Michigan Quantum Summer School

Ann Arbor, June 16-27, 2008.

Optical Semiconductor Dots for Quantum Information I

- Theory here. Phys fr experimental view - Prof Steel
- Qubit: a spin in a semiconductor quantum dot
- Preparation of a quantum state: optical initialization
- Quantum operations by optical control
  - universal if arbitrary one qubit rotations plus an entangling two-qubit operation
- Scaling up to a useful system, eg quantum computer
- Dissipative effects (tomorrow afternoon session)
  - Spin relaxation and decoherence

L. J. Sham
University of California San Diego

Supported by ARO/NSA-LPS, DARPA/AFOSR, NSF

Collaborators in Quantum Computer

UCSD

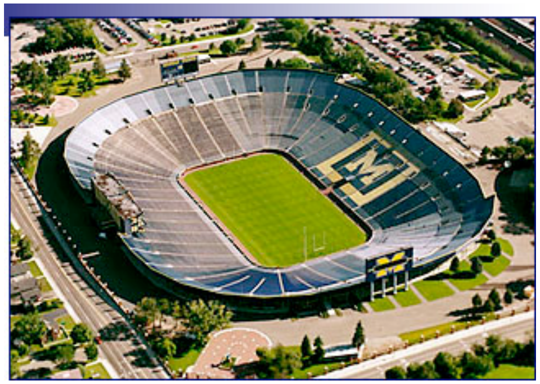
Theory

Pochung Chen
Carlo Piermarocchi
Yseulte Dale
Sophia Economou
Wang Yao
Renbao Liu
Michael Leuenberger
Clive Granger
Semion Saikin
Parin Dalal

Univ of Michigan

Quantum Optics

Duncan Steel
Gang Chen
Todd Stievater
Xiaoqin Li
Gurudev Dutt
Jun Cheng
Yanwen Wu
Xiaodong Xu
Bo Sun
Qiong Huang



Naval Res Lab

Fabrication & Characterization

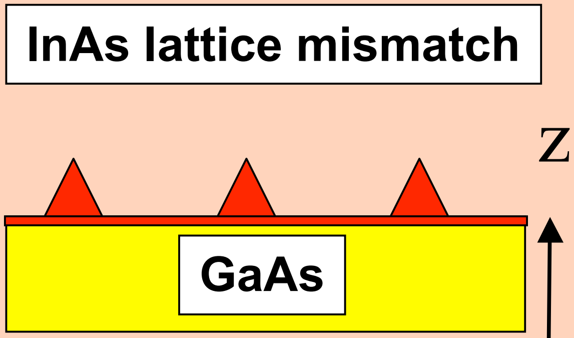
Dan Gammon
D.S. Katzer
D. Park
J.G. Tischler
A.S. Bracker
M.E. Ware
E.A. Stinaff
M.F. Doty
M. Schreibner

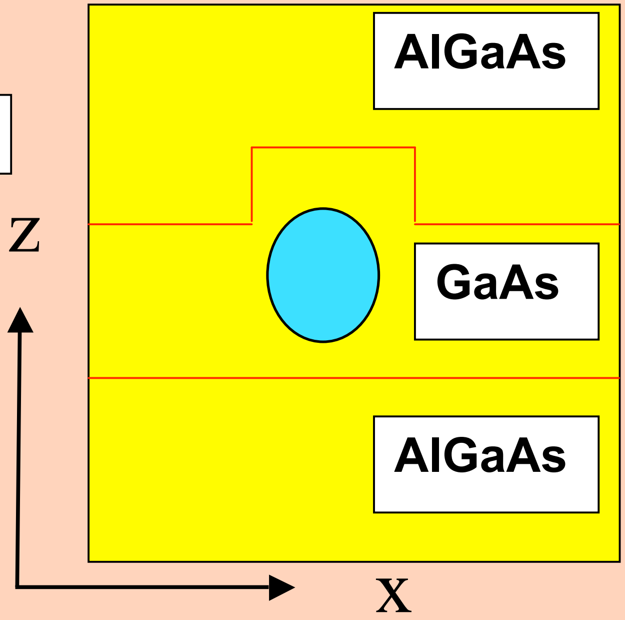
Three Kinds of Semiconductor Quantum Dots

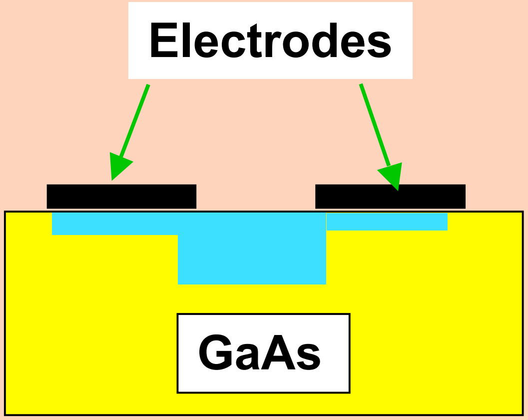
Self-assembled quantum dots
(3-10 nm)
$\Delta E \sim 100$ meV

Interface fluctuation quantum dots
($30 \times 40 \times 3$ nm^3)
$\Delta E \sim 10$ meV

Gated quantum dots
(100 nm)
$\Delta E \sim 1$ meV

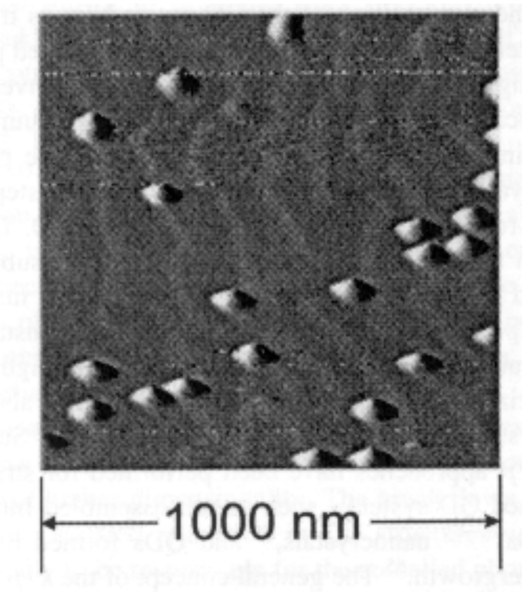






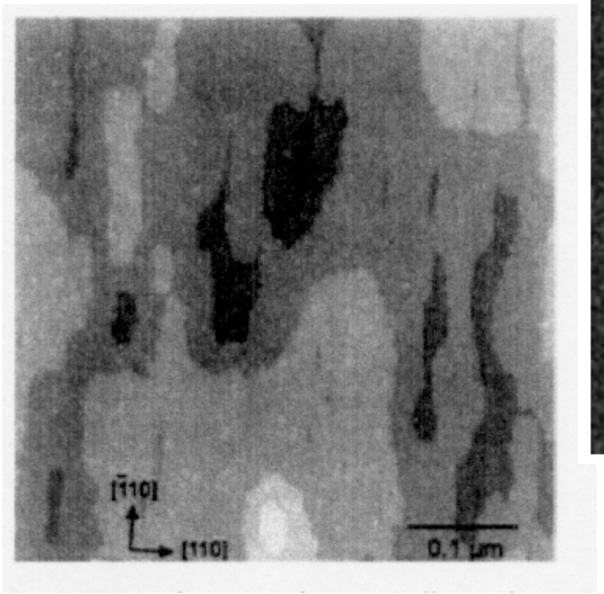
Experimentalists’ view of Quantum Dots

Self-assembled quantum dot



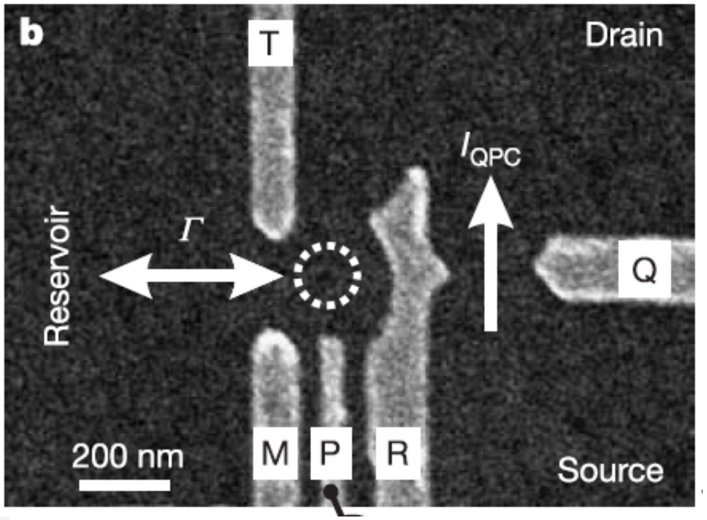
A. Zrenner, et al.
J.Chem.Phys. **112**, 7790 (2000).

Interface fluctuation quantum dot



D.Gammon, *et al.*,
PRL **76,** 3005 (1996).

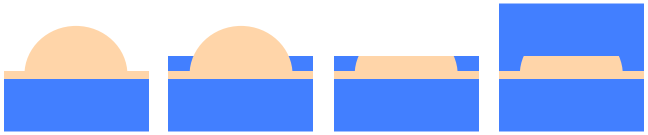
Gated quantum dot

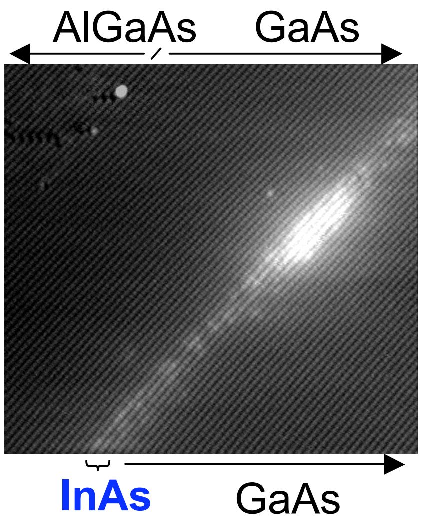


Elzerman et al.
Nature **430**, 431
(2004)

A real system: electron spin in a quantum dot

MBE-grown InAs/GaAs Dots

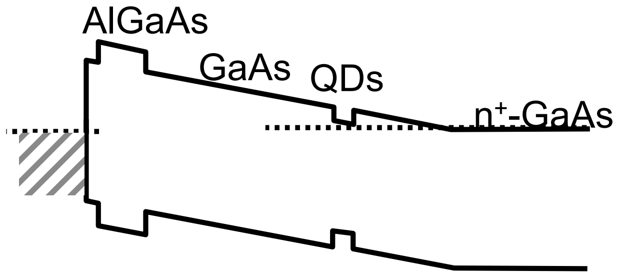




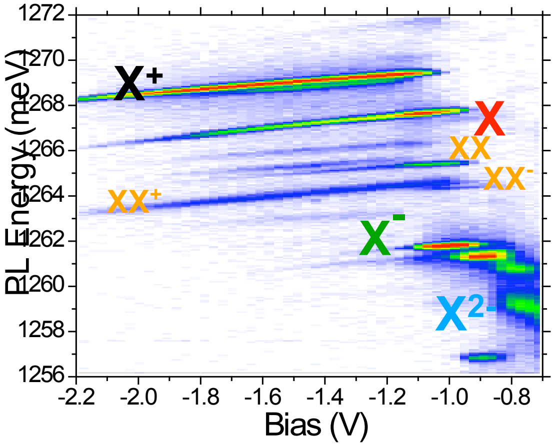
Cross-sectional STM image

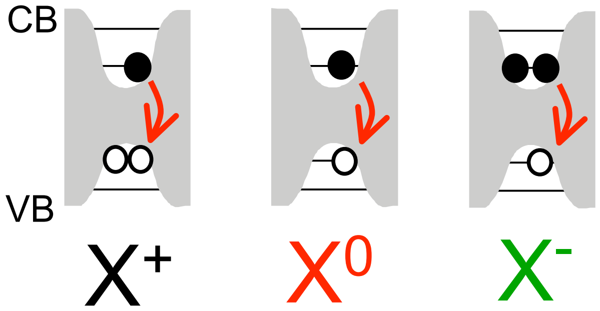
Quantum Dot: height ~ 2-4 nm
width ~ 10-30 nm

NRL group: D. Gammon, A. S. Bracker, M. F. Doty, M. Scheibner, E. A. Stinaff, J. G. Tischler, M. E. Ware



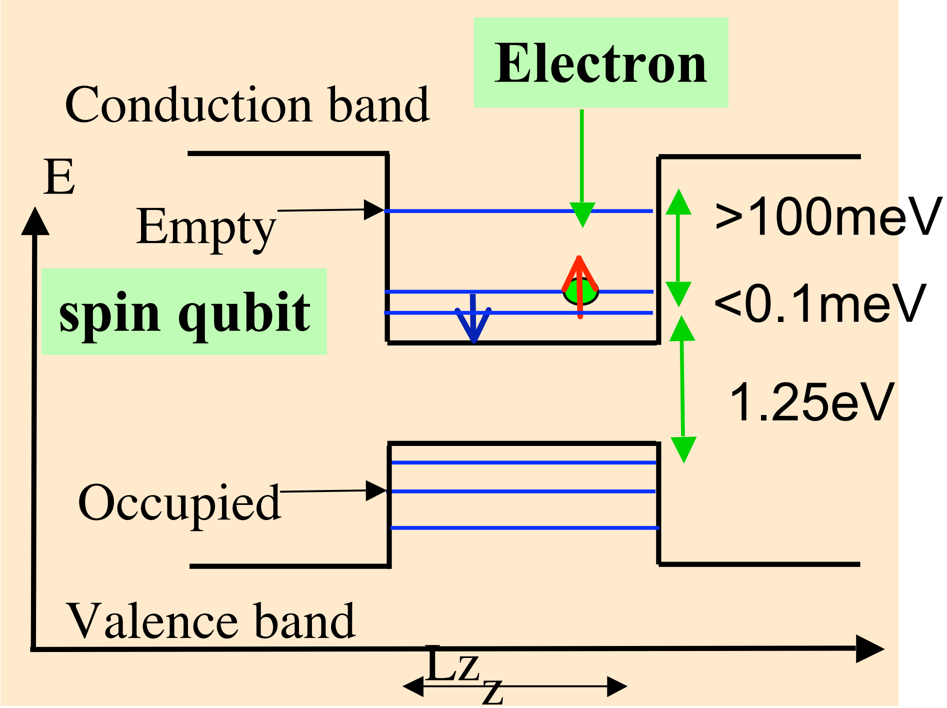
Optical spectrum provides excellent measure of charge state





Theorists' view of quantum dots

Square-well quantum dots



Conduction band

Electron

E

Empty

spin qubit

>100meV

<0.1meV

1.25eV

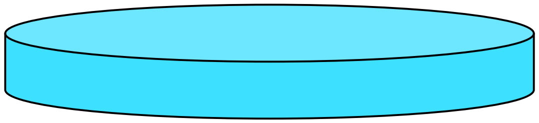
Occupied

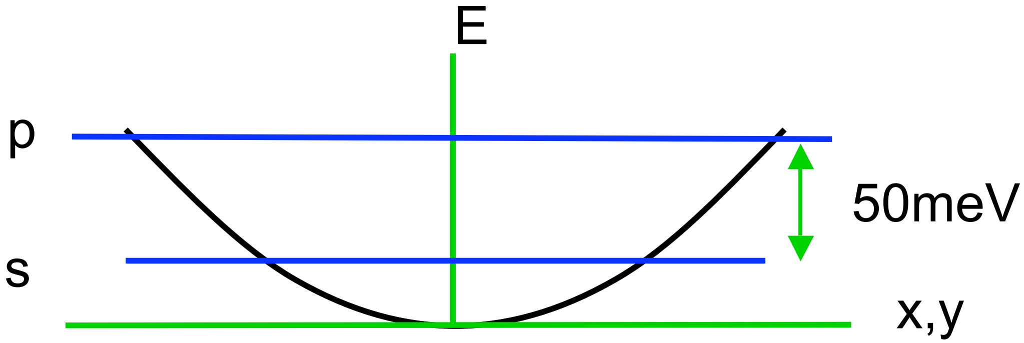
Valence band

Lz

z

Lateral harmonic well quantum dots





E

p

s

50meV

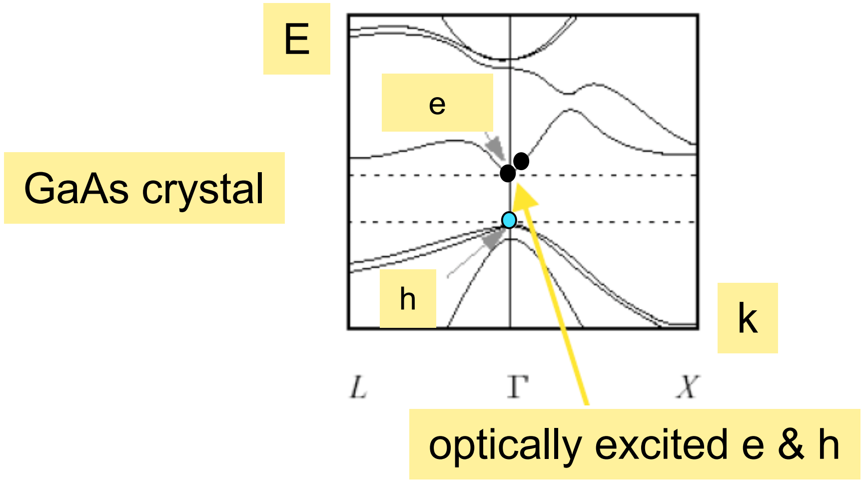
x,y

Spin qubit is robust

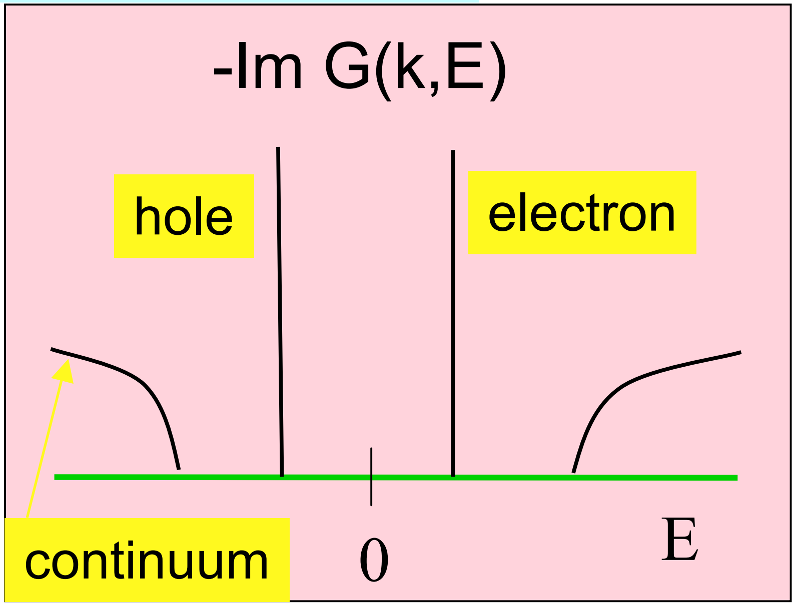
Is an electron in a dot isolated?

N+1 electron problem

- lattice symmetry ==> band gap
- e-e interaction ==>
  - renormalized mass
  - dielectric screening of interaction
  - e in filled valence band inert
  - life time infinite w/o excitations across the gap
- e-phonon interaction ==> finite life time, prolonged at low T
- Exciton (e-h) is an exact excited state w/o radiative interaction.
- Confinement of an electron in a quantum dot
- Residual decoh - op gen phonons & local electron polarizations



Electron spectral density



Kohn PR (1960), Sham, PR (1966), Sham and Rice, PR (1966).

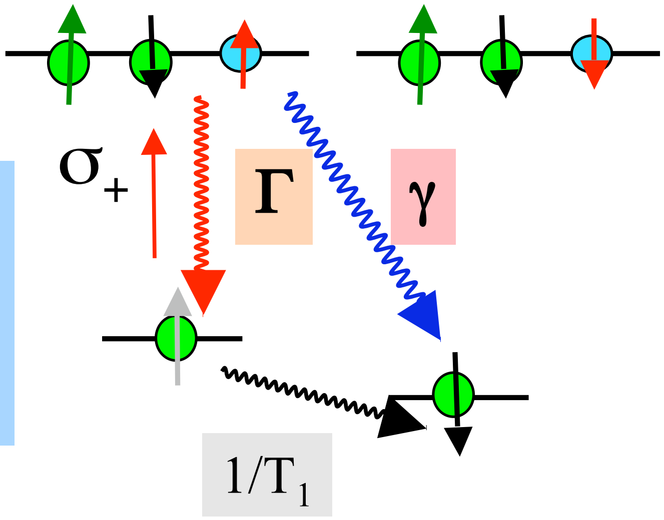
Single spin state preparation by optical pumping

Multi-particle states

A. Kastler (1952)

allowed **Γ > γ** forbidden

Trion



Magnetic field along z, the optical axis

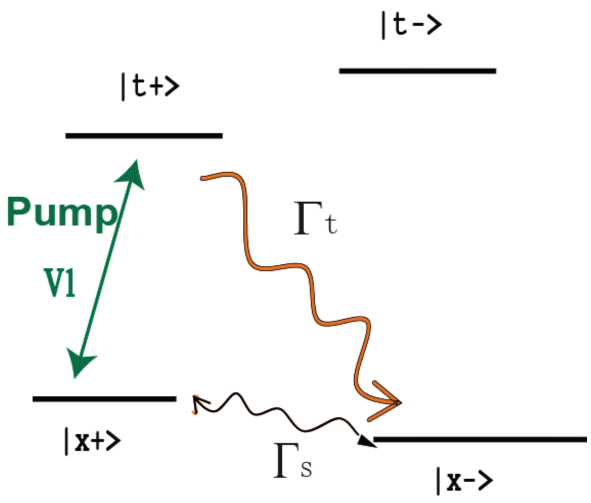
- Expt of SAQD InAs in GaAs
- Resonant laser excitation for a time (~300 ms) >> 1/ γ (1 μs) but less than T_1 due to tunneling
- Fidelity 0.998 at 0.3T (or spin T~20 mK for Zeeman~ 4K) - at op temp of 4K, B~62T, it would takes forever at spin flip rate $1/T_1$ to equilibrate

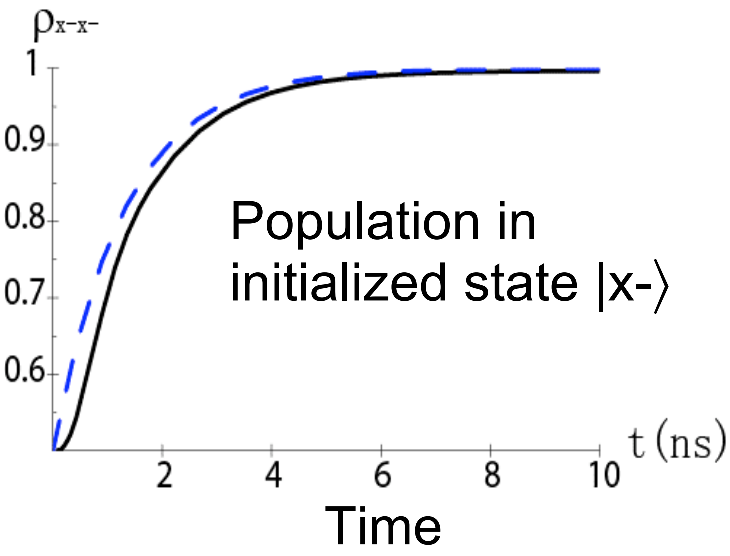
Atatüre, Dreiser, Badolato, Högele, Karrai, Imamoğlu, Science 2006

Theory

Fast spin initialization in a singly charged quantum dot

Magnetic field along x, normal to the optical axis





Population in initialized state |x-⟩

- Population of |x-> states as a function of time.
- The blue dash cures and the solid lines are the analytical and numerical results, respectively
- Near-unity fidelity is approached around 10 ns.
- The Rabi frequency is taken to be equal to the trion decay rate.

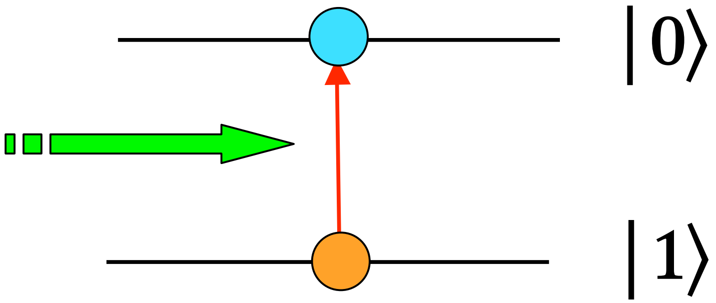
Theory:Emary, Xudong Xu, Steel, Saikin, Sham, PRL 2007

Experiment (in Steel's lectures): Xu, Wu, Sun, Huang, Cheng, Steel, Bracker, Gammon, and Emary, Sham, PRL 2007

Quantum Operation on a Single Qubit

Not q-op

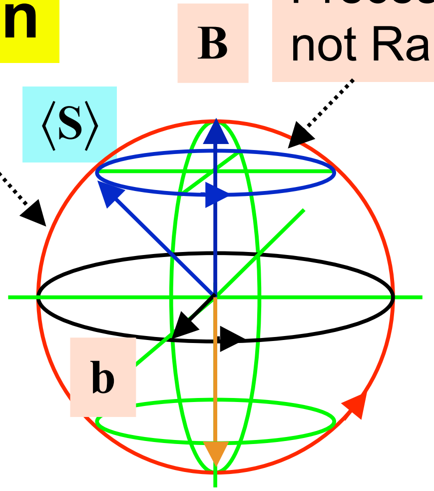
Optical Excitation by Fermi golden rule



Excitation EM field with incoherent bandwidth

Rabi rotation

Precession not Rabi



Rotating frame

$$\hat{H} = B\sigma_z + b[\cos(\omega t)\sigma_x + \cos(\omega t)\sigma_y]$$

A coherent b pulse of the duration may rotation the spin in an angle α about a fixed axis in the rotating frame

$$\mathrm{R}(\boldsymbol{\alpha}, \mathbf{n}) = \begin{bmatrix} \cos(\frac{\alpha}{2}) - in_z \sin(\frac{\alpha}{2}) & -i(n_x - in_y)\sin(\frac{\alpha}{2}) \\ -i(n_x + in_y)\sin(\frac{\alpha}{2}) & \cos(\frac{\alpha}{2}) + in_z \sin(\frac{\alpha}{2}) \end{bmatrix}$$

Theory

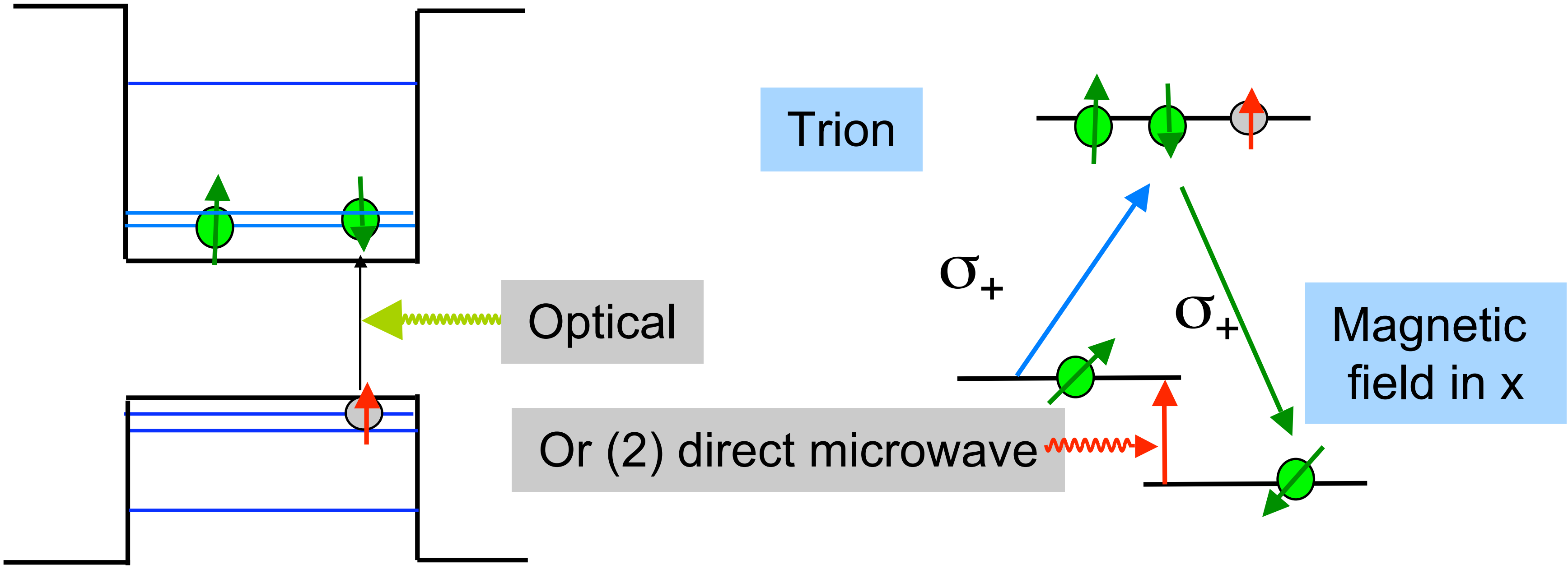
One electron spin state in a SAQD

Single particle levels

Spin and trion states

Spin State to Trion

(1) Adiabatic NR Raman Spin-flip Process



Trion

Optical

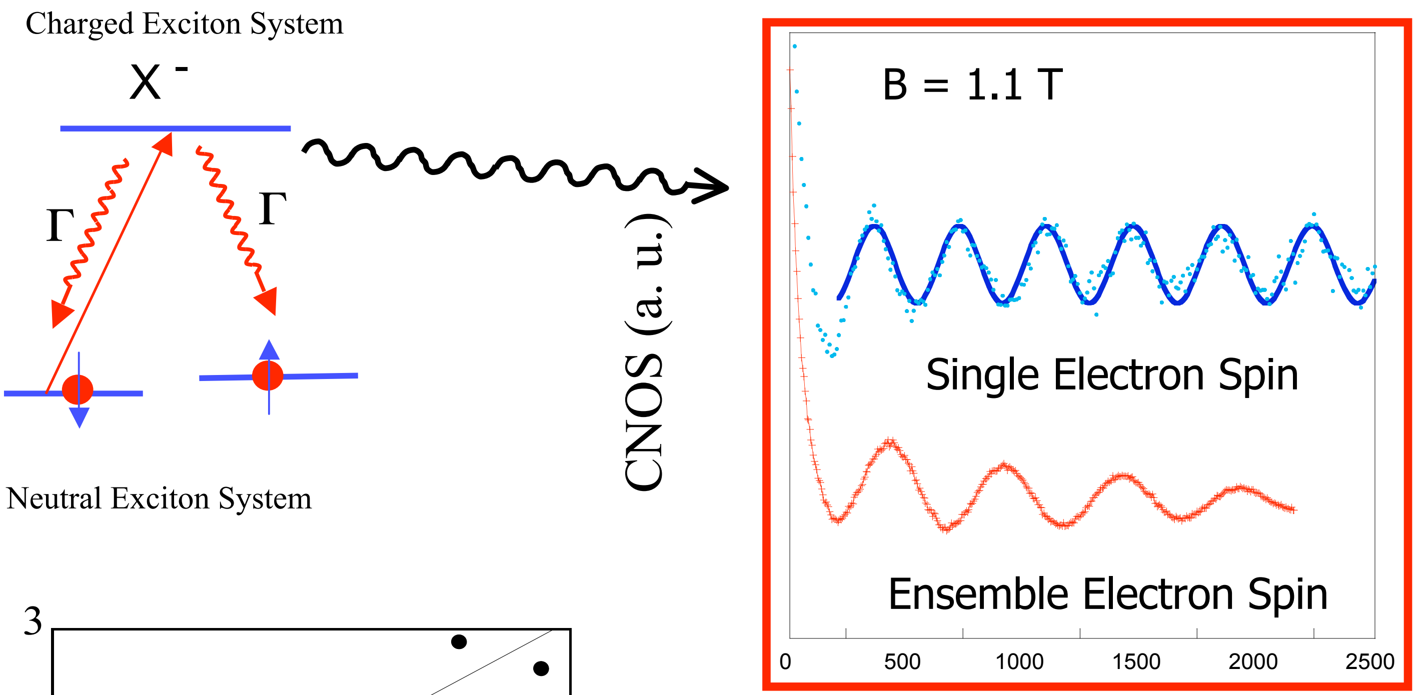
Magnetic field in x

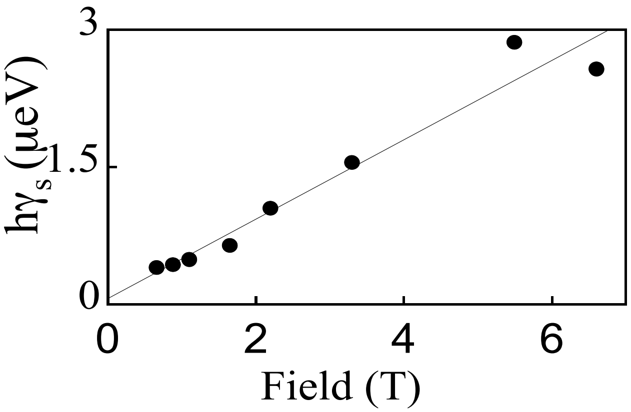
Or (2) direct microwave

Arbitrary rotation of the spin state -- single qubit gate

Pochung Chen, C. Piermarocchi, L.J. Sham, D. Gammon, and D.G. Steel, PRB **69**, 075320 (2004)

Single Electron Spin Coherence: Raman Quantum Beats



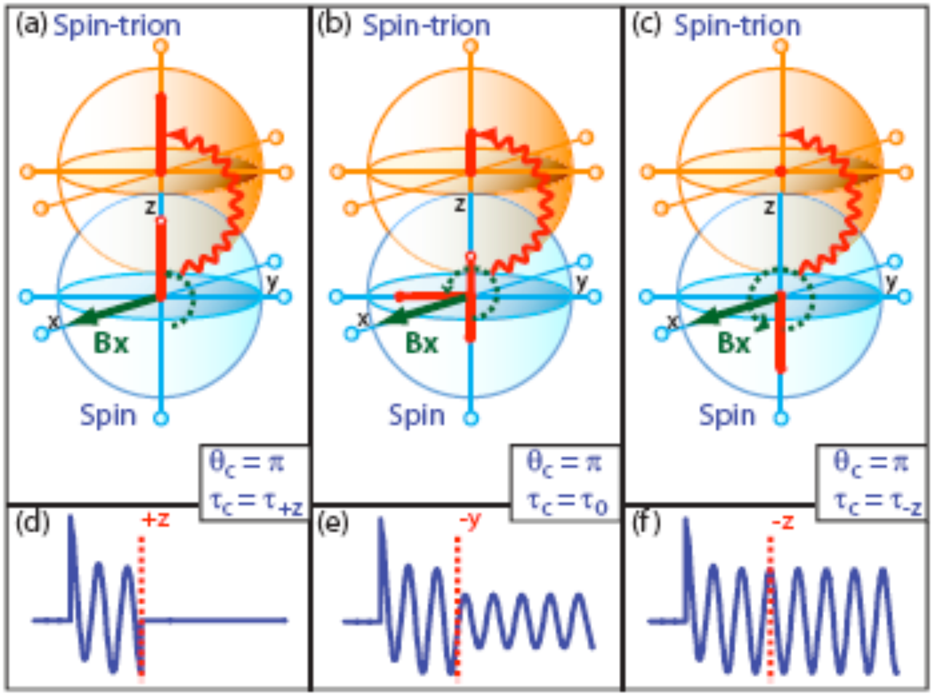


T_2^* >10 nsec at B=0

Dutt *et al Phys. Rev. Let.* - 2005
Petta et al Science 2005
Bracker et al. 2005

Rabi rotation

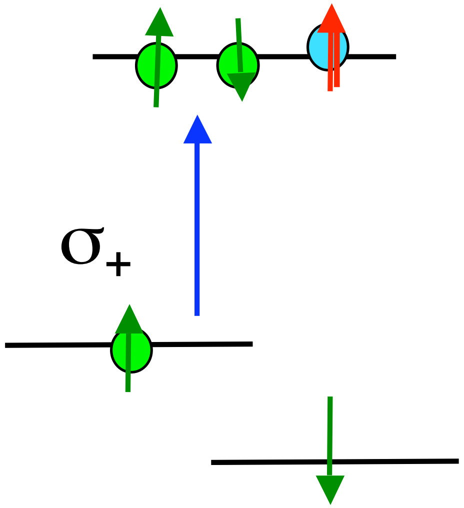
Control of single spin with single optical pulse



+z

Precession about x, measurement along z, pulse rotation about z

Trion



σ_+

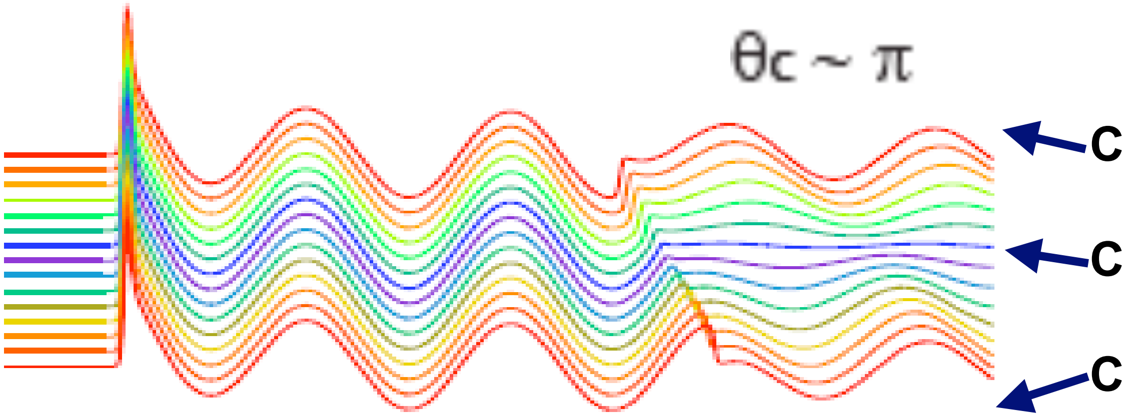
Magnetic field in x

S. Economou, L. J. Sham, Yanwen Wu, and D. G. Steel, PRB 2006

Expt

Spin Coherence Modulated Trion Excitation (taking background into account)

Calculated

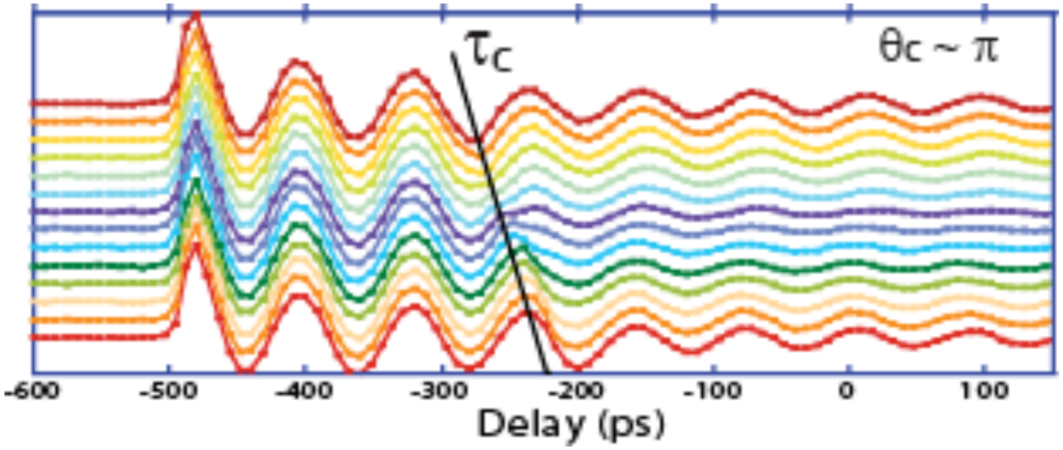


Contol at the -Z

Control at the -Y

Contro at the +Z

Measured



Yanwen Wu, Erik Kim, Xiaodong Xu, Jun Cheng, Steel, Bracker, Gammon, Economou & Sham, PRL 2007

An entangling gate with two interacting qubits

In the computational basis $|00\rangle\ |01\rangle\ |10\rangle\ |11\rangle$

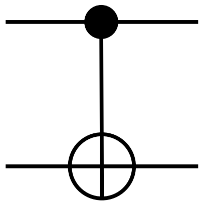
General SWAP

$$S(\alpha,\hat{y}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha/2) & -\sin(\alpha/2) & 0 \\ 0 & \sin(\alpha/2) & \cos(\alpha/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

SWAP ($\alpha = \pi$)

sqrt SWAP ($\alpha = \pi/2$)
entangling

Controlled-NOT Gate



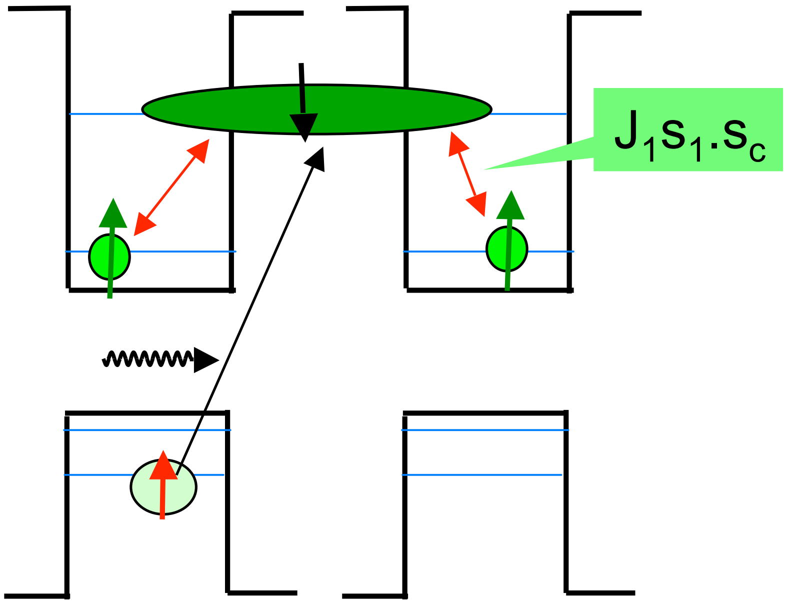
Rot q2 conditional on q1

$$\mathrm{CROT}_{12} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Theory

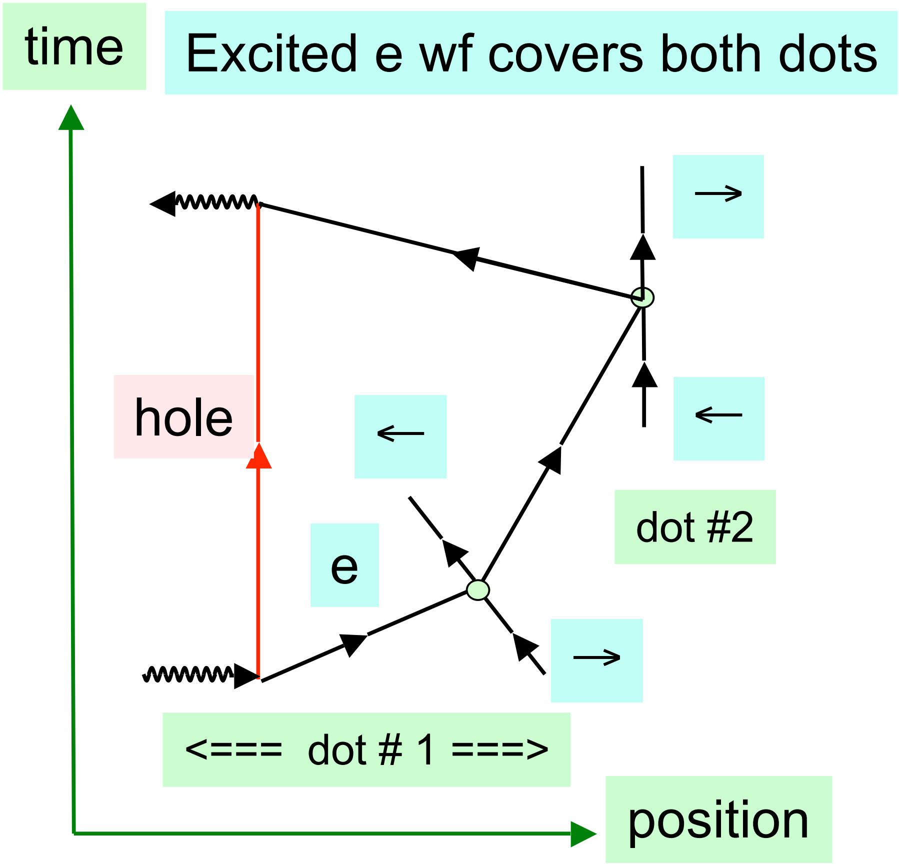
Controlling spin interaction between two electrons in two dots

Continuum or tunnel exciton



Single particle levels

ORKKY or Bloombergen-Roland



Excited e wf covers both dots

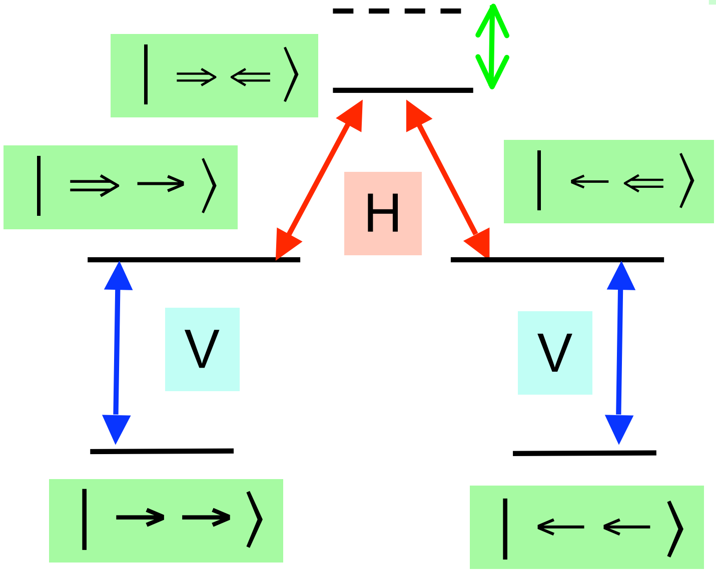
Pochung Chen, Piermarocchi, Sham, & Steel, PRL 02

Emary & Sham, PRB 07

Theory

Optical control of two dot-spins via 2 trions

Spin and trion states



$|\Rightarrow\Leftarrow\rangle$

$|\Rightarrow\rightarrow\rangle$

$|\leftarrow\Leftarrow\rangle$

H

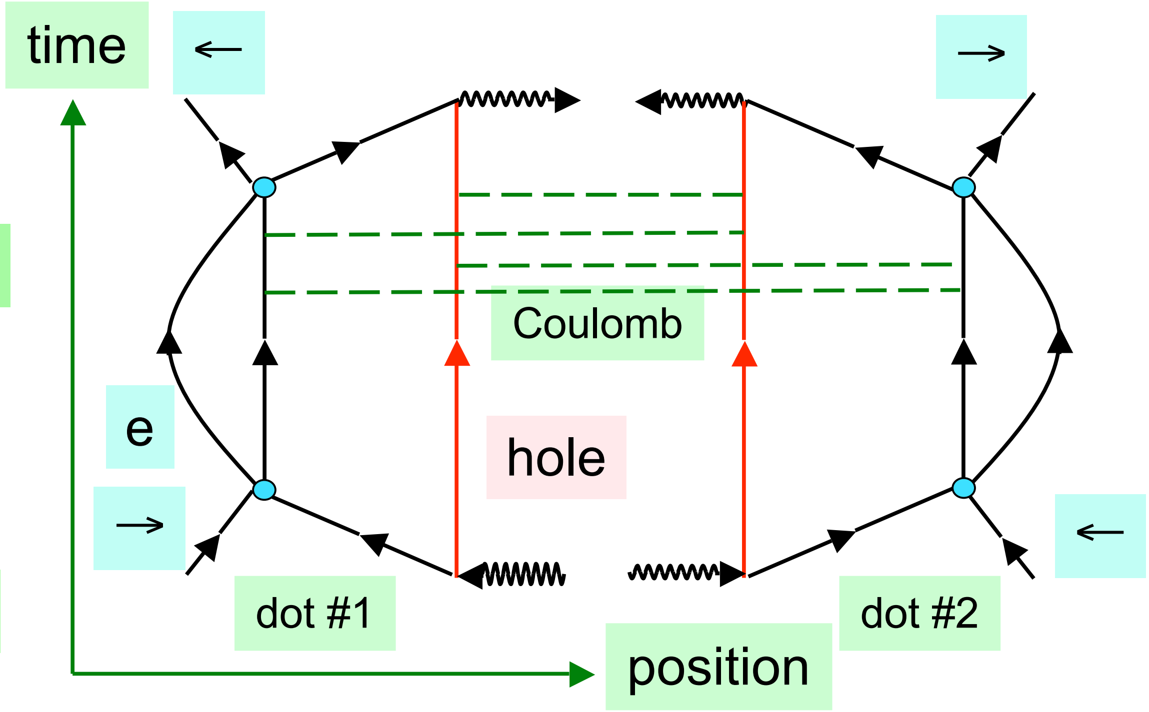
V

V

$|\rightarrow\rightarrow\rangle$

$|\leftarrow\leftarrow\rangle$

Two trions have Coulomb interaction



time

$\leftarrow$

$\rightarrow$

Coulomb

e

$\rightarrow$

hole

$\leftarrow$

dot #1

dot #2

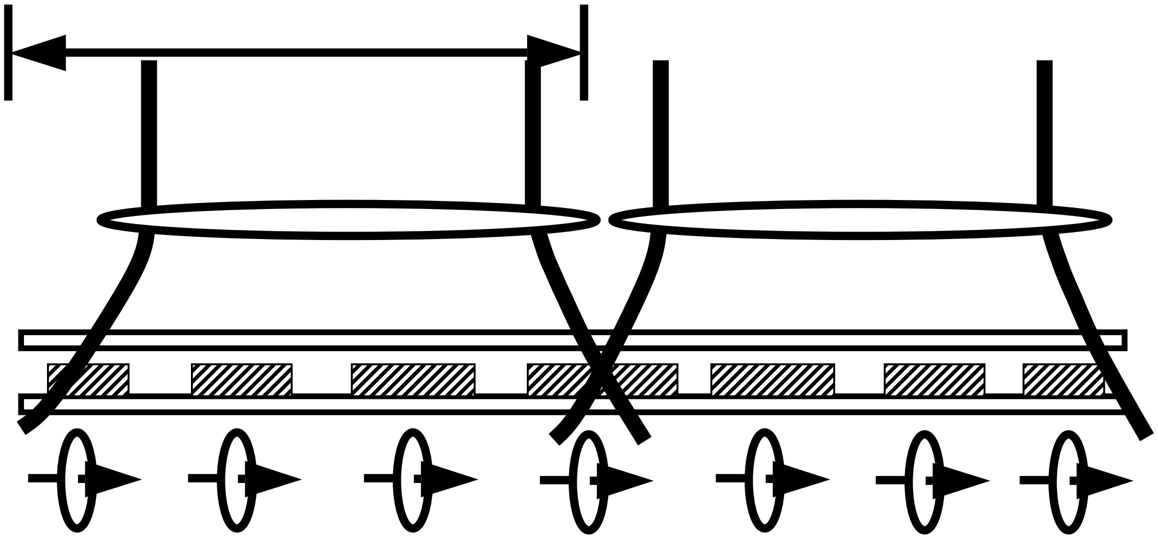
position

Four beams of light

e wfs confined to each dot

Cf. ORKKY: less demand on dot fabrication, more on optics

Scaling: Architecture of a 7 bit QC



Within each zipcode, each dot is addressed by its frequency

Resource estimate

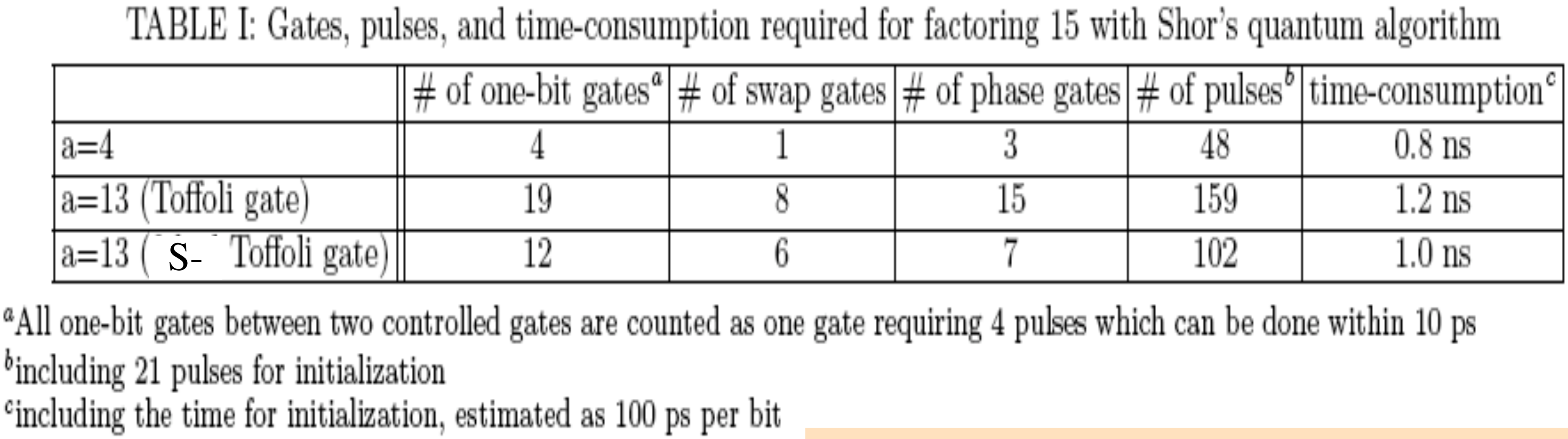


TABLE I: Gates, pulses, and time-consumption required for factoring 15 with Shor's quantum algorithm

	# of one-bit gates[a]	# of swap gates	# of phase gates	# of pulses[b]	time-consumption[c]
a=4	4	1	3	48	0.8 ns
a=13 (Toffoli gate)	19	8	15	159	1.2 ns
a=13 (S- Toffoli gate)	12	6	7	102	1.0 ns

[a] All one-bit gates between two controlled gates are counted as one gate requiring 4 pulses which can be done within 10 ps

[b] including 21 pulses for initialization

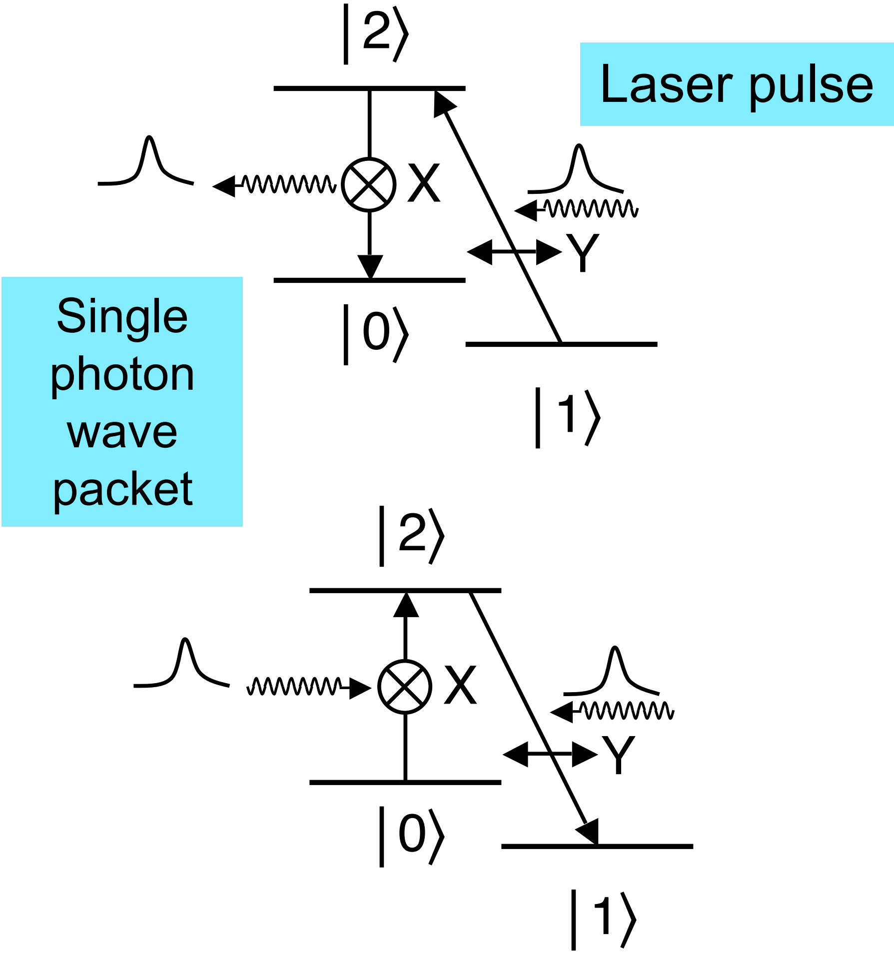
[c] including the time for initialization, estimated as 100 ps per bit

Renbao Liu and L. J. Sham, unpublished.

To built a scalable system: Qubit conversion

- CQED and Q-Net pioneered by
  - Cirac, Zoller, Kimble & Mabuchi, PRL 78, 3221 (1997).
- Control - deterministic in CQED
  - Adiabatic control: Fleischhaeur, Yelin & Lukin, Opt. Comm 179, 395 (2000)
  - Adiabatic control: Duan, Kuzmich & Kimble, PRA 67, 032305 (2003)
  - Non-adiabatic: Yao, Liu & Sham PRL 95, 030504 (2005)

Control processes deterministic

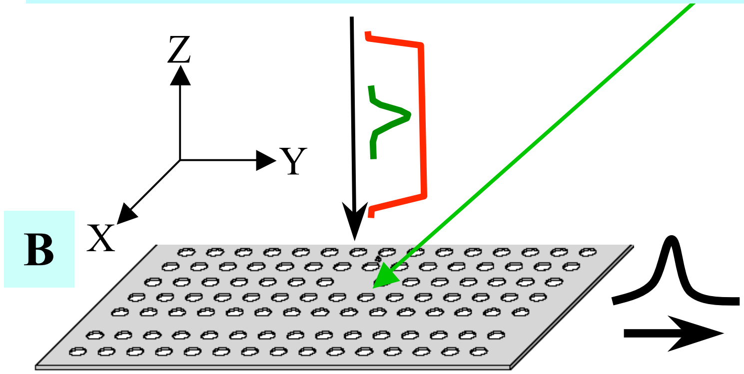


Laser pulse

Single photon wave packet

Cavity-dot-wave guide for solid state CQED

Cavity containing a cluster of dots



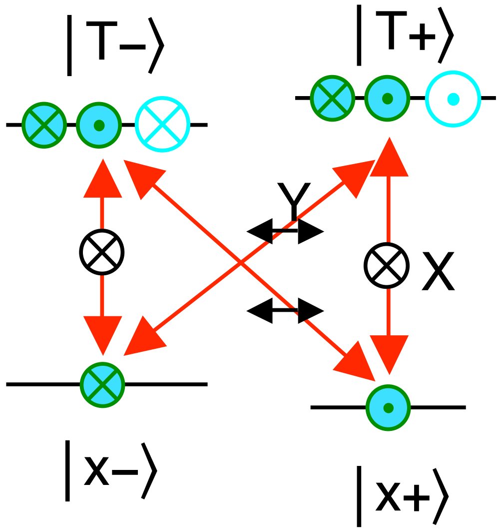
Deterministic process driven by Y pulse (2)

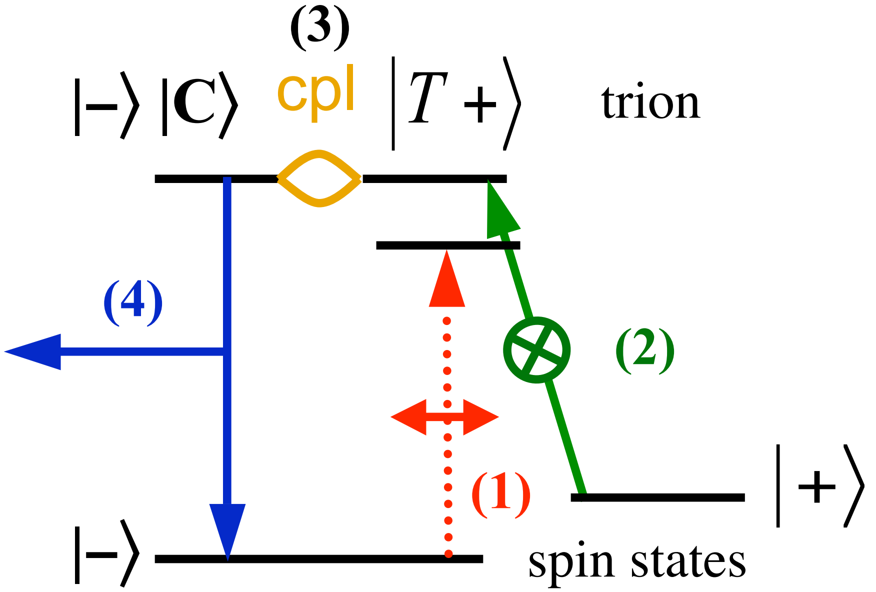
1. Pulse moves |T+⟩ to resonance with |–⟩ |C⟩
2. Y pulse (macro) transforms |+⟩ to |T+⟩
3. |T+⟩ evolves to |–⟩ |C⟩ generating a photon
4. **Photon (micro) moves along wave guide**

Spin-photon state swap

$|+\rangle\,|vac\rangle \rightarrow |-\rangle\,|\alpha\rangle$

Selection rules





Consequences of spin-photon swap

$$|+\rangle\,|\mathrm{vac}\rangle \rightarrow |-\rangle\,|\alpha\rangle$$

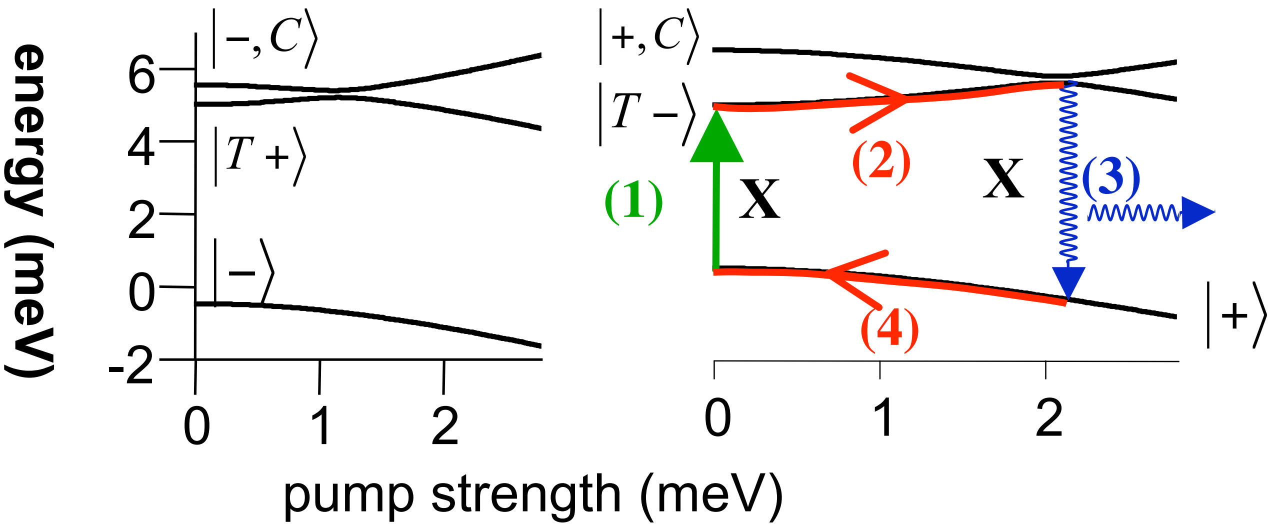
$$[\beta_+|+\rangle + \beta_-|-\rangle]\,|\mathrm{vac}\rangle \rightarrow |-\rangle\,[\beta_+|\alpha\rangle + \beta_-|\mathrm{vac}\rangle]$$

- A stationary qubit & a flying qubit exchanging info.
- Initialization
  - Reduce an unpolarized state to a spin state, $|-\rangle$, say.
  - Basic process: Wave guide serves as entropy dump
- Entanglement of a spin and a photon

$$|+\rangle|\mathrm{vac}\rangle \rightarrow [|+\rangle\,|\mathrm{vac}\rangle + |-\rangle|\alpha\rangle]/\sqrt{2}$$

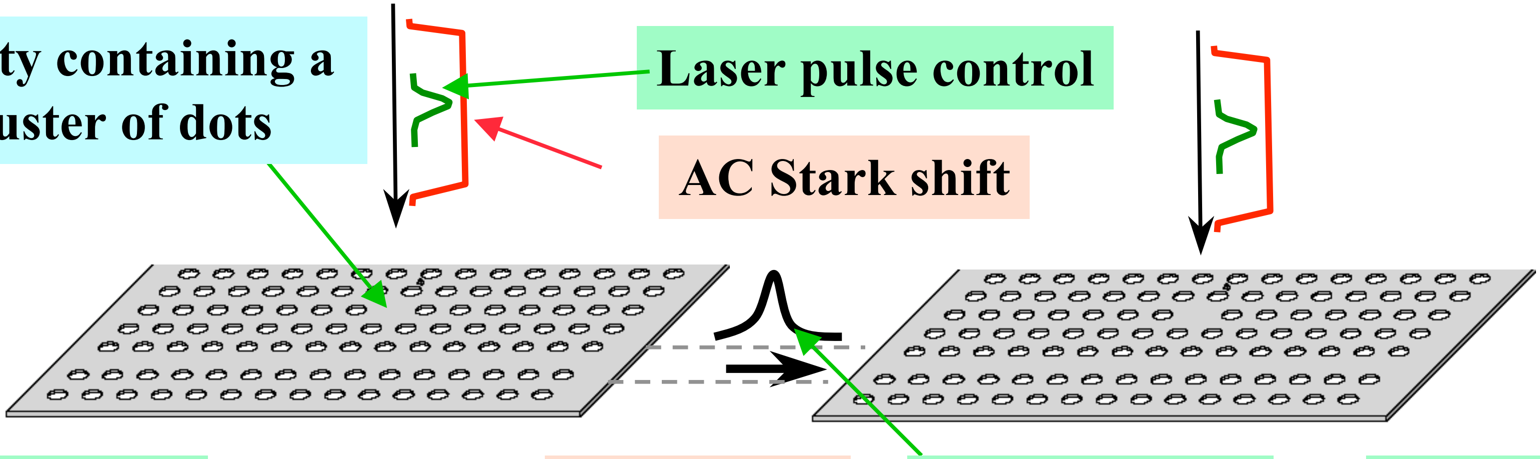
Liu, Yao, Sham, PRB **72,** 081306 (R) (2005)

Quantum Non-Demolition (QND) Measurement of n Spins



- Projective measurement
  - – If there is no photon output, the spin state is $|-\rangle$.
  - – If there is a photon, the spin state is $|+\rangle$.
- QND - The spin state unchanged between measurements.
  - – Hence, can be cycled many times to collect photons.
- Nonideal measurements can be analyzed by POVM.

Solid state quantum computing network



Cavity containing a cluster of dots

Laser pulse control

AC Stark shift

Sending

No actual gap

Photon packet

Receiving

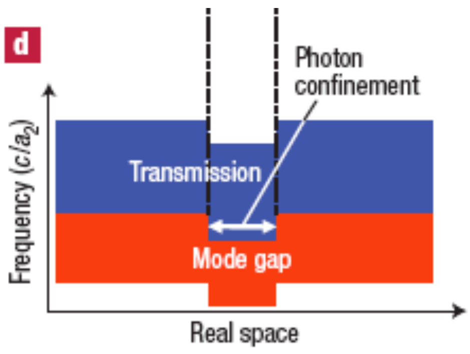
- Send: Optical control of the spin qubit in the dot via trion & cavity mode generates a photon wave packet
  - entangling the spin qubit with the (0,1) photon states
- Receive: reverse optical pulse to absorb photon completely
  - net: entangling sender spin qubit with receiver spin qubit
- Basis for distributed computation to scale up a Q computer

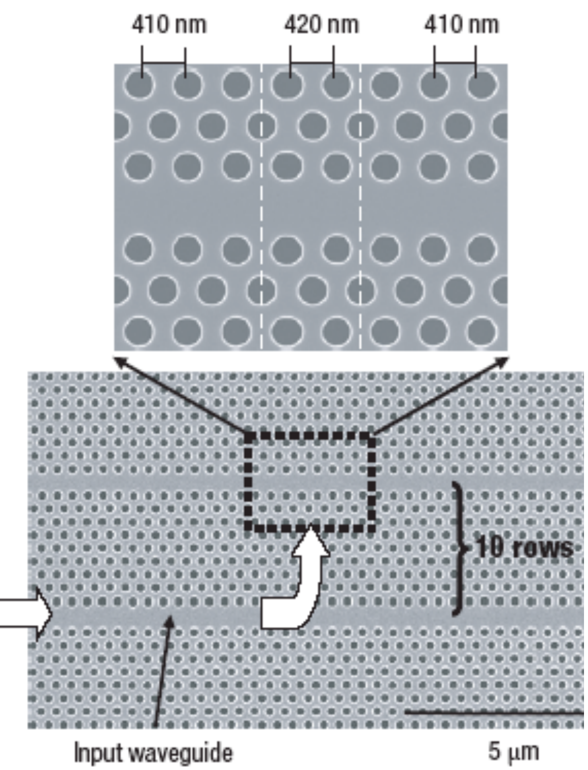
Wang Yao, Renbao Liu, and L. J. Sham, PRL **95**, 030504 (2005), PRB **72,** 081306 (R) (2005), J. Opt. B: Quantum Semiclass. Opt. **7,** S318 (2005).

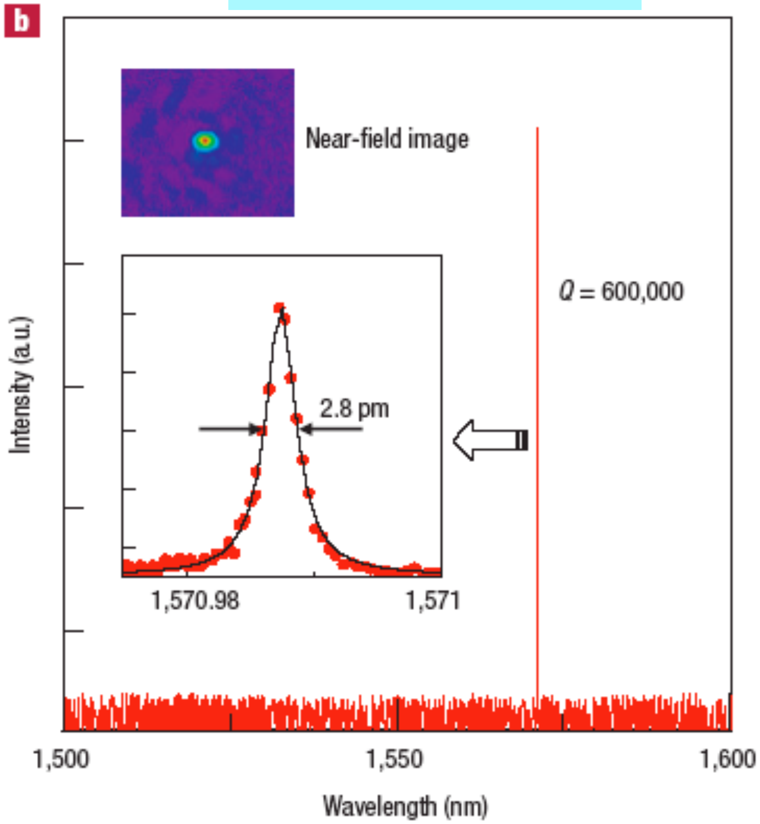
Cavity and wave guide in photonic lattice

Photonic double-heterostructure

$Q = 6 \times 10^5$







c = 250 nm

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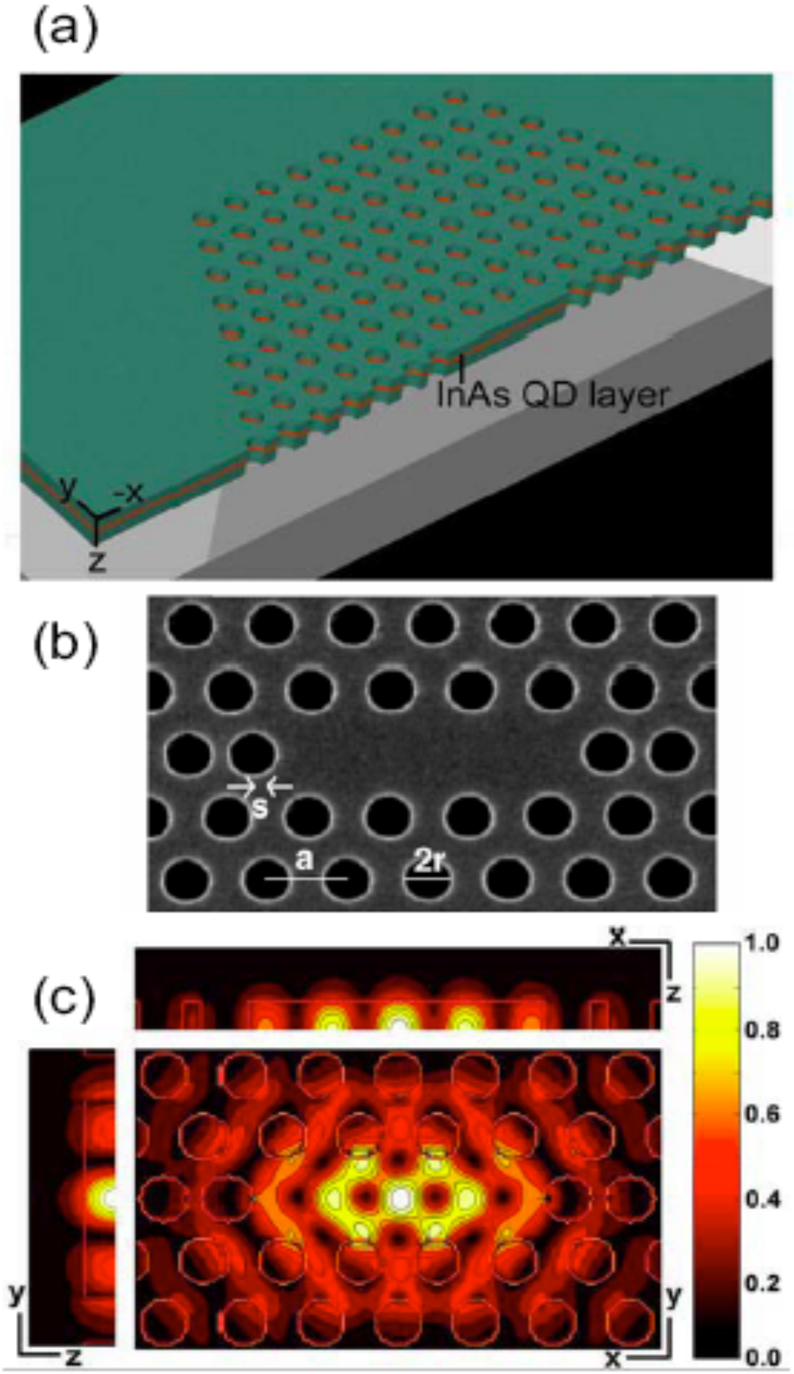
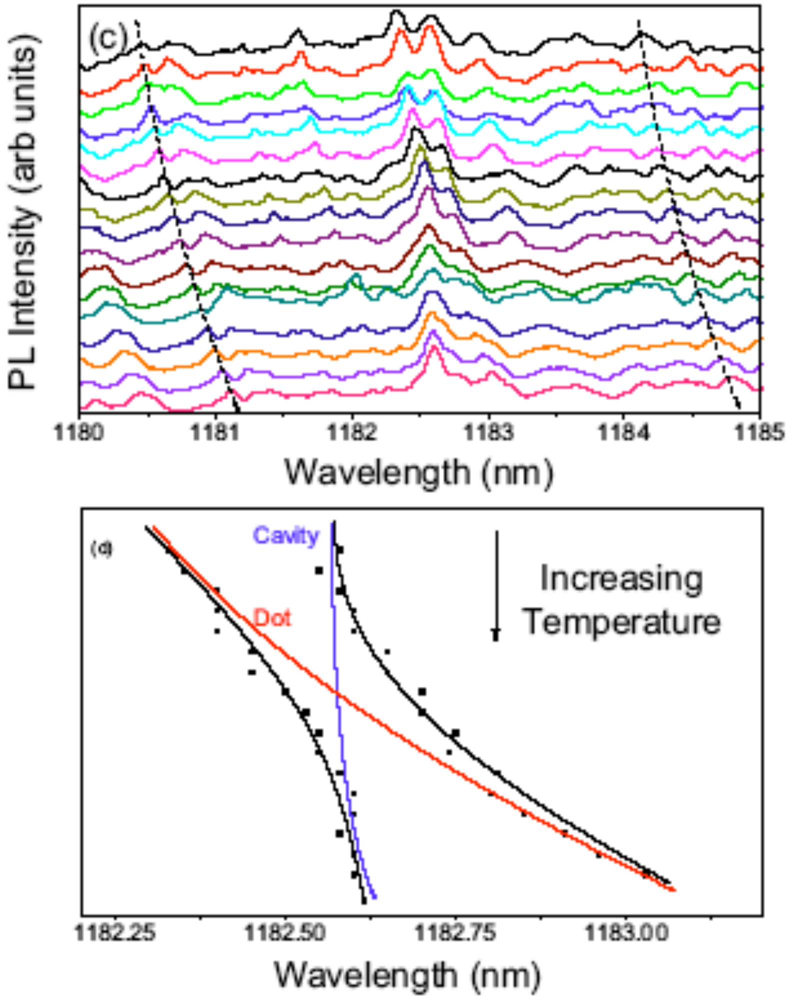
Evidence for Strong Coupling CQED





Yoshie, Schere, Hendrickson, Khitrova, Gibbs,Ruppe, Ell, Shchekin, Deppe , Nature 04