Calculation of Three-Loop Terms for the MSSM Higgs Mass

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in collaboration with

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Outline

Motivation

Higgs mass measurement at LHC

- $\mathrm{M_h}$ gives stringent bounds on new Physics
  especially for supersymmetry
- Higgs mass calculable for given superpartner masses
- exclude large regions of parameter space

keep in mind

- what are the uncertainties in the calculation?
- Higgs mass *logarithmically* sensitive to stop masses
  $\mathcal{O}(\mathrm{GeV})$ change in $M_h \Leftrightarrow \mathcal{O}(\mathrm{TeV})$ change in $m_{\tilde{t}}$
- precision calculation necessary

Higgs Sector of the MSSM

$$V = m_1^2 \left|H_1\right|^2 + m_2^2 \left|H_2\right|^2 + m_{12}^2 \left(\epsilon_{ab} H_1^a H_2^b + \epsilon_{ab} H_1^{a*} H_2^{b*}\right)$$
$$+ \frac{1}{8}\left({g_1}^2 + {g_2}^2\right)\left(\left|H_1\right|^2 - \left|H_2\right|^2\right)^2 + \frac{1}{2}{g_2}^2 \left|H_1^* H_2\right|^2$$

spontaneous symmetry breaking

H_1, H_2 aquire vacuum expectation values
$\Rightarrow$ gauge bosons and fermions aquire masses.

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difference to SM: quartic terms fixed by gauge couplings

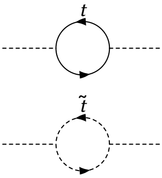
M_h can be predicted!

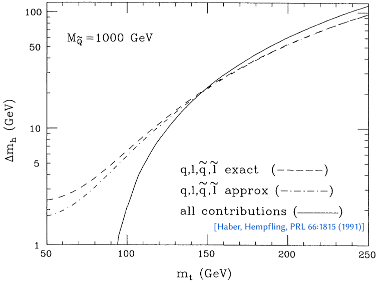
- tree level: $M_h \leq M_Z$
- large radiative corrections depending on superpartner spectrum
- measurement and calculation of M_h constrain SUSY parameters

One-Loop

- radiative corrections from heavy particles

[Ellis,Ridolfi,Zwirner 1991; Haber,Hempfling 1991; Okada,Yamaguchi,Yanagida 1991, Brignole ’92; Chankowski,Pokorski,Rosiek 1994; Dabelstein 1995; Bagger,Matchev,Pierce,Zhang 1997]





- most important contributions: top and stop loops $\propto m_t^4 \ln \frac{m_{\tilde{t}}}{m_t}$
- one-loop shift of the order of the tree-level value
- mild dependence on external momentum p^2

Corrections at Two Loops

- p^2=0 approximation:
  - $\alpha_t\ \alpha_s$ [Hempfling, Hoang '94; Heinemeyer, Hollik, Weiglein '98, '99; Espinosa, Zhang '99, Degrassi, Slavich, Zwirner '01]
  - α_t^2 [Hempfling, Hoang '94; Brignole, Degrassi, Slavich, Zwirner '01]
  - $\alpha_s\ \alpha_b$ [Brignole, Degrassi, Slavich, Zwirner '02]
  - $\alpha_t\ \alpha_b$, α_b^2 [Dedes, Degrassi, Slavich '03]

  implemented in public codes

  CPSuperH, FeynHiggs, softsusy, SPheno, SuSpect

[Lee, Pilaftsis, Carena, Choi, Drees, Ellis, Wagner; Degrassi, Frank, Hahn, Heinemeyer, Hollik, Slavich, Rzehak, Weiglein; Allanach; Porod Staub; Djouadi, Kneur, Moultaka]

- momentum dependence, electroweak effects [Martin '02, '04]
  not yet implemented in public code
  work in progress to include
  momentum dependence in FeynHiggs [Borowka, Heinrich '13]

Estimating Higher Order Effects

Detailed Study of the Precision of Two-Loop Codes

[Allanach, Djouadi, Kneur, Porod, Slavich '04]

- compares results from different codes

	SPS1a	SPS2	SPS4	SPS5	SPS9
SuSpect	112.1	116.8	114.1	116.1	117.5
FeynHiggs	113.8	118.3	116.1	118.5	118.3

- studies scale dependence
- projects effects of neglecting momentum from 1-loop

concludes

- remaining uncertainty: 3-5 GeV
- caveat: in 2004, focus on rather light stops $\approx$ 1 TeV
  expect larger uncertainties for heavier stops

Beyond Two Loops

- remaining uncertainty: $\approx 3 - 5\,\mathrm{GeV}$
  rising with superpartner masses

ATLAS	$125.5 \pm 0.2^{+0.5}_{-0.6}$ GeV
CMS	$125.7 \pm 0.3 \pm 0.3$ GeV

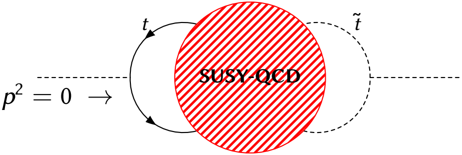
theory has to do better

Results Beyond Two Loops

- three-loop LL and NLL through renormalisation group [Martin '07]
- calculation of the $\alpha_t\ \alpha_s^2$ terms [Harlander, PK, Mihaila, Steinhauser '08 & '10]
- special cases from vacuum stability

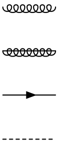
[Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia '12]

Leading Terms at Three Loops



- corrections from top and stops
- external momentum zero
- 3 loops, no legs
- > 30.000 diagrams

- needs regularisation consistent with SUSY
  - Dimensional Reduction
- many masses:



$m = 0$

$m_{\tilde{g}}$

$m_t\,,\ m_q = 0$

$m_{\tilde{t}_1}\,,\ m_{\tilde{t}_2}\,,\ m_{\tilde{q}}$

Methodology: Regularisation

Dimensional Regularisation (DREG)

- regulate divergencies via a shift in the dimension
  $4 \to D = 4 - 2\varepsilon$

Supersymmetry

- connects bosonic and fermionic degrees of freedom
- numbers of degrees of freedom has to match
- spoiled by DREG (vector fields)

What to do?

- either restore Ward identities by finite counterterms
- or use Dimensional Reduction (DRED) [Siegel '84]

Regularisation by Dimensional Reduction

change integration measure, leave the fields as they are

- compactify spacetime such that fields only depend on D components of spacetime
- partial derivatives and momenta are restricted to D dimensions
- vector fields are left "intact"
  introduce ε-Skalars to restore vector fields

Dimensional Reduction in Practise

bare lagrange density of Yang-Mills theory with fermions:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2\alpha}\left(\partial^\mu A_\mu\right)^2 + C^{a*}\partial^\mu D_\mu^{ab} C^b + i\bar{\psi}^\alpha \gamma^\mu D_\mu^{\alpha\beta}\psi^\beta$$

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DRED: $A_4^\mu \to A_D^\mu \oplus A_{2\epsilon}^\mu$, $\partial_4^\mu \to \partial_D^\mu \oplus 0_{2\epsilon}^\mu$

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$$+ \frac{1}{2}\left(D_\mu^{ab}A_\nu^b\right)^2 - g\bar{\psi}^\alpha\gamma_\mu R_{\alpha\beta}^a\psi^\beta A_\mu^a - \frac{1}{4}g^2 f^{abc}f^{ade}A_\mu^b A_\nu^c A_\mu^d A_\nu^e$$

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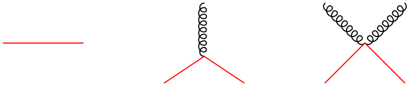
propagator of 2ϵ scalar fields,
gauge interaction ε-scalars



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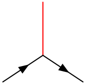
Yukawa-type interaction of fermion with ε-scalars



Dimensional Reduction in Practise

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Quartic self-interaction of ε-scalars

Methodology: Multi-Mass Diagrams

- multi-scale three-loop diagrams: $t, \tilde{t}_1, \tilde{t}_2, \tilde{q}, \tilde{g}$
- can't do integrals for arbitrary masses
  - assume fixed hierarchy among superpartner masses

$$m_q = 0, \quad m_t \ll m_{\tilde{t}_1} \approx m_{\tilde{t}_2} \approx m_{\tilde{g}} \approx m_{\tilde{q}}$$
$$m_t \ll m_{\tilde{t}_1} \ll m_{\tilde{t}_2} \approx m_{\tilde{g}} \ll m_{\tilde{q}}$$

- asymptotic expansion leads to one-scale integrals
- complication: hierarchy not known
- perform calculation for many hierarchies, choose the most appropriate

Asymptotic Expansion

asymptotic expansion: algorithmic way to disentangle scales

[Gorishnii ’87; Smirnov ’90; Tkachov ’93; Pivovarov ’93]

- partition diagram into subgraphs that
  - contain all the propagators with heaviest mass
  - are 1PI w.r.t. the other scales
- taylor expand the subgraphs in the other scales
- insert the taylor expansion as an effective vertex into the original diagram
- iterate with the next to heaviest mass

result: expansion in ratios and logarithms of the scales
coefficients are one-scale integrals

- automatisation: q2e, exp [Harlander, Seidensticker]

Sample Result: $m_{\tilde{t}_{1,2}} = m_{\tilde{g}} \ll m_{\tilde{q}}$ (on-shell)

$$
\begin{aligned}
\Delta M_h \quad = \quad & -\frac{3 G_F M_t^4}{\sqrt{2}\pi^2} \Bigg\{ - L_{tS} + \frac{\alpha_s}{\pi}\left[4 L_{tS} - 2 L_{tS}^2\right] + \left(\frac{\alpha_s}{\pi}\right)^2 \Bigg[-\frac{1091}{324} - \frac{1}{27}\pi^2 - \frac{1}{9}\zeta_3 \\
& + \left(\frac{1591}{108} + 3 L_{\mu t} - \frac{1}{3}\pi^2 + \frac{4}{9}\pi^2 \ln 2 - \frac{55}{18} L_{t\tilde{q}} - \frac{5}{6} L_{t\tilde{q}}^2\right) L_{tS} \\
& + \left(-\frac{19}{18} - \frac{3}{2} L_{\mu t} + \frac{5}{3} L_{t\tilde{q}}\right) L_{tS}^2 - \frac{53}{18} L_{tS}^3 \\
& + \left(-\frac{475}{108} + \frac{5}{9}\pi^2\right) L_{t\tilde{q}} + \frac{25}{36} L_{t\tilde{q}}^2 + \frac{5}{18} L_{t\tilde{q}}^3 \\
& + \mathcal{O}\left(\frac{M_S^2}{M_{\tilde{q}}^2}\right) \Bigg] \Bigg\},
\end{aligned}
$$

$$
L_{tS} = \ln \frac{M_t^2}{M_{SUSY}^2}, \qquad L_{\mu t} = \ln \frac{\mu^2}{M_t^2}, \qquad L_{t\tilde{q}} = \ln \frac{Mt^2}{M_{\tilde{q}}^2}
$$

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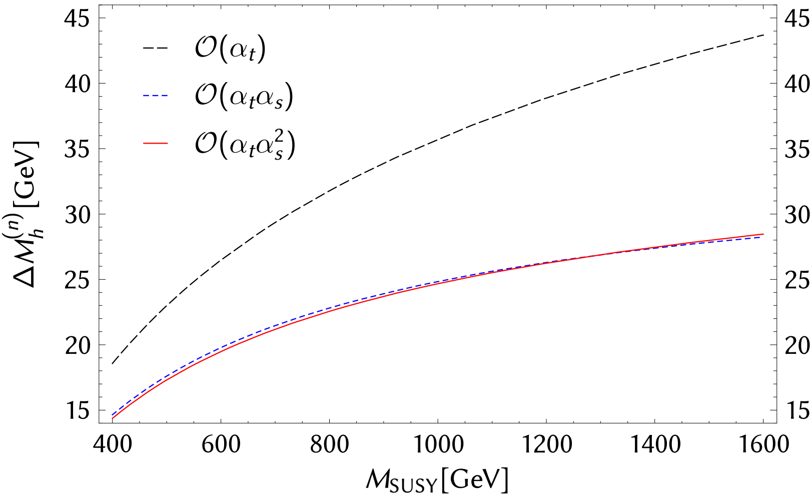

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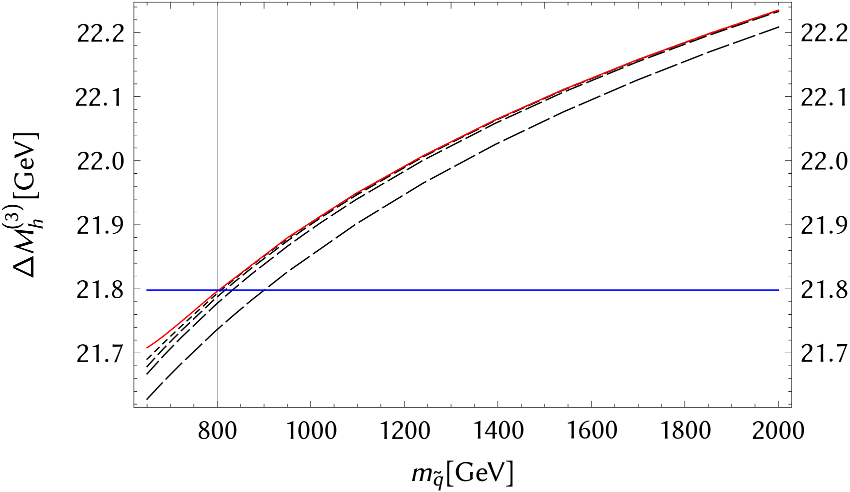

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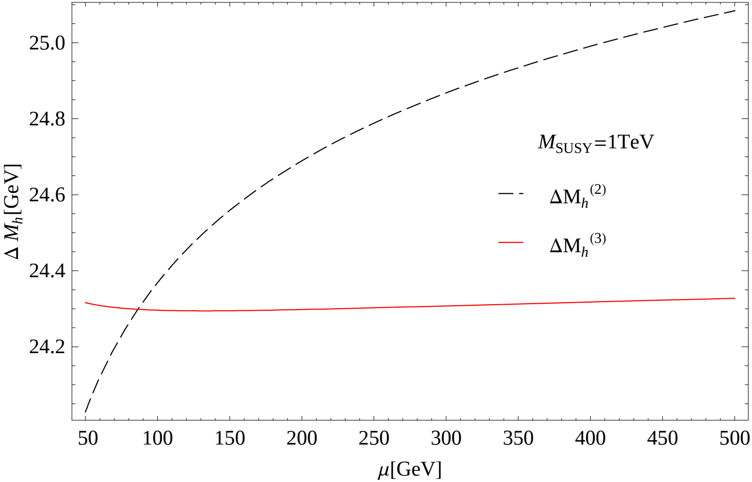

Renormalisation Scheme Dependence

- renormalisation scheme: minimal subtraction vs. on-shell

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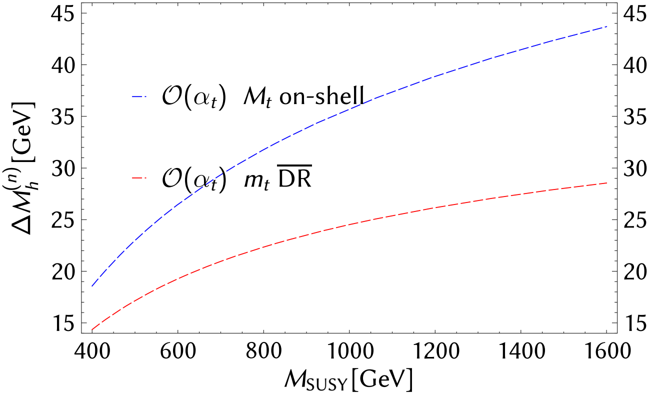

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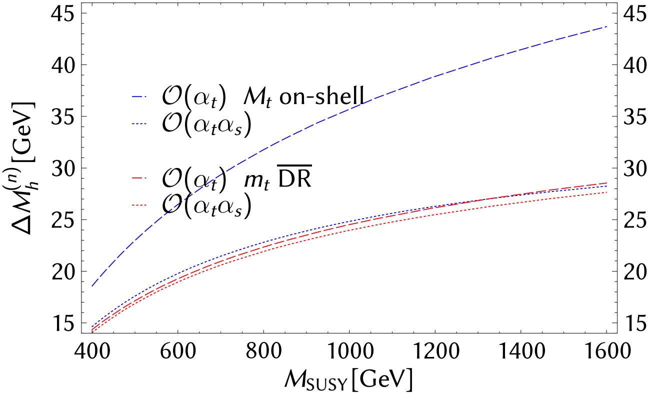

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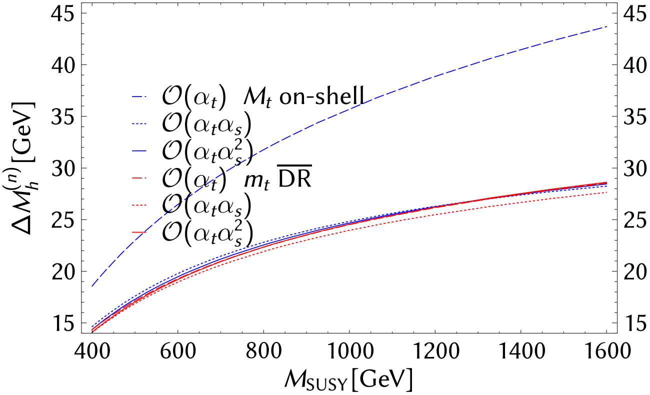
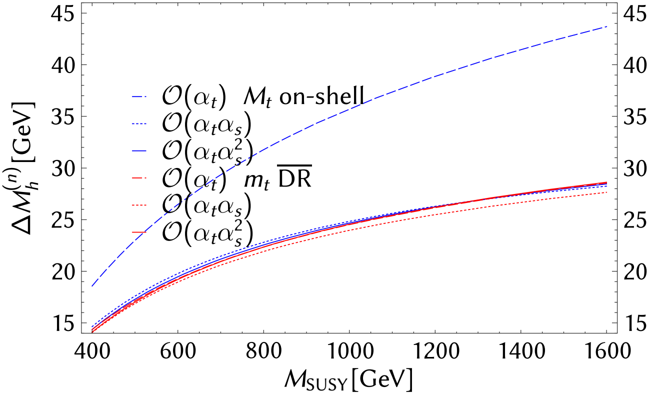


need two-loop conversion formula $m_t^{\overline{\text{DR}}}(m_t^{OS})$

[Martin '05]

Renormalisation Scheme Dependence

- renormalisation scheme: minimal subtraction vs. on-shell

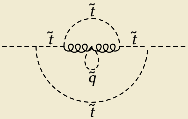


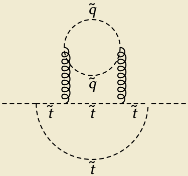
need two-loop conversion formula $m_t^{\overline{\mathrm{DR}}}(m_t^{OS})$ [Martin '05]

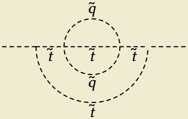
choice: minimal subtraction using Dimensional Reduction ($\overline{\mathrm{DR}}$)

Avoiding $\frac{M^2}{m^2}$ terms

consider these diagrams, for $m_{\tilde{t}} \ll m_{\tilde{q}}$







- generate terms of order $\frac{m_{\tilde{q}}^2}{m_{\tilde{t}}^2}$

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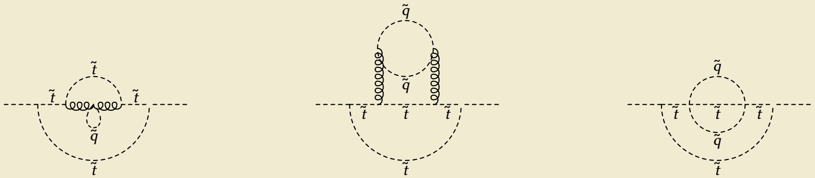
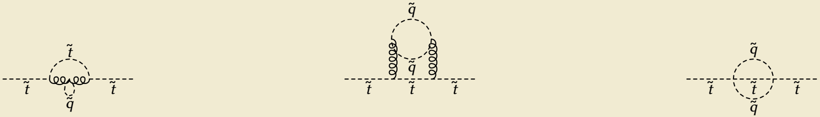
consider these diagrams, for $m_{\tilde{t}} \ll m_{\tilde{q}}$



- generate terms of order $\frac{m_{\tilde{q}}^2}{m_{\tilde{t}}^2}$
- have to renormalise corresponding counterterms on-shell



Checks

- agreement with literature
  two-loop [Degrassi, Slavich, Zwirner '01]
  3-loop LL and NLL [Martin '07]
- calculated in general covariant gauge
- calculation in unbroken SUSY: corrections vanish

Combining Results: H3m

combine with corrections from other sectors

- use existing "wheel": FeynHiggs
- consistent renormalisation of parameters:
  on-shell vs. modified minimal subtraction using DRED
- consistent values of parameters
  - spectrum generator via SUSY Les Huches interface
  - evolve α_S (RunDec, RunDecSUSY)

    [Chetyrkin, Kühn, Steinhauser '00; Harlander, Mihaila, Steinhauser '05,'07]
  - convert m_t to $\overline{\mathrm{DR}}$ scheme using four-loop running and two-loop decoupling [Kunz, Mihaila, in preparation]
- automatic choice of appropriate approximation
  - compare approximation at two loops with full two loop result

JHEP 1008 (2010) 104 http://www.ttp.kit.edu/Progdata/ttp10/ttp10-23

Calculating α_s and m_t

strong coupling

$$\alpha_s^{\overline{MS},QCD5}(m_Z) \rightarrow \alpha_s^{\overline{DR},MSSM}(\mu)$$

top mass

$$m_t^{\mathrm{OS}} \rightarrow m_t^{\overline{DR},MSSM}(\mu)$$

Calculating α_s and m_t

strong coupling

$$\alpha_s^{\overline{MS},QCD5}(m_Z) \to \alpha_s^{\overline{MS},QCD5}(\mu) \to \alpha_s^{\overline{DR},MSSM}(\mu)$$

- run to large scale (4 loops)
- couple to full theory, go to $\overline{DR}$ (2 loops)

top mass

$$m_t^{\mathrm{OS}} \qquad \to m_t^{\overline{DR},MSSM}(\mu)$$

Calculating α_s and m_t

strong coupling

$$\alpha_s^{\overline{MS},QCD5}(m_Z) \to \alpha_s^{\overline{MS},QCD5}(\mu) \to \alpha_s^{\overline{DR},MSSM}(\mu)$$

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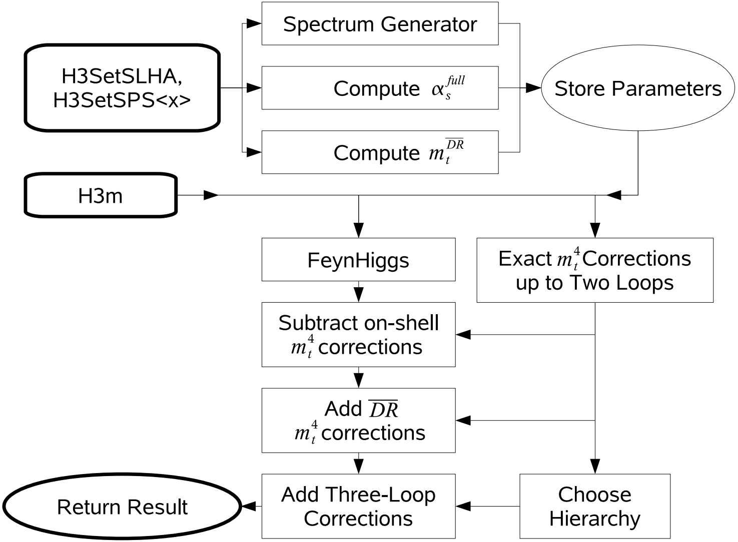
top mass

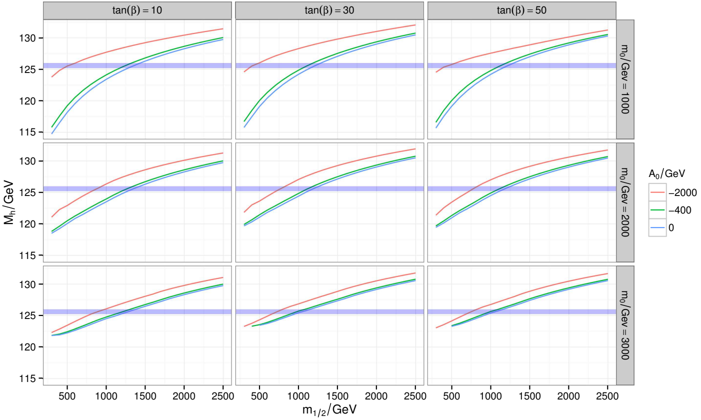
$$m_t^{\mathrm{OS}} \to m_t^{\overline{MS},QCD5}(\mu = m_t^{\mathrm{OS}}) \to m_t^{\overline{MS},QCD5}(\mu) \to m_t^{\overline{DR},MSSM}(\mu)$$

- convert on-shell mass to $\overline{MS}$ scheme (3 loops)
- run to large scale (4 loops)
- couple to full theory, go to $\overline{DR}$ (2 loops)

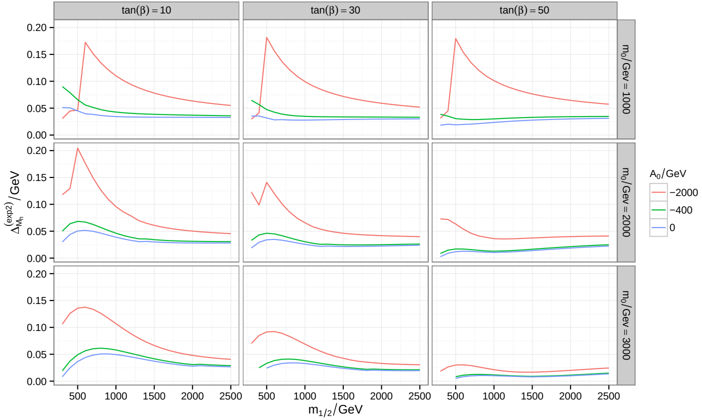
Program Organisation



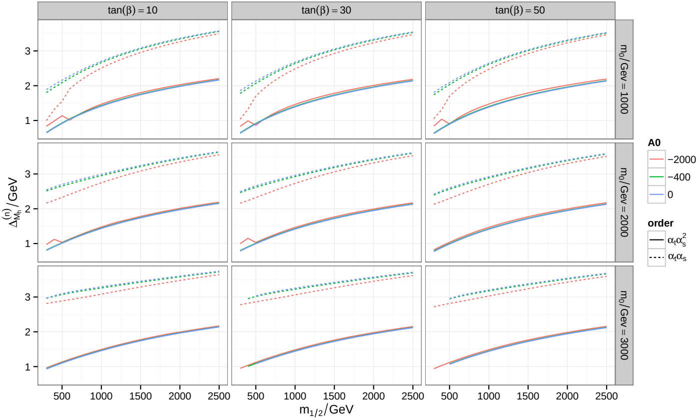
Numerics for constrained MSSM



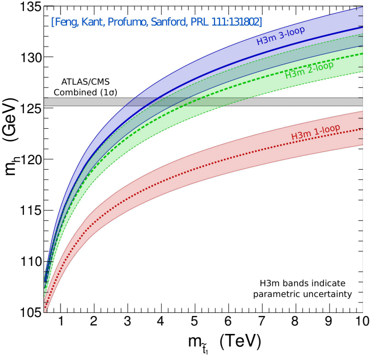
Error due to Asymptotic Expansion



Perturbative Behaviour



Numerics for Unmixed Stops



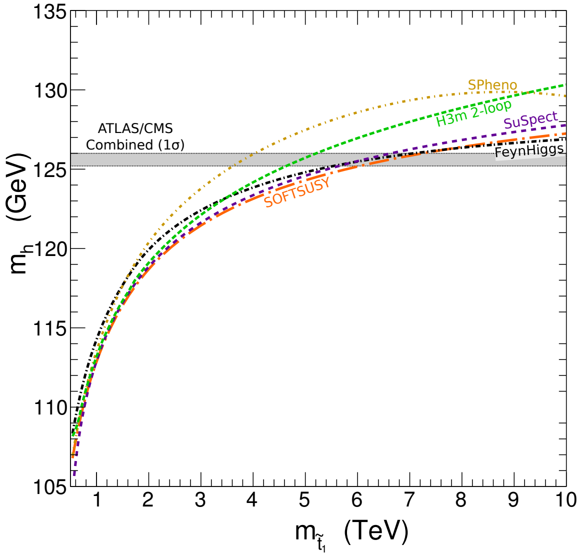
- error bands:
  parametric uncertainty

$$m_t^{\text{pole}} = 173.3 \pm 1.8\,\text{GeV}$$
$$\alpha_s(m_Z) = 0.1184 \pm 0.0007$$

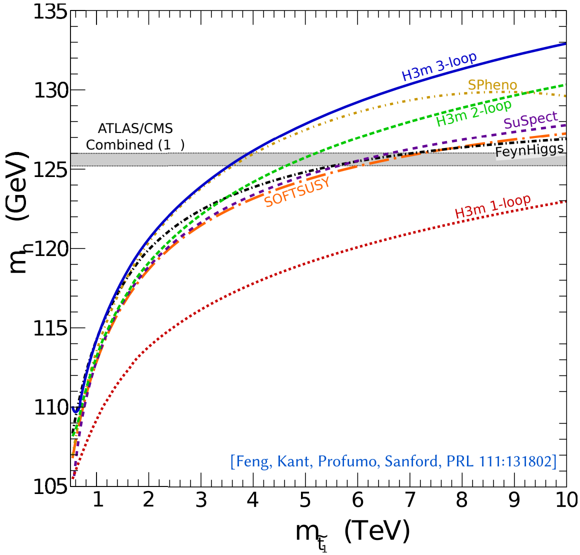
- m_t dominates
  conservative error
- size of three-loop terms grows with superpartner masses

for multi-TeV squarks, three-loop terms shift M_h by 0.5 to 3 GeV

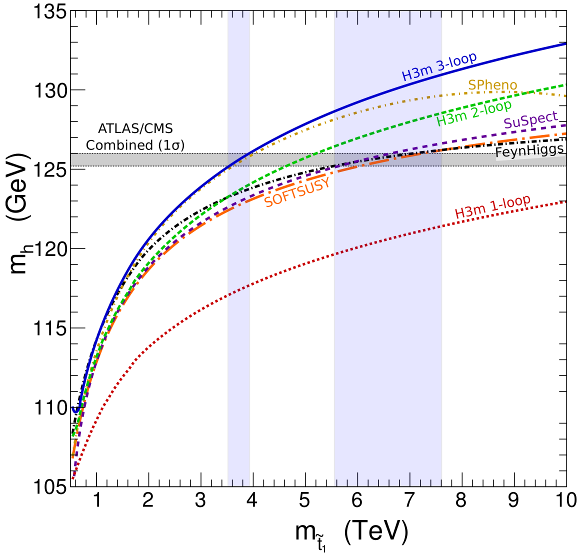
Comparison With 2-Loop Codes



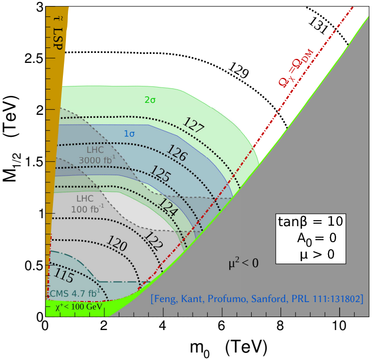
Comparison With 2-Loop Codes



Comparison With 2-Loop Codes

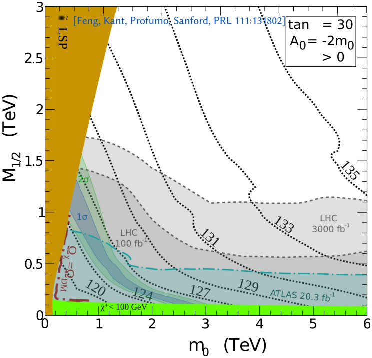


LHC Discovery Potential



- negligible stop mixing
- σ includes estimated perturbative error and parametric uncertainty due to M_t and α_s
- 3-4 TeV stop masses possible requiring $\Omega_\chi = \Omega_{DM}$
- even lighter whithout cosmological bounds

LHC Discovery Potential



- significant stop mixing
- favoured region completely accessible by 14 TeV LHC
- m_0 as low as 1 TeV preferred

Conclusions

- Higgs mass provides unique opportunity to constrain SUSY spectrum
  - for both light and heavy scales
- three-Loop Terms are important
  - M_h raised by as much as 3 GeV
  - lowers required scalar masses to 3-4 TeV
- future improvements
  - momentum-dependent terms at two-loop level
  - $\alpha_t^2\ \alpha_s$ terms
    cancellations in a particular scenario [Martin '07]
    is this a general feature?
- remaining uncertainties
  - induced uncertainty from M_t 0.5-2 GeV
  - missing higher orders roughly 1 GeV
    subject to size of SUSY masses