Improvements of Higgs mass predictions in supersymmetric theories

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FeynHiggs collaboration

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Calculation of Higgs masses in the MSSM

Two approaches:

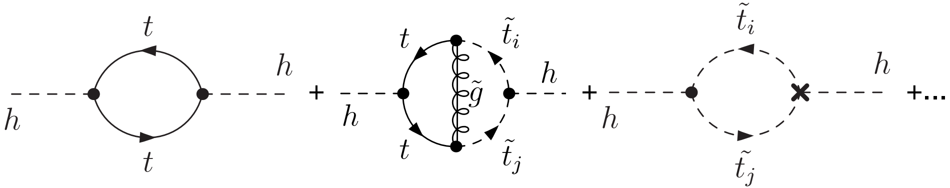
- Feynman diagrammatic approach

  (or effective potential approach for vanishing external momenta)

- renormalization group equation approach

Feynman diagrammatic approach

Calculate Feynman diagrams
which contribute to the Higgs-boson self energies $\hat{\Sigma}$:



1-loop level $\mathcal{O}(\alpha_t)$ 2-loop level $\mathcal{O}(\alpha_t \alpha_s)$ Counterterm contr.

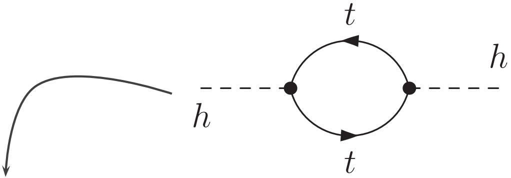
$\alpha_t \sim$ (top Yukawa coupl.)^2

Feynman diagrammatic approach

Two-point-function:

$$-i\hat{\Gamma}(p^2) = p^2 - \mathbf{M}(\mathbf{p^2})$$

with the matrix:



$$\mathbf{M}(\mathbf{p^2}) = \begin{pmatrix} M^2_{h_{\text{Born}}} - \hat{\Sigma}_{hh}(p^2) & -\hat{\Sigma}_{Hh}(p^2) \\ -\hat{\Sigma}_{Hh}(p^2) & M^2_{H_{\text{Born}}} - \hat{\Sigma}_{HH}(p^2) \end{pmatrix}$$

(CP-conserving case: mixing only between

CP-even Higgs bosons h, H)

Feynman diagrammatic approach

Calculate the zeros of the determinant of $\hat{\Gamma}$:

$$\det[p^2 - \mathbf{M(p^2)}] = 0$$

or calculate the eigenvalues $\lambda(p^2)$ of $\mathbf{M(p^2)}$ (`FeynHiggs` approach):

$$\det[\lambda(p^2) - \mathbf{M(p^2)}] = 0$$

and solve iteratively:

$$p^2 - \lambda(p^2) = 0$$

$\Rightarrow$ loop-corrected Higgs mass values

Renormalization group equation approach

Assumption: all SUSY particles and the CP-odd Higgs boson mass M_A being heavy $\sim M_S$

(i) Match quartic Higgs coupling λ at scale M_S

(ii) Use SM-RGE running to obtain λ at scale m_t

(iii) Higgs mass is given by $m_h^2(m_t) = 2\lambda(m_t)v^2$ with $v \approx 174$ GeV being the Higgs vacuum expectation value

Approach can be refined to allow for different scales

Advantages

- Feynman diagrammatic approach:

  All log- and non-log terms are taken into account at a certain order of perturbation theory:

  Especially important for lower mass scales

- Renormalization group equation approach:

  Resummation of potentially large log-terms:

  Especially important for larger mass scales

$\Rightarrow$ Combine both approaches

Combination

- Feynman diagrammatic part: from `FeynHiggs`
- Renormalization group equation (RGE) part: 2-loop RGE for running

[Espinosa, Quiros '91]

$$\mu \frac{d\lambda(\mu)}{d\mu} = \frac{1}{(16\pi^2)} \left[12\lambda^2 + 12{y_t}^2\lambda - 12{y_t}^4\right]$$
$$+ \frac{1}{(16\pi^2)^2} \left[-78\lambda^3 + 60{y_t}^6 - 3\lambda{y_t}^4 - 64{g_s}^2{y_t}^4 + 80\lambda{g_s}^2{y_t}^2 - 72\lambda^2{y_t}^2\right]$$

$$\mu \frac{dg_s(\mu)}{d\mu} = \frac{g_s}{(16\pi^2)} \left[-7{g_s}^2\right] + \frac{g_s}{(16\pi^2)^2} \left[-26{g_s}^4 - 2{g_s}^2{y_t}^2\right]$$

$$\mu \frac{dy_t(\mu)}{d\mu} = \frac{y_t}{(16\pi^2)} \left[\frac{9}{2}{y_t}^2 - 8{g_s}^2\right]$$
$$+ \frac{y_t}{(16\pi^2)^2} \left[-12{y_t}^4 + \frac{3}{2}\lambda^2 - 6\lambda{y_t}^2 + 36{g_s}^2{y_t}^2 - 108{g_s}^4\right]$$

quartic Higgs coupling
strong coupling
top Yukawa coupling

Combination

- Renormalization group equation (RGE) part: (continued)

  Matching at scale $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$: [Carena, Haber, Heinemeyer, Hollik, Wagner, Weiglein, hep-ph/0001002]

$$\lambda(M_S) = \frac{3{y_t}^4}{8\pi^2} \frac{X_t^2}{M_S^2} \left[1 - \frac{X_t^2}{M_S^2} \right]$$

$m_{\tilde{t}_i}$ = stop masses

$X_t = A_t - \mu \cot\beta$ = squark mixing parameter

$\Rightarrow$ leading + next-leading log ($\ln \frac{M_S}{m_t}$) resummation

Combination

- Combination of both approaches:

  Avoid double counting of logs

  $\Rightarrow$ Subtract logs from the Feynman diagrammatic (FD) result:

$$\Delta M_h^2 = (\Delta M_h^2)^{\text{FD}}(X_t^{\text{OS}}) - (\Delta M_h^2)^{\text{FD,log}}(X_t^{\text{OS}}) + (\Delta M_h^2)^{\text{RGE}}(X_t^{\overline{\text{MS}}})$$

  ⋆ Both approaches use a $\overline{\text{MS}}$ top quark mass

  ⋆ FD: X_t in on-shell scheme, RGE: X_t in $\overline{\text{MS}}$ scheme:
  Conversion needed:

$$X_t^{\overline{\text{MS}}} = X_t^{\text{OS}} \left[1 + \ln \frac{M_S^2}{m_t^2} \left(\frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi} \right) \right]$$

Combination

For $M_A \gg M_Z$:

$$\hat{\Sigma}_{\phi_u\phi_u} \approx (\sin\beta)^{-2}\Delta M_h^2$$

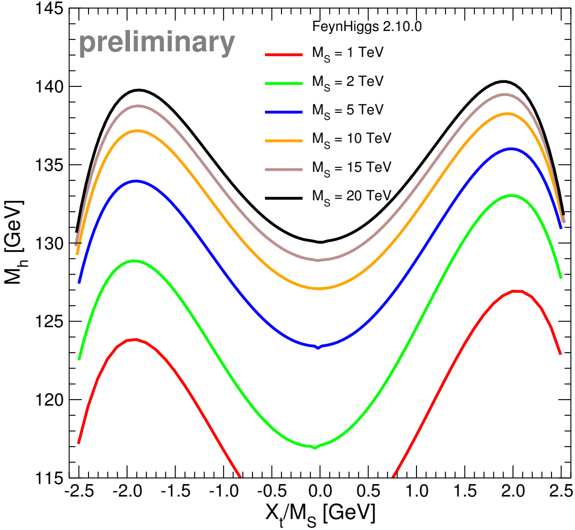
self energy of the
interaction eigenstates ϕ_u

ϕ_u couples to up-type quarks

Correction can be incorporated into the self energy matrix

Results

Higgs mass dependence on X_t/M_S and M_S:



lower scales:

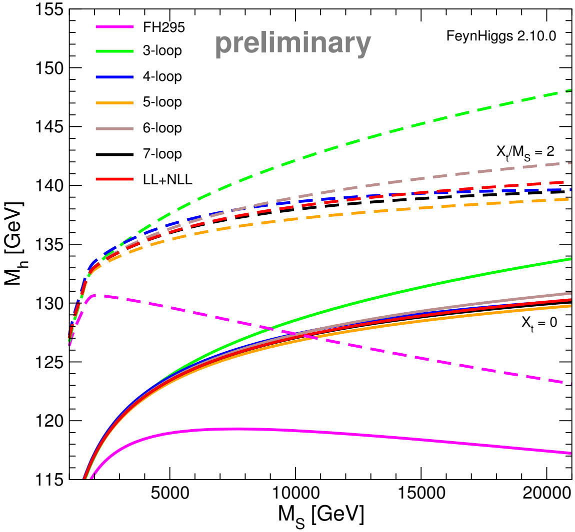
maxima: large difference in size due to non-log terms

larger scales:

differences between maxima become smaller (still sizeable in between)

$M_A = M_2 = \mu = 1$ TeV, $m_{\tilde{g}} = 1.6$ TeV, $\tan\beta = 10$

Results

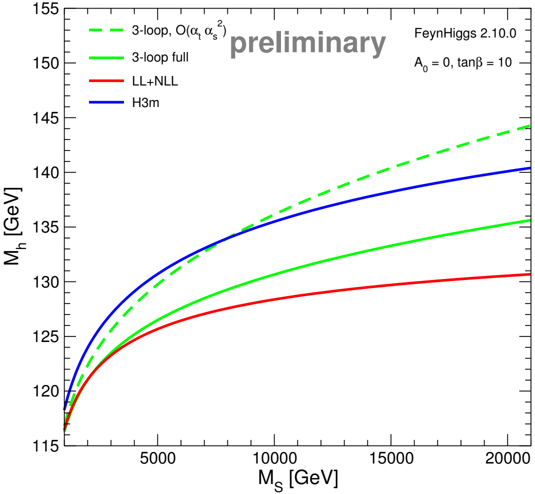


Comparison of:

- ⋆ old FeynHiggs reliable up to $M_s = \mathcal{O}(1\text{TeV})$
- ⋆ analyt. solution of RGE: 3-loop . . . 7-loop level
- ⋆ numerical solution: logs resummed to all orders

$M_A = M_2 = \mu = 1$ TeV, $m_{\tilde{g}} = 1.6$ TeV, $\tan\beta = 10$

Results



Comparison with `H3m`:

[Kant, Harlander, Mihaila, Steinhauser, arXiv:1005.5709]

3-loop: $\mathcal{O}(\alpha_t\alpha_s^2)$, $\mathcal{O}(\alpha_t^2\alpha_s)$, $\mathcal{O}(\alpha_t^3)$

- ⋆ only leading and next-to leading logs
- ⋆ single scale M_S

H3m: ⋆ complete $\mathcal{O}(\alpha_t\alpha_s^2)$ result
- ⋆ different scales

At 2-loop: different ren. schemes

CMSSM: $m_0 = m_{1/2} = 200 \ldots 15000$ GeV, $A_0 = 0$, $\tan\beta = 10$, $\mu > 0$, spectra generation with `SoftSUSY` [Allanach, hep-ph/0104145]

Conclusion

Lower SUSY scales: Feynman diagrammatic approach
(or effective potential approach)

Large SUSY scales: Renormalization group equation approach

⇒ Consistent combination of both approaches:

Good prediction for all scales

Further refinements: Allow for:

- ⋆ smaller CP-odd Higgs boson masses
- ⋆ large stop mass splitting