BATS-R-US: a Multi-Physics and Multi-Application Code

Bart van der Holst, Gábor Tóth

Center for Space Environment Modeling

Center for RAdiative Shock Hydrodynamics

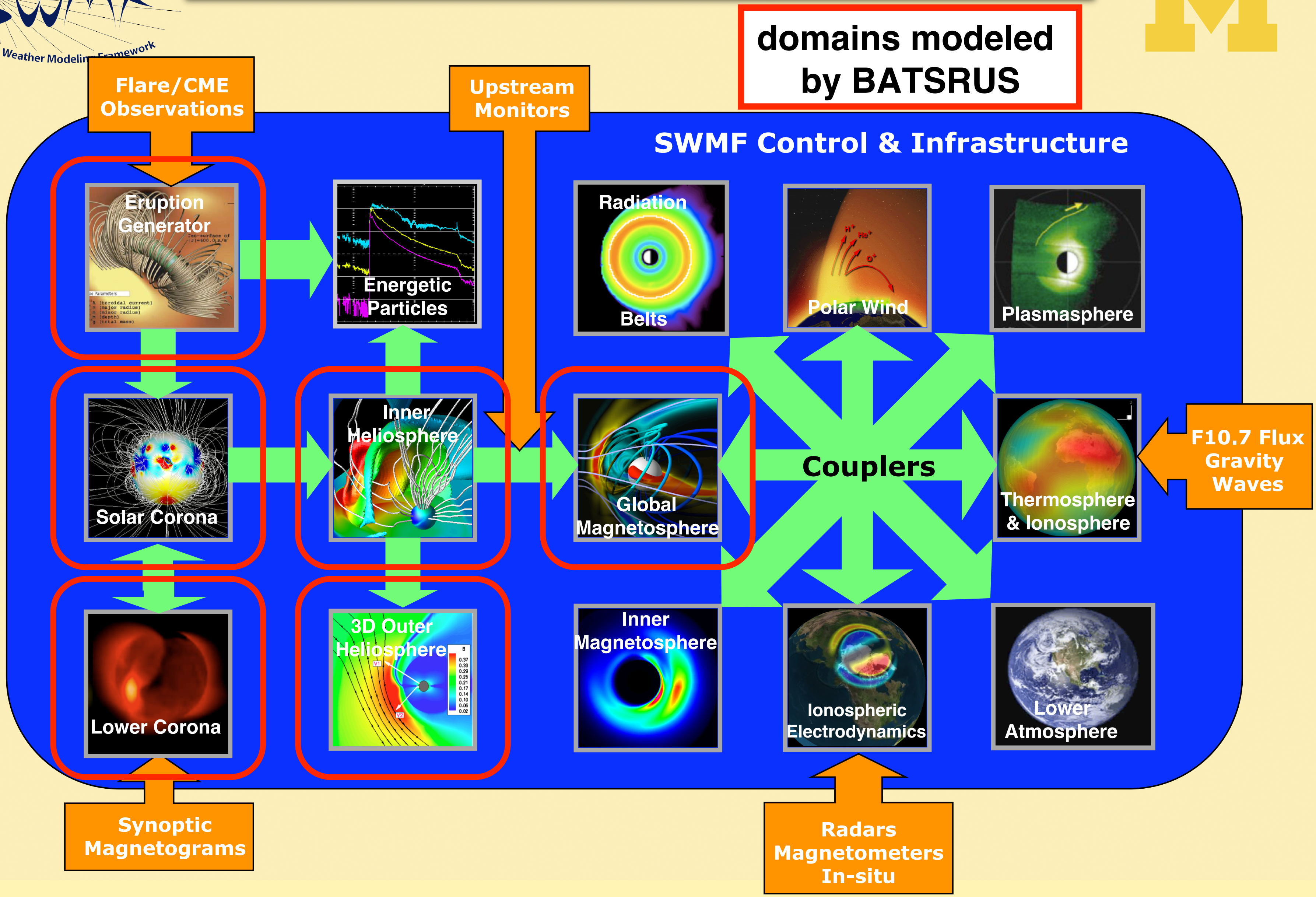
University Of Michigan

http://csem.engin.umich.edu

Outline

- **Space Weather Modeling Framework**
- **BATS-R-US**
- **MHD equations with anisotropic proton pressure**
  - Magnetosphere application
- **Electron physics**
  - Solar wind application
- **Summary**

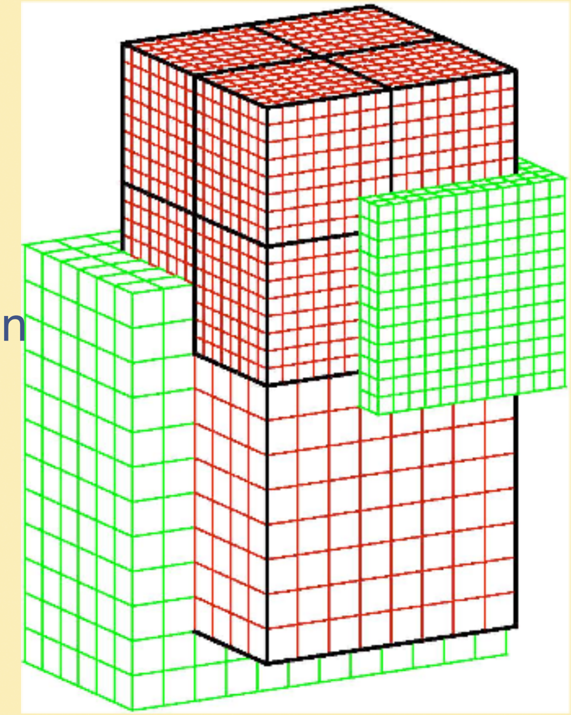
Space Weather Modeling Framework



SWMF is freely available at http://csem.engin.umich.edu and via CCMC

BATS-R-US

Block Adaptive Tree Solar-wind Roe Upwind Scheme



- **Physics**
  - Classical, semi-relativistic and Hall MHD
  - Multi-species, multi-fluid, anisotropic ion pressure
  - (Anisotropic) heat conduction, Alfvén wave turbulence
  - Radiation hydrodynamics with grey/multigroup diffusion
  - Multi-material, non-ideal equation of state
- **Numerics**
  - Conservative finite-volume discretization
  - Parallel block-adaptive grid
  - Cartesian and generalized coordinates
  - Splitting the magnetic field into $B_0 + B_1$
  - Divergence B control: 8-wave, CT, projection, parabolic/hyperbolic cleaning
  - Shock-capturing TVD schemes: Rusanov, HLLE, AW, Roe, HLLD
  - Explicit, point-implicit, semi-implicit, fully implicit time stepping
- **Applications**
  - Sun, heliosphere, magnetospheres, unmagnetized planets, moons, comets…
- **100,000+ lines of Fortran 90 code with MPI parallelization**

Anisotropic MHD

- **Different pressures parallel and perpendicular to the magnetic field**
- **Space physics applications**
  - Reconnection
  - Magnetosphere
  - Coupling with inner magnetosphere models
  - Solar wind heating
- **Difficulties**
  - What is a proper conservative form?
  - Physical instabilities: fire-hose, mirror, proton cyclotron
- **Combinations with more physics**
  - Separate electron pressure
  - Hall MHD, semi-relativistic, multi-ion

Resistive MHD with electrons and anisotropic ion pressure

Mass conservation: $$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0$$

Momentum: $$\frac{\partial \rho\mathbf{u}}{\partial t} + \nabla\cdot(\rho\mathbf{u}\mathbf{u} + P) = \mathbf{J}\times\mathbf{B}$$

$$P = (p_\perp + p_e)I + (p_\parallel - p_\perp)\mathbf{b}\mathbf{b} \qquad p = \frac{2p_\perp + p_\parallel}{3}$$

Induction: $$\frac{\partial \mathbf{B}}{\partial t} + \nabla\times\mathbf{E} = 0$$

Pressure:

$$\frac{\partial p_\perp}{\partial t} + \nabla\cdot(p_\perp\mathbf{u}) = \frac{1}{3\tau}(p_\parallel - p_\perp) + \frac{2}{\tau_{ie}}(p_e - p) - p_\perp\nabla\cdot\mathbf{u} + p_\perp\mathbf{b}\cdot(\nabla\mathbf{u})\cdot\mathbf{b}$$

$$\frac{\partial p}{\partial t} + \nabla\cdot(p\mathbf{u}) = -(\gamma-1)[p\nabla\cdot\mathbf{u} + \eta\mathbf{J}^2] + \frac{2}{\tau_{ie}}(p_e - p)$$

$$\frac{\partial p_\parallel}{\partial t} + \nabla\cdot(p_\parallel\mathbf{u}) = \frac{2}{3\tau}(p_\perp - p_\parallel) + \frac{2}{\tau_{ie}}(p_e - p) - 2p_\parallel\mathbf{b}\cdot(\nabla\mathbf{u})\cdot\mathbf{b}$$

$$\tau_{ie} = \frac{2}{3}\frac{M_i}{\eta e^2 n_e}$$

Electron pressure:

$$\frac{\partial p_e}{\partial t} + \nabla\cdot(p_e\mathbf{u}) = (\gamma-1)\left[-p_e\nabla\cdot\mathbf{u} + \eta\mathbf{J}^2 + \nabla\cdot(\kappa\mathbf{b}\mathbf{b}\cdot\nabla T_e)\right] + \frac{2}{\tau_{ie}}(p - p_e)$$

Electric field: $$\mathbf{E} = -\mathbf{u}\times\mathbf{B} + \eta\mathbf{J} \qquad \mathbf{b} = \mathbf{B}/B$$

Current: $$\mathbf{J} = \nabla\times\mathbf{B}$$

Conservative Form?

- **Shock capturing schemes require conservation laws to get proper jump conditions**
  - Double adiabatic invariants are not the right conservative variables
  - Energy conservation only replaces one of the two pressure equations
  - The anisotropy behind a shock is determined by instabilities
- **We use the energy equation and instability criteria to get proper jump conditions.**

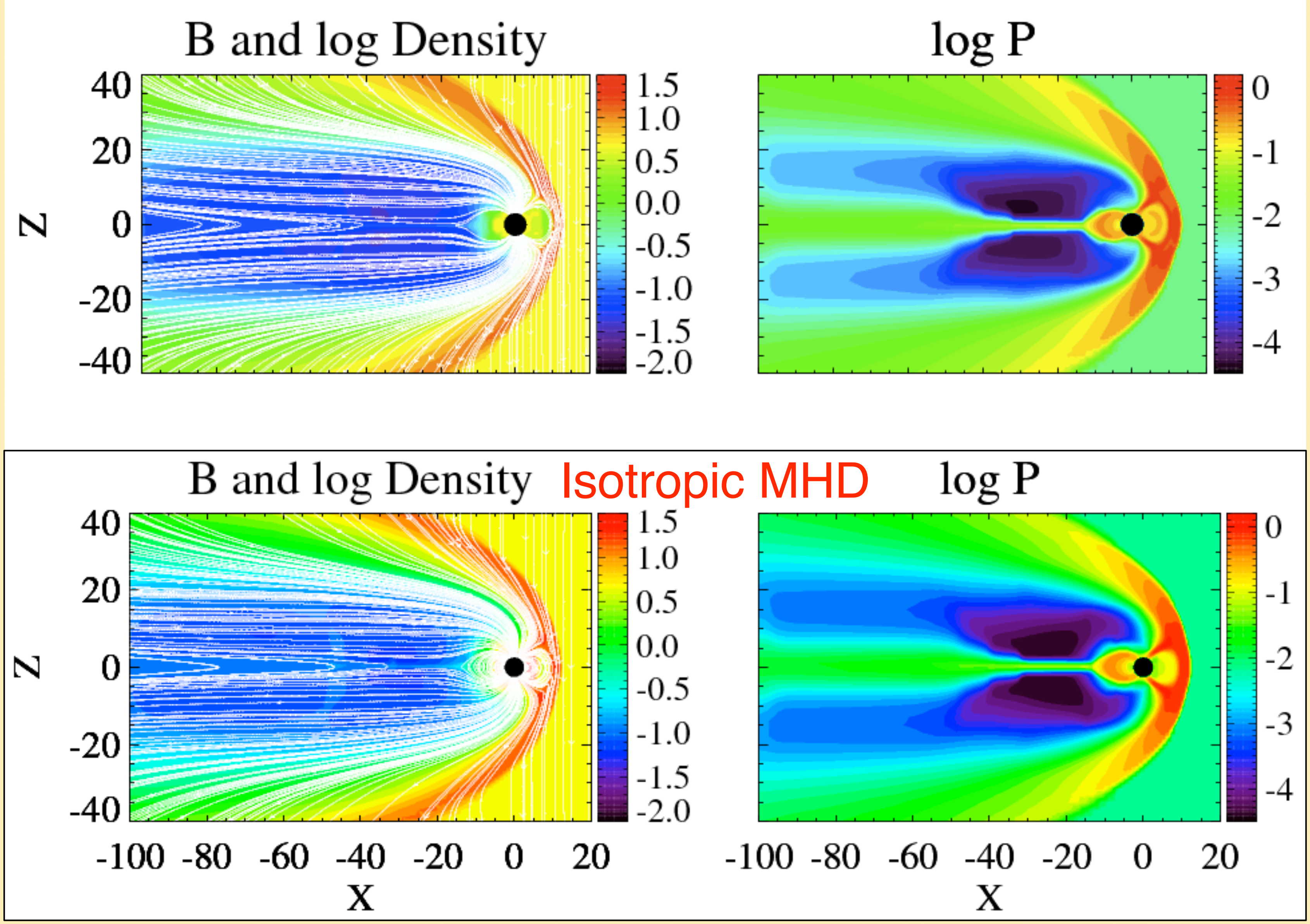
Limiting the Anisotropy

- **Instabilities**
  - Fire-hose: $\frac{p_\parallel}{p_\perp} > 1 + \frac{B^2}{p_\perp}$ (destabilized Alfvén wave)
  - Mirror: $\frac{p_\perp}{p_\parallel} > 1 + \frac{B^2}{2p_\perp}$
  - Proton cyclotron: $\frac{p_\perp}{p_\parallel} > 1 + 0.847 \left(\frac{B^2}{2p_\parallel}\right)^{0.48}$
  - In unstable regions we reduce anisotropy so it becomes stable
- **Ion-ion, ion-electron and/or wave-ion interactions:**
  - Push ion pressure towards isotropic distribution with time rate τ

Idealized Steady Magnetosphere Run (Xing Meng)

- dipole axis aligned with Z
- Steady solar wind: n = 5 /cc, v = 400 km/s, B_Z = -5 nT
- Solve for energy and parallel pressure near bow shock
- Enforce stability conditions
- Relaxation rate towards isotropy: τ = 20 s

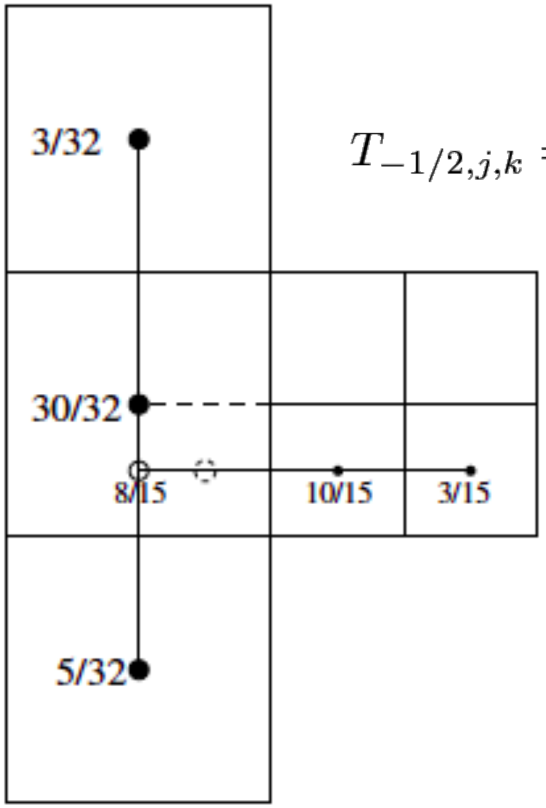
Idealized Steady Magnetosphere Run (Xing Meng)



Electron thermal heat conduction

$$C_V \frac{\partial T_e}{\partial t} = -\nabla \cdot \left[\kappa_e T_e^{5/2} \frac{\mathbf{BB}}{B^2} \cdot \nabla T_e \right]$$

- At resolution changes: interpolation of Te at fine AMR block has to be third order to make the scheme second order. Use finite difference approach.



- At block faces:

$$T_{-1/2,j,k} = \frac{5T_{-1/2,j-3/2,k-3/2} + 30T_{-1/2,j+1/2,k+1/2} - 3T_{-1/2,j+5/2,k+5/2}}{32}$$

$$T_{0,j,k} = \frac{8T_{-1/2,j,k} + 10T_{1,j,k} - 3T_{2,j,k}}{15}$$

- Similar interpolation schemes are used for the block edges

Electron Physics in Solar Wind

- Collisional electron heat conduction from inner coronal boundary to approximately $5R_{sun}$, smoothly diminishes between $5R_{sun}$ and $10R_{sun}$
- Heating of protons by Alfvén wave dissipation
- Heating of electrons by collisional coupling with protons

$$\frac{\partial p_\mathrm{i}}{\partial t} + \nabla \cdot (p_\mathrm{i}\mathbf{u}) + (\gamma - 1)p_\mathrm{i}\nabla \cdot \mathbf{u} = (\gamma - 1)\left[\qquad Q_\mathrm{i} + \lambda_\mathrm{ei}(T_\mathrm{e} - T_\mathrm{i})\right],$$

$$\frac{\partial p_\mathrm{e}}{\partial t} + \nabla \cdot (p_\mathrm{e}\mathbf{u}) + (\gamma - 1)p_\mathrm{e}\nabla \cdot \mathbf{u} = (\gamma - 1)\left[-\nabla \cdot \mathbf{q}_\mathrm{e} + \lambda_\mathrm{ei}(T_\mathrm{i} - T_\mathrm{e})\right],$$

$$\mathbf{q}_\mathrm{e} = -\kappa_\mathrm{e} T_\mathrm{e}^{5/2}\frac{\mathbf{BB}}{B^2} \cdot \nabla T_\mathrm{e}$$

WKB Alfvén Waves in Solar Wind

- Wind acceleration: work done by wave pressure force
- Coronal heating: formulation of the Kolmogorov dissipation by Hollweg (1986)

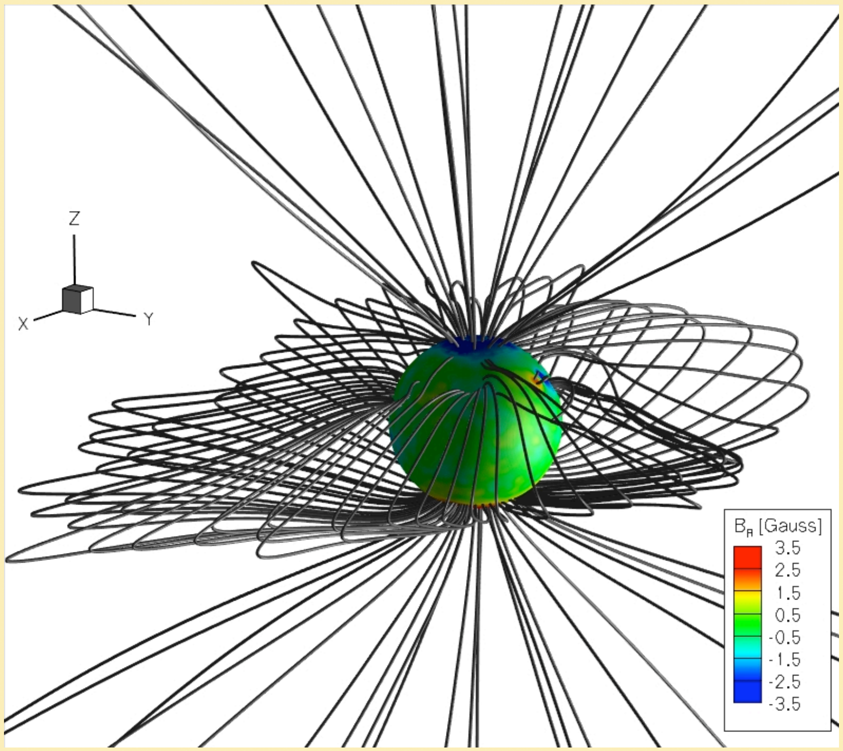
$$\frac{\partial E_{\rm w}^{+}}{\partial t} + \nabla \cdot \left[E_{\rm w}^{+} (\mathbf{u} + \mathbf{u}_{\rm A}) \right] + p_{\rm w}^{+} \nabla \cdot \mathbf{u} = -Q^{+},$$

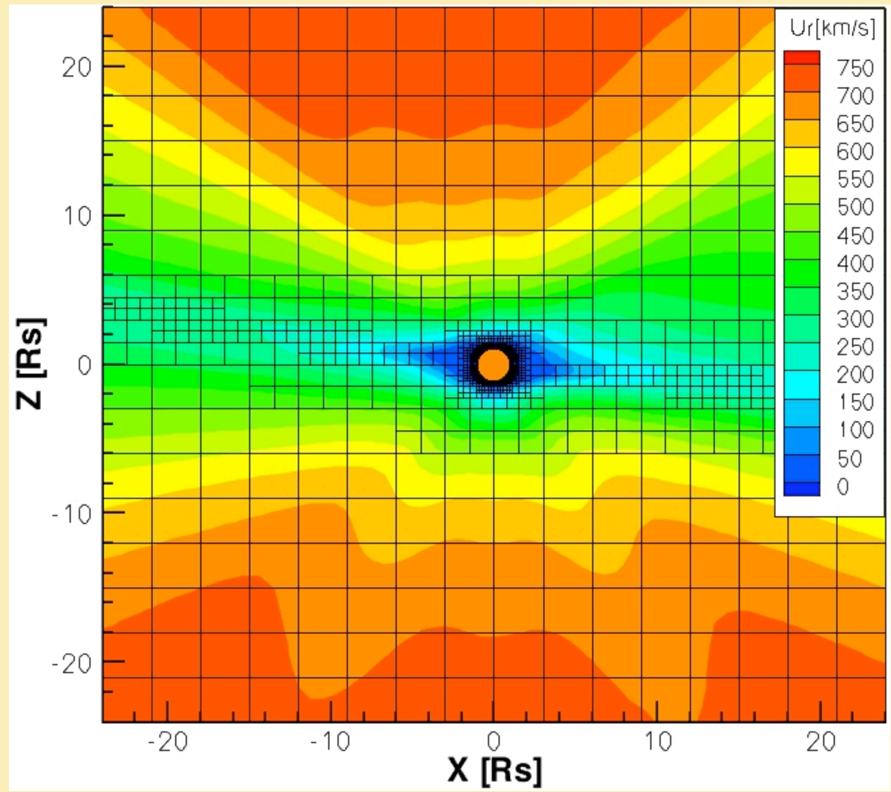
$$\frac{\partial E_{\rm w}^{-}}{\partial t} + \nabla \cdot \left[E_{\rm w}^{-} (\mathbf{u} - \mathbf{u}_{\rm A}) \right] + p_{\rm w}^{-} \nabla \cdot \mathbf{u} = -Q^{-},$$

Ion heating $\quad Q_{\rm i} = Q^{+} + Q^{-} = \frac{{E_{\rm w}^{+}}^{3/2}}{L\sqrt{\rho}} + \frac{{E_{\rm w}^{-}}^{3/2}}{L\sqrt{\rho}}, \quad L = C/\sqrt{B},$

- Free parameter C in heating scale height L

December 2008 Solar Wind

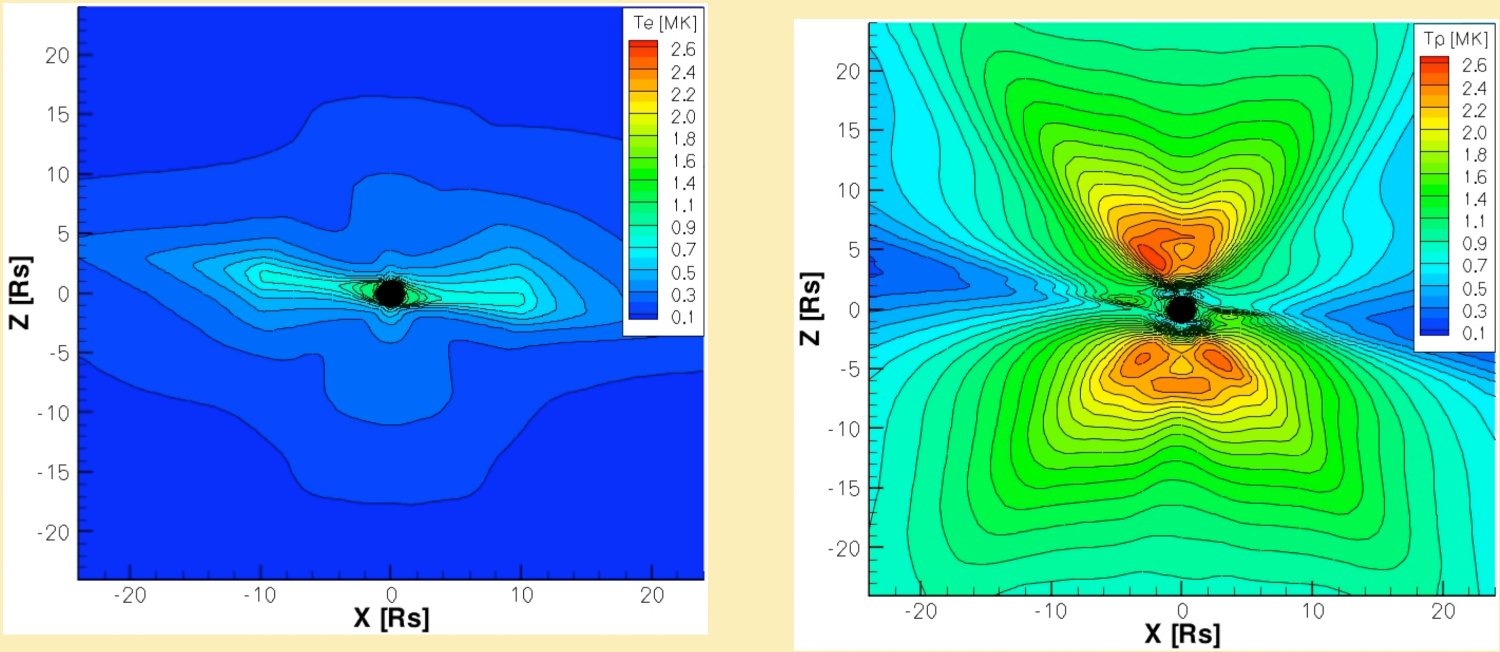




Selected field lines showing streamer belt

Meridional slice showing bimodal wind due to Alfvén waves

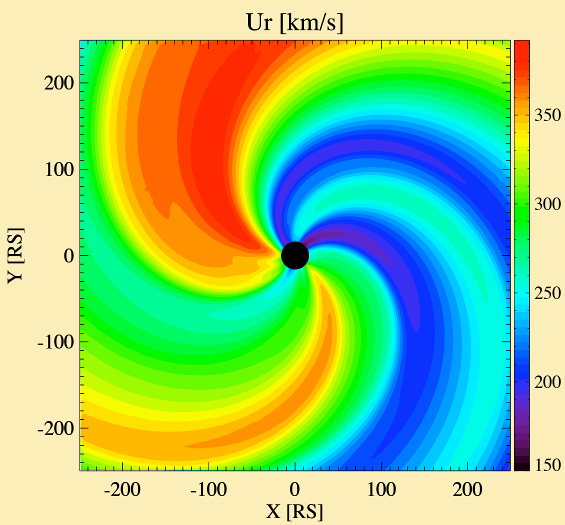
December 2008 Solar Wind

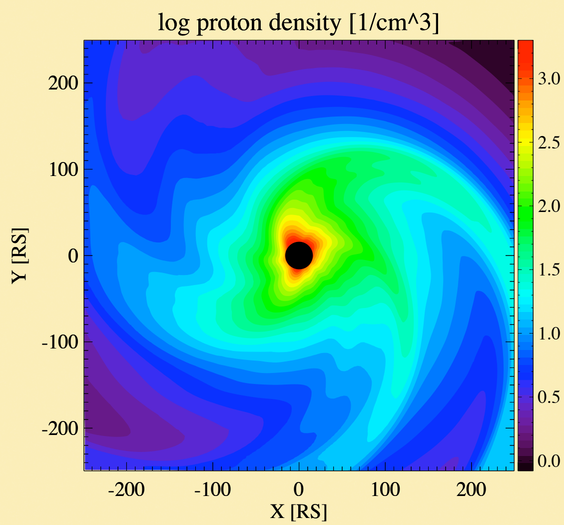


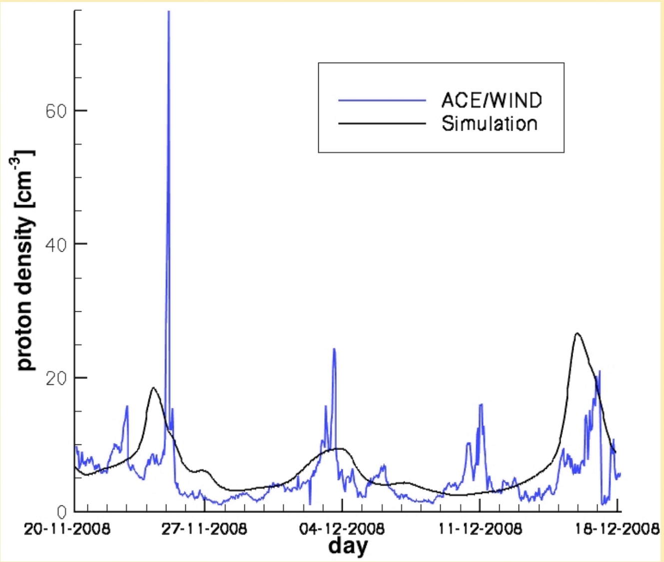
- **High electron temperature above streamer due to heat conduction, cool electrons in fast wind due to adiabatic expansion**
- **Protons mostly heated in coronal hole due to Alfvén waves**

Corotating Interaction Regions in Inner Heliosphere

- Non-alignment of magnetic- and rotation-axis gives appearance of fast and slow streams
- Result: compression of plasma seen as spiral arms
- Comparison with ACE satellite at L1 point







Summary

- **Anisotropic proton pressures in BATSRUS**
  - Enforcing stability limits
  - Optional (ad hoc) relaxation term
  - Comparison of magnetosphere simulations with observations in progress
- **Electron Physics in BATSRUS**
  - Separate electron pressure and electron thermal heat conduction
  - Comparison of solar wind at 1AU with observations is rather good
- **Plans**
  - Combined electron pressure and anisotropic proton pressure in both the solar wind and magnetosphere
  - Include counter-propagating Alfvén waves (partial reflection due to inhomogeneities in background)