N} \sim \alpha^\nu$$

density in trap center

: scaling exponent remains invariant

Comparison with experiments

Comparison with experiment

- finite trap: Gaussian density distribution

- cigar shaped trap:

  in the crossover between 3D and 1D?

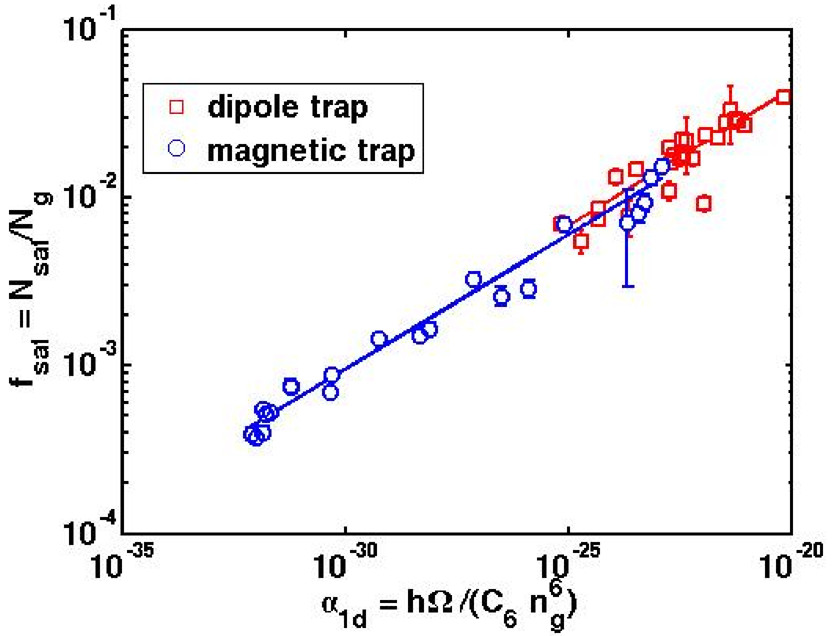
- single parameter fit is is consistent with 1D result

- depending on the variation, shows 1D or 3D scaling

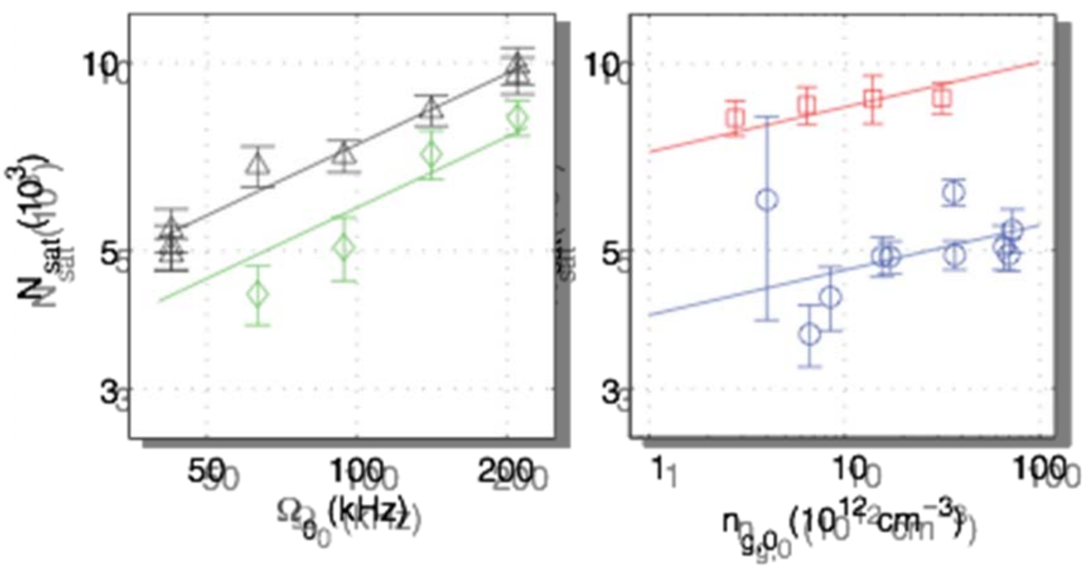
$N_e \sim \Omega^{0.39}$: consistent with 3D scaling (0.4)

$N_e \sim n^{0.07}$: consistent with 1D scaling (0.077)

additonal length scale
transverse trapping?
laser coherence?



Heidemann *et al*, PRL (2007)



Coherent evolution: derivation of Master equation

Time evolution

Corrections to Mean-field theory

$$H = \sum_i H_i + \frac{1}{2}\sum_{i\neq j} g(\mathbf{r}_i, \mathbf{r}_j)\left(P_i^e - f_e\right)\left(P_j^e - f_e\right)\frac{n^{6/d}}{|\mathbf{r}_i - \mathbf{r}_j|^6}$$

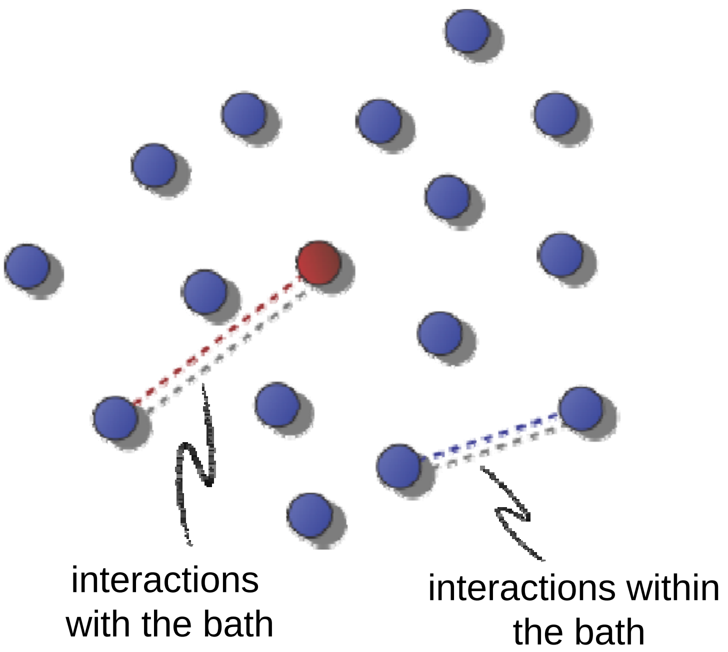
Local hamiltonian

$$H_i = \frac{\alpha}{2}\sigma_i^z + P_i^e h_z$$

- mean-field solution
- coherent oscillations
- no damping/decoherence

Quadratic fluctuations

- corrections to the mean-field theory
- coupling of the different local Hamiltonians
- introduces damping



Spin bath

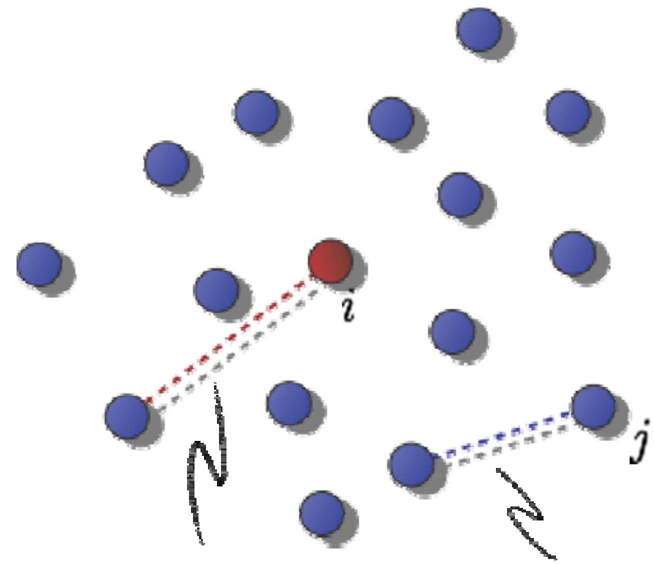
local Hamiltonian
for fixed particle

Hamiltonian for all
remaining particles

$$H = H_{\text{sytem}} + H_{\text{int}} + H_{\text{bath}}$$

coupling system-bath

$$\frac{1}{2}\sum_j g(\mathbf{r}_i, \mathbf{r}_j)\,(P_i^e - f_e)\,(P_j^e - f_e)\,\frac{n^{6/d}}{|\mathbf{r}_i - \mathbf{r}_j|^6}$$



derivation of
master equation $\rho^{(i)}$

- rotating wave approximation
- self-consistency

  $f_e(t)$

- time evolution of the particles in the bath
  is equal to the time evolution of the system

Master equation

Reduced density matrix

- interaction picture

$$\partial_t \rho^{(t)} = \sum_{\omega,\omega'} \gamma_{\omega\omega'}(t) \left(\hat{A}_\omega \rho^{(t)} \hat{A}^\dagger_{\omega'} - \frac{1}{2}\left\{ \hat{A}^\dagger_{\omega'} \hat{A}_\omega, \rho^{(t)} \right\} \right)$$

representation of P_i^e in the eigenbasis of H_i

coupling rates determinedself-consistently

Solution:

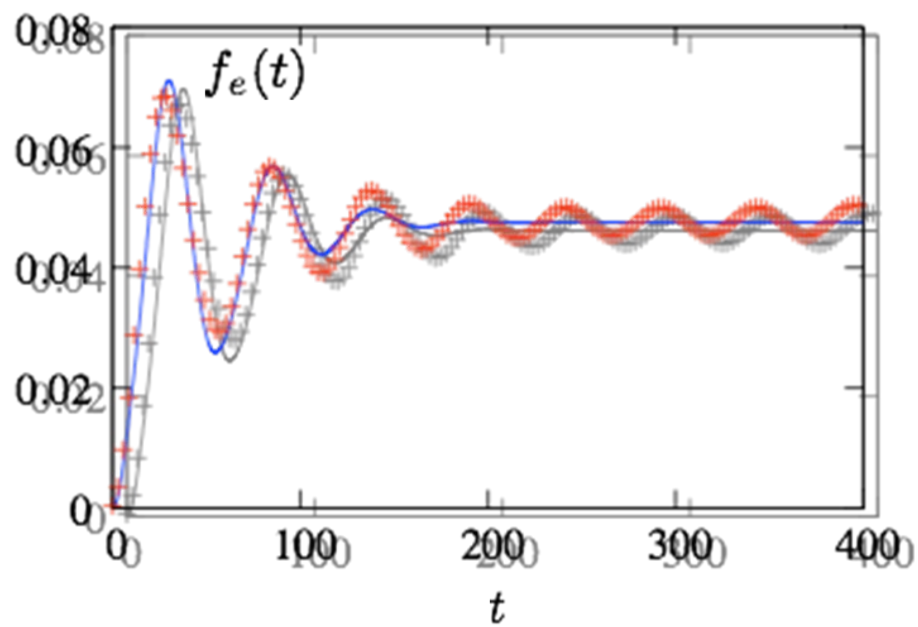
- stationary state is given by the mean-field solution

- independent Rabi oscillations on short times

- damping of oscillations due to interactions between the particles

- sinlge fitting parameter: shape of $g(\mathbf{r})$

Comparison with exact numerical intergration:

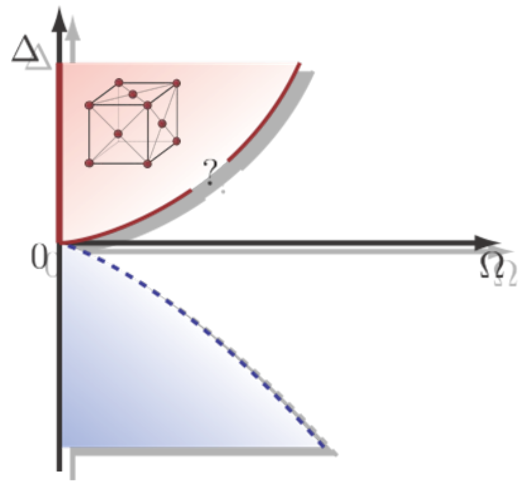


Conclusion and Outlook

Van der Waals blockade

- complex quantum many-body system

- critical phenomena with universal scaling exponents



Methods

- mean-field theory

- effective master equation to describe the dynamics