Introdu 00000	Etion         Beyond linear dispersive           00         0000	Results 000	Conclusion O
	Circuit quantum electrodyn	amics : beyond	the
	linear dispersive	regime	

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June 23<sup>th</sup>, 2008



Boissonneault, Gambetta and Blais, *Phys. Rev. A* 77 060305 (R) (2008)

Introduction	Beyond linear dispersive	Results	Conclusion
0000000	0000	000	0

1 Introduction

#### Atom and cavity

- Cavity QED
- Charge qubit and coplanar resonator
- Circuit QED
- The linear dispersive limit
- Circuit VS cavity QED
- 2 Beyond linear dispersive
  - Understanding the dispersive transformation
  - The dispersive limit
  - Dissipation in the system
  - Dissipation in the transformed basis

#### 3 Results

- Reduction of the SNR
- Measurement induced heat bath
- The case of the transmon

#### 4 Conclusion

Conclusion

Introduction ••••••	Beyond linear dispersive	Results	Conclusion O
Atom and cavity			
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Energy	 ;		

Two-levels system Hamiltonian

$$H = \frac{\omega_a}{2} \sigma_z \qquad \sigma_z = \begin{pmatrix} 1 & 0\\ 0 & -1 \end{pmatrix}$$





Maxime Boissonneault



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Université de Sherbrooke

Introduction	Beyond linear dispersive	Results	Conclusion O
Cavity QED			
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Cavity QED			
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Ator	n-cavity interaction		
	$H_I = -\vec{D} \cdot \vec{E} \approx a(a^{\dagger} + a)\sigma_T$	$\approx a(a^{\dagger}\sigma_{-} + a\sigma_{+})$	

$$g(z) = -d_0 \sqrt{\frac{\omega}{V\epsilon_0}} \sin kz$$

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Cavity QED			
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Atom-cavit	ty interaction		

$$H_I = -\vec{D} \cdot \vec{E} \approx g(a^{\dagger} + a)\sigma_x \approx g(a^{\dagger}\sigma_- + a\sigma_+)$$
$$g(z) = -d_0 \sqrt{\frac{\omega}{V\epsilon_0}} \sin kz$$

## Jaynes-Cummings Hamiltonian

$$H = \frac{\omega_a}{2}\sigma_z + \omega_r a^{\dagger}a + g(a^{\dagger}\sigma_- + a\sigma_+)$$

Jaynes and Cummings, *Proc. IEEE* **51** 89-109 (1963) Raimond, Brune and Haroche, *Rev. Mod. Phys.* **73** 565–582 (2001) Mabuchi and Doherty, *Science* **298** 1372-1377 (2002)

Introduction

Introduction	Beyond linear dispersive	Results	Conclusion
00000	0000	000	0
Charge qubit and coplanar resonato			



# Classical Hamiltonian $H = 4E_C(n - n_g)^2 - E_J \cos \delta$ $E_C = \frac{e^2}{2(C_g + C_J)}, \qquad n_g = \frac{C_g V_g}{2e}$ $E_J = \frac{I_0 \Phi_0}{2\pi}$

Introduction	Beyond linear dispersive	Results	Conclusion
00000	0000	000	0
Charge qubit and coplanar resonator			





## Classical Hamiltonian $H = 4E_C (n - n_g)^2 - E_J \cos \delta$ $E_C = \frac{e^2}{2(C_g + C_J)}, \qquad n_g = \frac{C_g V_g}{2e}$ $E_J = \frac{I_0 \Phi_0}{2\pi}$

## Quantum Hamiltonian

$$\begin{split} H &= \sum_{n} 4E_{C}(n-n_{g})^{2} \left| n \right\rangle \left\langle n \right| \\ &- \sum_{n} \frac{E_{J}}{2} (\left| n \right\rangle \left\langle n+1 \right| + \text{h.c.}) \end{split}$$

Restricting to  $n_g \in [0,1]$  :  $H = \omega_a \sigma_z/2$ 

Shnirman, Schön and Hermon, *Phys. Rev. Lett.* **79** 2371–2374 (1997) Bouchiat *et al.*, *Physica Scripta* **T76** 165-170 (1998) Nakamura, Pashkin and Tsai, *Nature (London)* **398** 786 (1999)

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Introduction	Beyond linear dispersive	Results	Conclusion
00000	0000	000	0
Charge qubit and coplanar resonator			





## Quantum Hamiltonian

$$H = \sum_{n} 4E_C (n - n_g)^2 |n\rangle \langle n|$$
$$-\sum_{n} \frac{E_J}{2} (|n\rangle \langle n + 1| + \text{h.c.})$$

Restricting to  $n_g \in [0,1]$  :  $H = \omega_a \sigma_z/2$ 

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Introduction	Beyond linear dispersive	Results	Conclusion	

#### Charge qubit and coplanar resonator





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Classical Hamiltonian $H = \frac{\Phi^2}{2L_r} + \frac{1}{2}C_rV^2$  $\omega_r = \sqrt{\frac{1}{L_rC_r}}$ 

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Classical Hamiltonian
$$H = \frac{\Phi^2}{2L_r} + \frac{1}{2}C_rV^2$$
$$\omega_r = \sqrt{\frac{1}{L_rC_r}}$$

## Quantum Hamiltonian

$$V = \sqrt{\frac{\omega_r}{2C_r}}(a^{\dagger} + a), \qquad \Phi = i\sqrt{\frac{\omega_r}{2L_r}}(a^{\dagger} - a)$$

$$H = \omega_r \left( a^{\dagger} a + \frac{1}{2} \right)$$

Quantum Fluctuations in Electrical Circuits, M. H. Devoret, Les Houches Session LXIII, Quantum Fluctuations p. 351-386 (1995).

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#### Parameters

- g : Qubit-cavity interaction
- $\omega_a$  : Qubit frequency
- $\omega_r$  : Resonator frequency
- $\Delta = \omega_a \omega_r$  : Detuning

$$H = \frac{\omega_a}{2}\sigma_z + \omega_r a^{\dagger}a + g(a^{\dagger}\sigma_- + a\sigma_+)$$

- Blais et al., Phys. Rev. A 69 062320 (2004)
- Wallraff et al., Nature 431 162 (2004)
- Wallraff et al., Phys. Rev. Lett. 95 060501 (2005)
- Leek et al., Science 318 1889 (2007)
- Schuster et al., Nature 445 515 (2007)
- Houck et al., Nature 449 328 (2007)
- Majer *et al.*, *Nature* **449** 443 (2007)

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Circuit QED			
Paramet	ters	Blais et al Phys Rev A 69 0	62320 (2004)
■ g : G	Qubit-cavity interaction	<ul> <li>Wallraff et al., Nature 431 162</li> </ul>	2 (2004)
$\blacksquare \omega_a$ :	Qubit frequency	<ul> <li>Wallraff et al., Phys. Rev. Lett (2005)</li> </ul>	. <b>95</b> 060501
$lacksquare$ $\omega_r$ :	Resonator frequency	Leek et al., Science <b>318</b> 1889	(2007)
$\bullet \ \Delta =$	$\omega_a - \omega_r$ : Detuning	Schuster et al., Nature 445 51	5 (2007)
	$\omega_a$ , $\dagger$ , $(\dagger$ , )	■ Houck et al., Nature 449 328	(2007)

- $H = \frac{\omega_a}{2}\sigma_z + \omega_r a^{\dagger}a + g(a^{\dagger}\sigma_- + a\sigma_+)$
- Majer et al., Nature 449 443 (2007)

Introduction	Beyond linear dispersive	Results	Conclusion		
0000000	0000	000	0		
The linear dispersive limit					

## Jaynes-Cummings $H = \omega_r a^{\dagger} a + \omega_a \frac{\sigma_z}{2} + g(a^{\dagger} \sigma_- + a \sigma_+)$ Small parameter $\lambda = g/\Delta$

Qubit control and readout - 10 GHz Cavity: super conducting 1D transmission line resonator

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Introduction	Beyond linear of	lispersive	Results	Conclusion
0000000	0000			0
The linear dispersive limit				
Javnes-Cumm	nings	Linear	r dispersive	
Jaynes-Cumm	nings	Linear	r dispersive	

$$\begin{split} H = \omega_r a^\dagger a + \omega_a \frac{\sigma_z}{2} + g(a^\dagger \sigma_- + a \sigma_+) \\ \text{Small parameter } \lambda = g/\Delta \end{split}$$



Linear dispersive  

$$H^{D} = (\omega_{a} + \chi) \frac{\sigma_{z}}{2} + (\omega_{r} + \chi \sigma_{z}) a^{\dagger} a$$
Lamb shift  $(\chi = g\lambda = g^{2}/\Delta)$ 
Stark shift or cavity pull  
Valid if  $\bar{n} \ll n_{crit.}$ , where  $n_{crit.} = 1/4\lambda^{2}$ .



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transmission line resonator

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Maxime Boissonneault

Introduction	Beyond linear dispersive	Results	Conclusion
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Circuit VS cavity QED			

Symbol	Optical cavity	Microwave cavity	Circuit
$\omega_r/2\pi$ or $\omega_a/2\pi$	350 THz	51 GHz	10 GHz
$g/\pi$	220 MHz	47 kHz	100 MHz
$g/\omega_r$	$3 \times 10^{-7}$	$10^{-7}$	$5 \times 10^{-3}$

Hood et al., Science 287 1447 (2000)

Raimond, Brune and Haroche, Rev. Mod. Phys. 73 565-582 (2001)

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## Motivation

• Circuit QED is harder than cavity QED on the dispersive limit ( $n_{\rm crit.}$  is smaller)

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Introduction	Beyond linear dispersive	Results	Conclusion
000000	0000	000	0
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- Circuit QED is harder than cavity QED on the dispersive limit ( $n_{\rm crit.}$  is smaller)
- The SNR is low, we want to measure harder... how does higher order terms affect measurement ?

Introduction	Beyond linear dispersive	Results	Conclusion
000000	0000	000	0
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## Motivation

- Circuit QED is harder than cavity QED on the dispersive limit ( $n_{\rm crit.}$  is smaller)
- The SNR is low, we want to measure harder... how does higher order terms affect measurement ?
- Must consider higher order corrections in perturbation theory

Introduction	Beyond linear dispersive	Results	Conclusion
0000000	●000	000	0
Understanding the dispersiv	e transformation		



$$\blacksquare H = \omega_r a^{\dagger} a + \omega_a \sigma_z / 2 + g(a^{\dagger} \sigma_- + a \sigma_+)$$



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## J-C : block diagonal

$$\blacksquare H = \omega_r a^{\dagger} a + \omega_a \sigma_z / 2 + g(a^{\dagger} \sigma_- + a \sigma_+)$$

- 1x1 block :  $H_0 = -\omega_a \mathbb{I}/2$
- 2x2 blocks :  $H_n = \frac{\Delta}{2}\sigma_z^n + g\sqrt{n}\sigma_x^n$
- Total Hamiltonian  $H = H_0 \oplus H_1 \oplus H_2 \cdots \oplus H_{\infty}$  $|0\downarrow\rangle$  $|0\uparrow\rangle$  $|1\downarrow\rangle$  $|1\uparrow\rangle$  $|2\downarrow\rangle$  $\omega_a$  $\langle 0 \downarrow |$ 0 0 0 0 . . . 2  $\frac{\omega_a}{2}$  $\langle 0 \uparrow |$ 0 0 0 g. . . \_\_\_\_  $\frac{\omega_a}{2}$  $H=\left. \left\langle 1\downarrow \right| \right.$ 0 0 0 g $\langle 1 \uparrow |$  $\frac{\omega_a}{2}$  $\sqrt{2}g$ 0 0 0 . . .  $\sqrt{2}g$  $\omega_a$  $\langle 2 \downarrow |$ 0 0 0 ... ÷ ÷ ÷ ٠.

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Introduction 0000000	Beyond linear dispersive ●○○○	Results	Conclusion O
Understanding the dispers	ive transformation		
J-C : bloc	k diagonal	ZA	(齐

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$$\theta_n = \arctan(2g\sqrt{n}/\Delta)$$

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Introduction	Beyond linear dispersive	Results	Conclusion
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Understanding the disper	sive transformation		
J-C : bloc	ck diagonal	Z <b>↑</b> ∡∄	

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## Diagonalization

- Rotation around Y axis
- In all subspaces  $\mathcal{E}_n$   $\mathcal{E}_0 = \{|g, 0\rangle\}$  $\mathcal{E}_n = \{|g, n\rangle, |e, n - 1\rangle\} \equiv \{|g^n\rangle, |e^n\rangle\}$



$$\theta_n = \arctan(2g\sqrt{n}/\Delta)$$



Introduction	Beyond linear dispersive	Results	Conclusion
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Understanding the dispersive transformation			
$I_{\rm C} \cdot hloc$	sk diagonal		

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- The qubit is now part photon and vice-versa



Introduction	Beyond linear dispersive	Results	Conclusion
0000000	0000	000	0
Understanding the dispersive transformation			

## J-C : block diagonal

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### Diagonalization

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Introduction	Beyond linear dispersive	Results	Conclusion
000000	0000	000	0
The dispersive limit			

## **Dispersive** limit

Jaynes-Cummings hamiltonian

$$H = \omega_r a^{\dagger} a + \omega_a \frac{\sigma_z}{2} + g(a^{\dagger} \sigma_- + a \sigma_+)$$

- Exact transformation : D

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Introduction	Beyond linear dispersive	Results	Conclusion
000000	0000	000	0
The dispersive limit			

## **Dispersive** limit

Jaynes-Cummings hamiltonian

$$H = \omega_r a^{\dagger} a + \omega_a \frac{\sigma_z}{2} + g(a^{\dagger} \sigma_- + a \sigma_+)$$

- Exact transformation : D
- Small parameter  $\lambda = g/\Delta 4\lambda^2 \bar{n} \ll 1$





Introduction	Beyond linear dispersive	Results	Conclusion
0000000	○●○○		O
The dispersive limit			

## **Dispersive** limit

Jaynes-Cummings hamiltonian

$$H = \omega_r a^{\dagger} a + \omega_a \frac{\sigma_z}{2} + g(a^{\dagger} \sigma_- + a\sigma_+)$$

- Exact transformation : D
- $\begin{tabular}{ll} \mbox{Small parameter } \lambda = g/\Delta \\ 4\lambda^2 \bar{n} \ll 1 \end{tabular} \end{tabular}$

Result at order 
$$\lambda$$
  
 $H^D = (\omega_a + \chi) \frac{\sigma_z}{2} + (\omega_r + \chi \sigma_z) a^{\dagger} a$   
Lamb shift  $(\chi = g\lambda = g^2/\Delta)$   
Stark shift or cavity pull

Result at order 
$$\lambda^2$$
  
 $H^D = (\omega_a + \chi') \frac{\sigma_z}{2} + [\omega_r + (\chi' - \zeta a^{\dagger} a) \sigma_z] a^{\dagger} a$   
 $\chi' = \chi(1 - \lambda^2) \zeta = \lambda^2 \chi$   
The cavity pull decrease :  $\langle CP \rangle = \chi' - \zeta \langle a^{\dagger} a \rangle$ 



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Introduction	Beyond linear dispersive	Results	Conclusion
0000000	0000	000	0
Dissipation in the system			



## Model for dissipation

• Coupling to a bath 
$$\begin{split} H_{\kappa} &= \int_{0}^{\infty} \sqrt{g_{\kappa}(\omega)} [b_{\kappa}^{\dagger}(\omega) + b_{\kappa}(\omega)] [a^{\dagger} + a] d\omega \\ H_{\gamma} &= \int_{0}^{\infty} \sqrt{g_{\gamma}(\omega)} [b_{\gamma}^{\dagger}(\omega) + b_{\gamma}(\omega)] \sigma_{x} d\omega \end{split}$$

### Parameters

- $\kappa$  : Rate of photon loss
- $\gamma_1$  : Transverse decay rate



#### Model for dissipation

• Coupling to a bath 
$$\begin{split} H_{\kappa} &= \int_{0}^{\infty} \sqrt{g_{\kappa}(\omega)} [b_{\kappa}^{\dagger}(\omega) + b_{\kappa}(\omega)] [a^{\dagger} + a] d\omega \\ H_{\gamma} &= \int_{0}^{\infty} \sqrt{g_{\gamma}(\omega)} [b_{\gamma}^{\dagger}(\omega) + b_{\gamma}(\omega)] \sigma_{x} d\omega \end{split}$$

## Parameters

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#### Model for dissipation

- Coupling to a bath  $\begin{aligned} H_{\kappa} &= \int_{0}^{\infty} \sqrt{g_{\kappa}(\omega)} [b_{\kappa}^{\dagger}(\omega) + b_{\kappa}(\omega)] [a^{\dagger} + a] d\omega \\ H_{\gamma} &= \int_{0}^{\infty} \sqrt{g_{\gamma}(\omega)} [b_{\gamma}^{\dagger}(\omega) + b_{\gamma}(\omega)] \sigma_{x} d\omega \end{aligned}$
- Dephasing  $H_{\varphi} = \eta f(t) \sigma_z$

### Parameters

- $\kappa$  : Rate of photon loss
- $\gamma_1$  : Transverse decay rate
- $\gamma_{\varphi}$  : Pure dephasing rate

Dissipation in the transfo	rmed basis		
0000000	0000	000	0
Introduction	Beyond linear dispersive	Results	Conclusion





Introduction	Beyond linear dispersive	Results	Conclusion		
000000	0000	000	0		
Dissipation in the transformed basis					



## Transformation of system-bath hamiltonian

$$a \xrightarrow{D} a + \lambda \sigma_{-} + \mathcal{O}\left(\lambda^{2}\right)$$

$$\bullet \ \sigma_{-} \xrightarrow{D} \sigma_{-} + \lambda a \sigma_{z} + \mathcal{O}\left(\lambda^{2}\right)$$

$$\sigma_z \xrightarrow{D} \sigma_z - 2\lambda(a^{\dagger}\sigma_- + a\sigma_+) + \mathcal{O}\left(\lambda^2\right)$$



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Introduction	Beyond linear dispersive	Results	Conclusion	
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Discipation in the transformed basis				



## Transformation of system-bath hamiltonian

$$a \xrightarrow{D} a + \lambda \sigma_{-} + \mathcal{O}\left(\lambda^{2}\right)$$

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• 
$$\sigma_z \xrightarrow{D} \sigma_z - 2\lambda(a^{\dagger}\sigma_- + a\sigma_+) + \mathcal{O}(\lambda^2)$$

## Method

- Transform the system-bath hamiltonian
- Trace out heat bath and cavity degrees of freedom (Gambetta et al., Phys. Rev. A 77 012112 (2008))

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0000000		000	0
Introduction	Beyond linear dispersive	Results	Conclusion



## Transformation of system-bath hamiltonian

$$a \xrightarrow{D} a + \lambda \sigma_{-} + \mathcal{O}\left(\lambda^{2}\right)$$

$$\sigma_{-} \xrightarrow{D} \sigma_{-} + \lambda a \sigma_{z} + \mathcal{O}\left(\lambda^{2}\right)$$

$$\sigma_z \xrightarrow{D} \sigma_z - 2\lambda(a^{\dagger}\sigma_- + a\sigma_+) + \mathcal{O}\left(\lambda^2\right)$$

### Method

- Transform the system-bath hamiltonian
- Trace out heat bath and cavity degrees of freedom (Gambetta et al., Phys. Rev. A 77 012112 (2008))

New rates (assuming white noises) •  $\gamma_{\downarrow} = \gamma_1 \left[ 1 - 2\lambda^2 \left( \bar{n} + \frac{1}{2} \right) \right] + \gamma_{\kappa} + 2\lambda^2 \gamma_{\varphi} \bar{n}$ •  $\gamma_{\uparrow} = 2\lambda^2 \gamma_{\varphi} \bar{n}$ •  $\gamma_{\kappa} = \lambda^2 \kappa$ 

Introduction	Beyond linear dispersive	Results	Conclusion
000000	0000	•00	0
Reduction of the SNR			

Number of measurement photons

 $\mathrm{SNR}\sim$  Num. phot.

Introduction	Beyond linear dispersive	Results	Conclusion
000000	0000	•00	0
Reduction of the SNR			

- Number of measurement photons
- Output rate :  $\kappa$

 $SNR \sim \frac{\kappa}{Num. phot.}$ 

Introduction	Beyond linear dispersive	Results	Conclusion
000000	0000	•00	0
Reduction of the SNR			

- Number of measurement photons
- Output rate :  $\kappa$
- **Fraction of photons detected** :  $\eta$

 $SNR \sim \frac{\kappa \times \eta \times Num. \text{ phot.}}{\kappa \times \eta \times Num. \text{ phot.}}$ 

Introduction	Beyond linear dispersive	Results	Conclusion
000000	0000	●00	0
Reduction of the SNR			

- Number of measurement photons
- Output rate :  $\kappa$
- Fraction of photons detected :  $\eta$
- Info per photon : cavity pull

 $SNR \sim \frac{\kappa \times \eta \times Num. \text{ phot.} \times \text{Info per phot.}}{\kappa \times \eta \times Num. \text{ phot.} \times \text{Info per phot.}}$ 

Introduction	Beyond linear dispersive	Results	Conclusion
000000	0000	●00	0
Reduction of the SNR			

- Number of measurement photons
- Output rate :  $\kappa$
- Fraction of photons detected :  $\eta$
- Info per photon : cavity pull
- Mixing rate :

$$\gamma_{\downarrow} + \gamma_{\uparrow} = \gamma_1 \left[ 1 - 2\lambda^2 \left( \bar{n} + \frac{1}{2} \right) \right] + \gamma_{\kappa} + 4\lambda^2 \gamma_{\varphi} \bar{n}$$

 $\mathrm{SNR} \sim \frac{\kappa \times \eta \times \mathsf{Num. \ phot.} \times \mathsf{Info \ per \ phot.}}{\mathsf{Mixing \ rate}}$ 

Introduction	Beyond linear dispersive	Results	Conclusion
0000000		•00	O
Reduction of the SNR			

- Number of measurement photons
- Output rate :  $\kappa$
- Fraction of photons detected :  $\eta$
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- $\begin{array}{l} \blacksquare \mbox{ Mixing rate }: \\ \gamma_{\downarrow} + \gamma_{\uparrow} = \gamma_1 \left[ 1 2\lambda^2 \left( \bar{n} + \frac{1}{2} \right) \right] + \gamma_{\kappa} + 4\lambda^2 \gamma_{\varphi} \bar{n} \end{array}$

 $\mathrm{SNR} \sim \frac{\kappa \times \eta \times \mathsf{Num. \ phot.} \times \mathsf{Info \ per \ phot.}}{\mathsf{Mixing \ rate}}$ 

## Conclusion

- SNR levels off with non-linear effects !
- Explains low experimental SNR
- Applies to all dispersive homodyne measurement



 $\begin{array}{l} \Delta/2\pi = 1.7 \; {\rm GHz}, \, g/2\pi = 170 \; {\rm MHz} \\ \kappa/2\pi = 34 \; {\rm MHz}, \, \gamma_1/2\pi = 0.1 \; {\rm MHz} \\ \gamma_\varphi = 0.1 \; {\rm MHz}, \, \eta = 1/80 \\ n_{\rm crit.} = 1/4\lambda^2 = 25 \end{array}$ 

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Introduction	Beyond linear dispersive	Results	Conclusion
0000000		• 0 0	O
Reduction of the SNR			

- Number of measurement photons
- Output rate :  $\kappa$
- Fraction of photons detected :  $\eta$
- Info per photon : cavity pull
- $\begin{array}{l} \blacksquare \mbox{ Mixing rate :} \\ \gamma_{\downarrow} + \gamma_{\uparrow} = \gamma_1 \left[ 1 2\lambda^2 \left( \bar{n} + \frac{1}{2} \right) \right] + \gamma_{\kappa} + 4\lambda^2 \gamma_{\varphi} \bar{n} \end{array}$

 $\mathrm{SNR} \sim \frac{\kappa \times \eta \times \mathsf{Num. \ phot.} \times \mathsf{Info \ per \ phot.}}{\mathsf{Mixing \ rate}}$ 

## Conclusion

- SNR levels off with non-linear effects !
- Explains low experimental SNR
- Applies to all dispersive homodyne measurement



 $\begin{array}{l} \Delta/2\pi = 1.7 \; {\rm GHz}, \, g/2\pi = 170 \; {\rm MHz} \\ \kappa/2\pi = 34 \; {\rm MHz}, \, \gamma_1/2\pi = 0.1 \; {\rm MHz} \\ \gamma_\varphi = 0.1 \; {\rm MHz}, \, \eta = 1/80 \\ n_{\rm crit.} = 1/4\lambda^2 = 25 \end{array}$ 

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Measurement induced heat bath			

## Mixing rates

- Downward rate :  $\gamma_{\downarrow}(\bar{n})$
- Upward rate :  $\gamma_{\uparrow}(\bar{n})$
- Heat bath with temperature  $T(\bar{n}) = (\hbar \omega_r / k_B) / \log(1 + 1/\bar{n})$



Introduction	Beyond linear dispersive	Results	Conclusion
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The case of the transmon			





Koch et al., Phys. Rev. A 76 042319 (2007)

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Conclusion			

## Main results

- Simple model describing the physics of measurement
- Side-effect of measuring harder : you heat your qubit (even with photons that can't be directly absorbed)
- Side-effect of measuring harder : each photon you add carries less information than the previous one
- Measuring harder  $\neq$  bigger SNR

## Coming soon

- The transmon (3 level system) (Koch et al., Phys. Rev. A 76 042319 (2007))
- Taking advantage of the non-linearity
- Comparison with experiments

More information : Boissonneault, Gambetta and Blais, *Phys. Rev. A* 77 060305 (R) (2008)





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