

# On the higher loop Corrections to the Higgs Mass in the MSSM

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Work done in collaboration with P. Draper and G. Lee, to appear soon

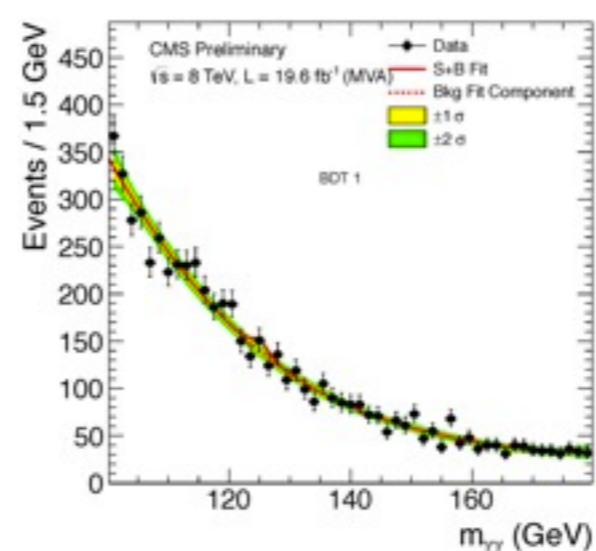
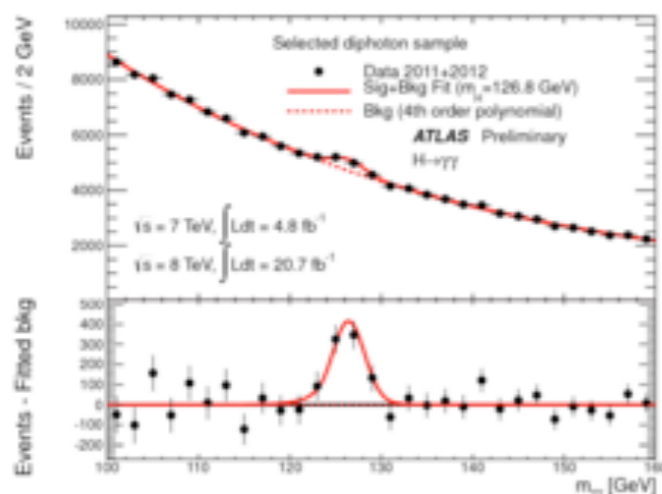
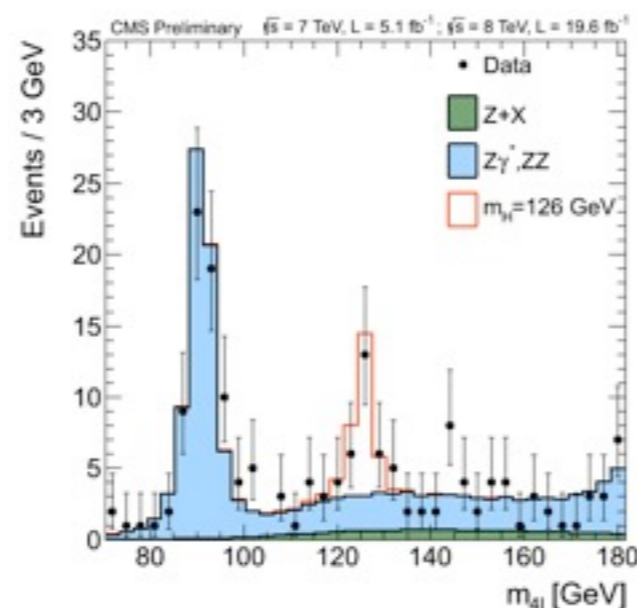
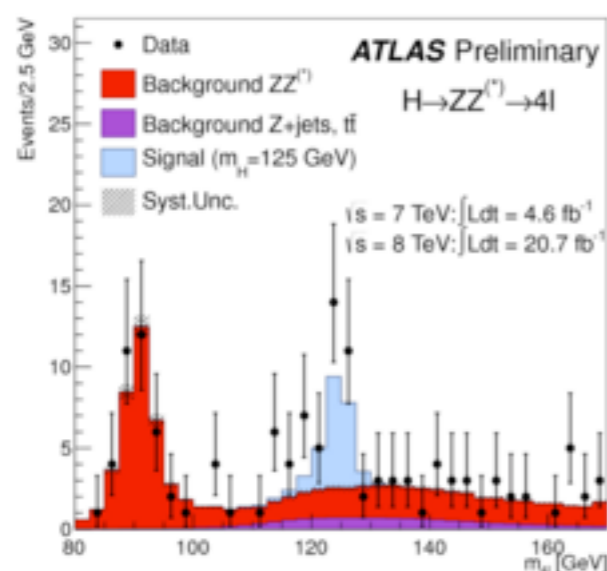
Higgs Mass Workshop, Ann Arbor, Michigan, 12.14.13

# A Standard Model-like Higgs particle has been discovered by the ATLAS and CMS experiments at CERN

We see evidence of this particle in multiple channels.

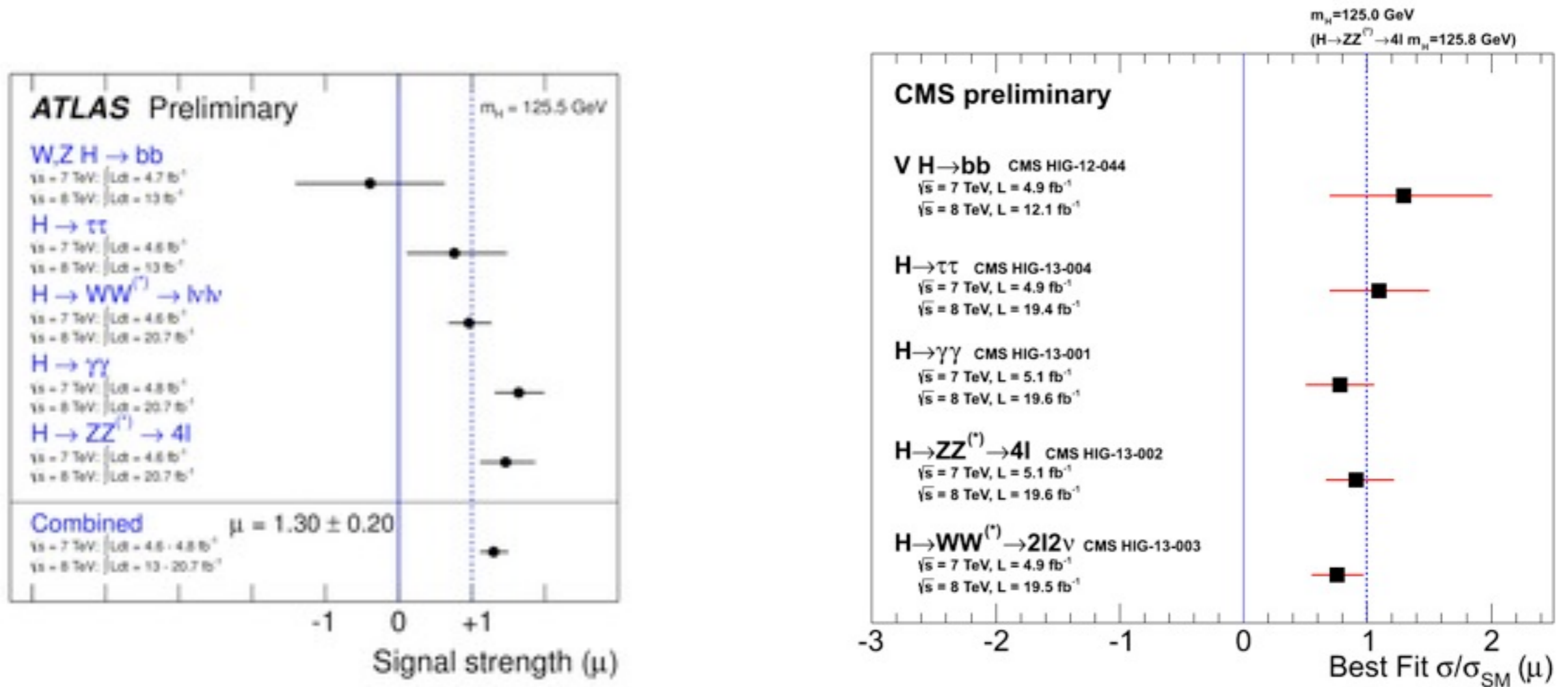
We can reconstruct its mass and we know that is about 125 GeV.

The rates are consistent with those expected in the Standard Model.



But we cannot determine the Higgs couplings very accurately

Large Variations of Higgs couplings are still possible



As these measurements become more precise, they constrain possible extensions of the SM, and they could lead to the evidence of new physics.

It is worth studying what kind of effects one could obtain in well motivated extensions of the Standard Model, like SUSY.

Marcela talking to Peter Higgs  
Fabiola Gianotti listening



# With Oscar Stal, Fabiola Gianotti, Lars Brink and Paul Langacker



With legendary t'Hooft and his wife



Thursday, December 19, 2013

With Brout's wife



I claim no resemblance at all





# With Discovery Announcers





Thursday, December 19, 2013

# Banquet

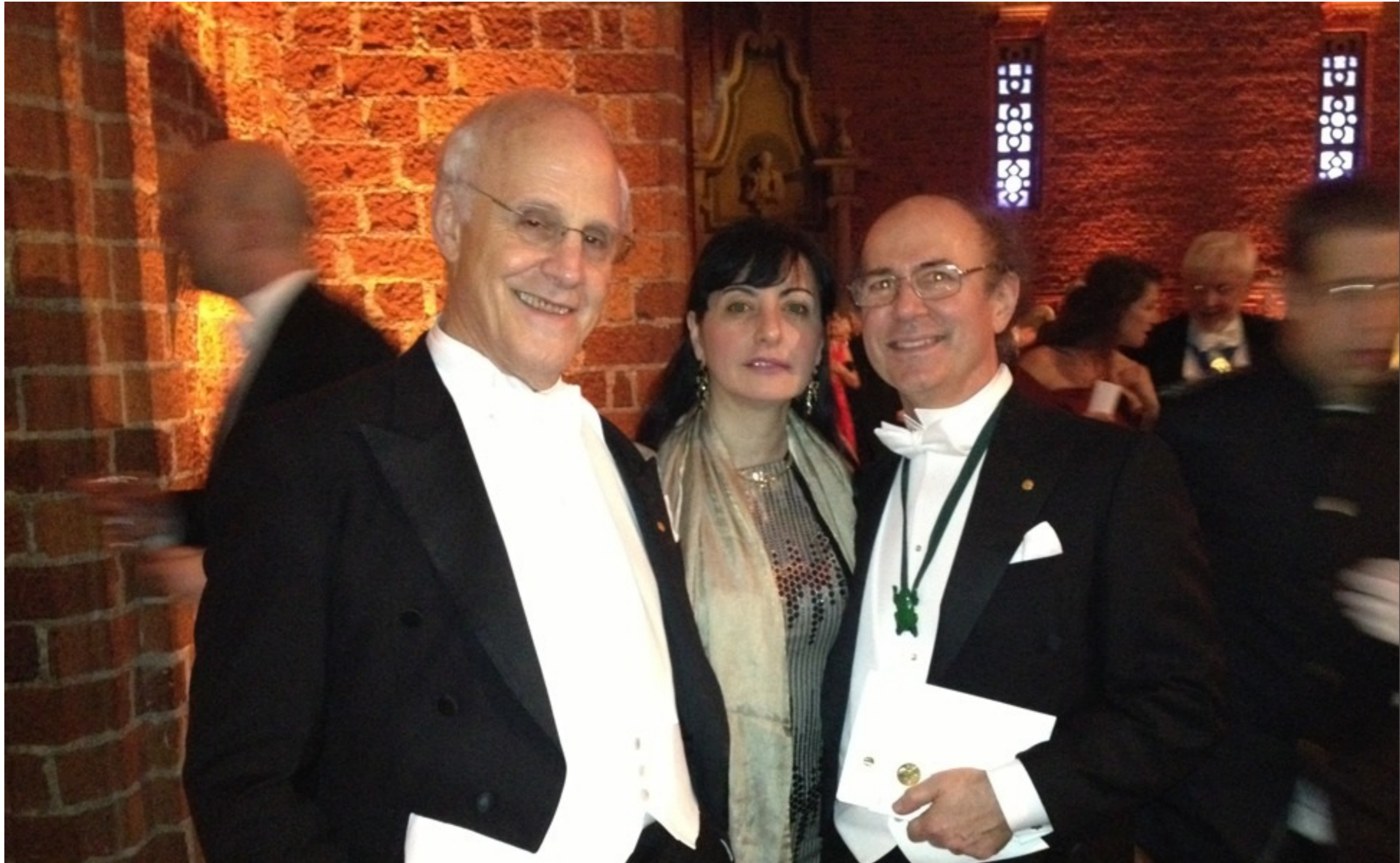


# The Beast and the Beauty



Thursday, December 19, 2013

# Marcela feeling asymptotically free



# The Ballroom



# Lightest SM-like Higgs mass strongly depends on:

- \* CP-odd Higgs mass  $m_A$
- \*  $\tan \beta$
- \* the top quark mass

- \* the stop masses and mixing

$$\mathbf{M}_{\tilde{t}}^2 = \begin{pmatrix} \mathbf{m}_Q^2 + \mathbf{m}_t^2 + \mathbf{D}_L & \mathbf{m}_t \mathbf{X}_t \\ \mathbf{m}_t \mathbf{X}_t & \mathbf{m}_U^2 + \mathbf{m}_t^2 + \mathbf{D}_R \end{pmatrix}$$

$M_h$  depends logarithmically on the averaged stop mass scale  $M_{SUSY}$  and has a quadratic and quartic dep. on the stop mixing parameter  $X_t$ . [ and on sbotton/stau sectors for large  $\tan\beta$ ]

For moderate to large values of  $\tan \beta$  and large non-standard Higgs masses

$$m_h^2 \cong M_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \left[ \frac{1}{2} \tilde{X}_t + t + \frac{1}{16\pi^2} \left( \frac{3}{2} \frac{m_t^2}{v^2} - 32\pi\alpha_3 \right) (\tilde{X}_t t + t^2) \right]$$

$$t = \log(M_{SUSY}^2 / m_t^2) \quad \tilde{X}_t = \frac{2X_t^2}{M_{SUSY}^2} \left( 1 - \frac{X_t^2}{12M_{SUSY}^2} \right) \quad \underline{X_t = A_t - \mu / \tan \beta} \rightarrow \text{LR stop mixing}$$

M.Carena, J.R. Espinosa, M. Quiros, C.W.'95

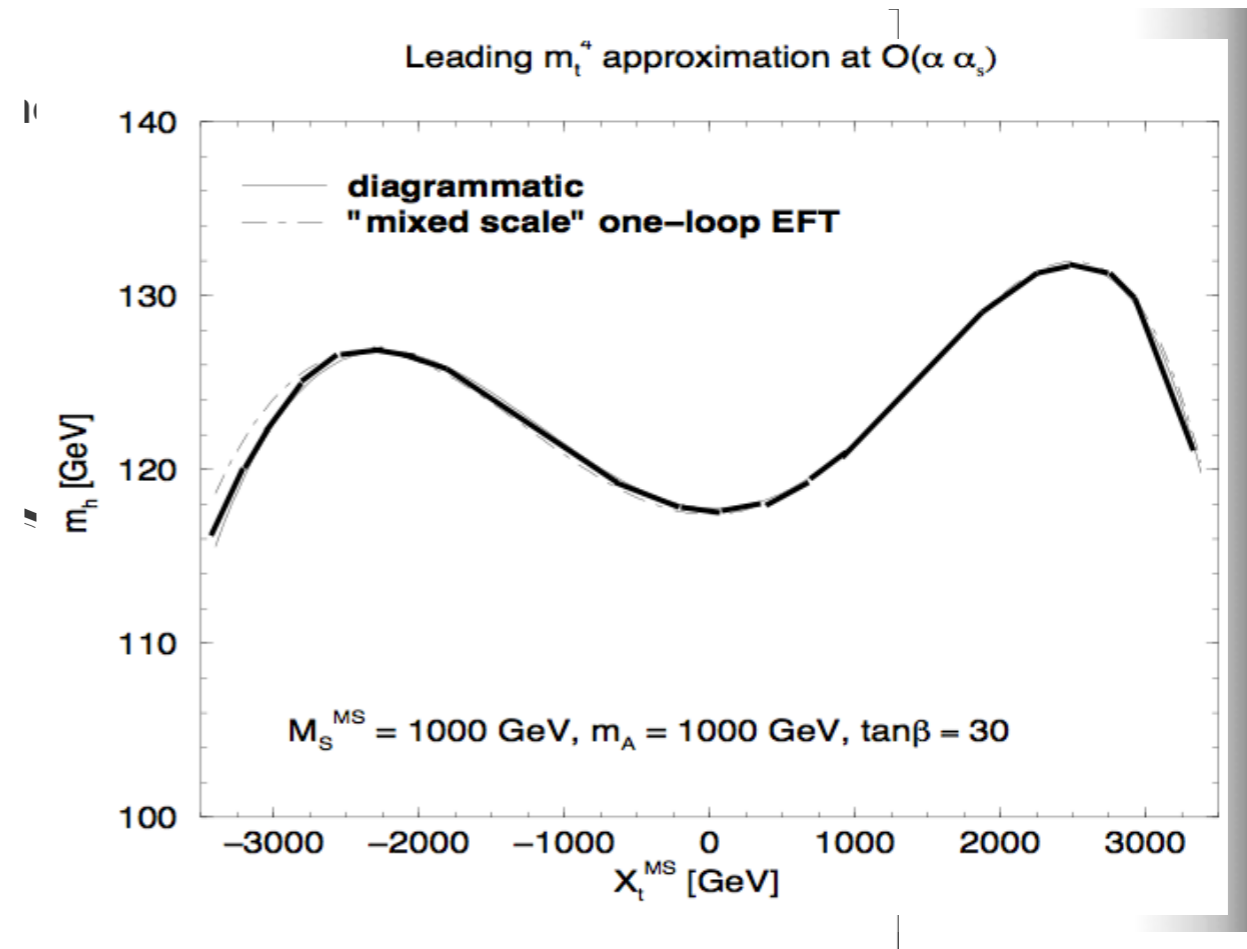
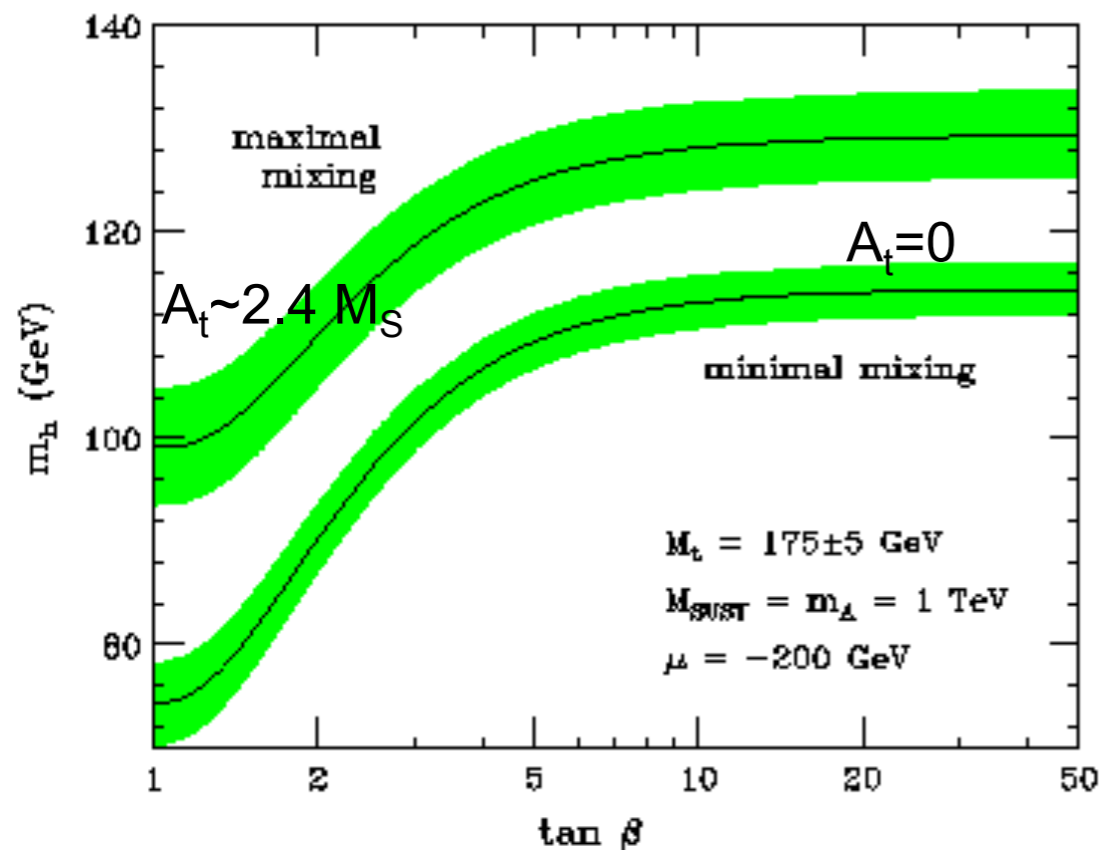
M. Carena, M. Quiros, C.W.'95

Analytic expression valid for  $M_{SUSY} \sim m_Q \sim m_U$

# Standard Model-like Higgs Mass

Long list of two-loop computations: Carena, Degrandi, Ellis, Espinosa, Haber, Harlander, Heinemeyer, Hempfling, Hoang, Hollik, Hahn, Martin, Pilaftsis, Quiros, Ridolfi, Rzehak, Slavich, C.W., Weiglein, Zhang, Zwirner

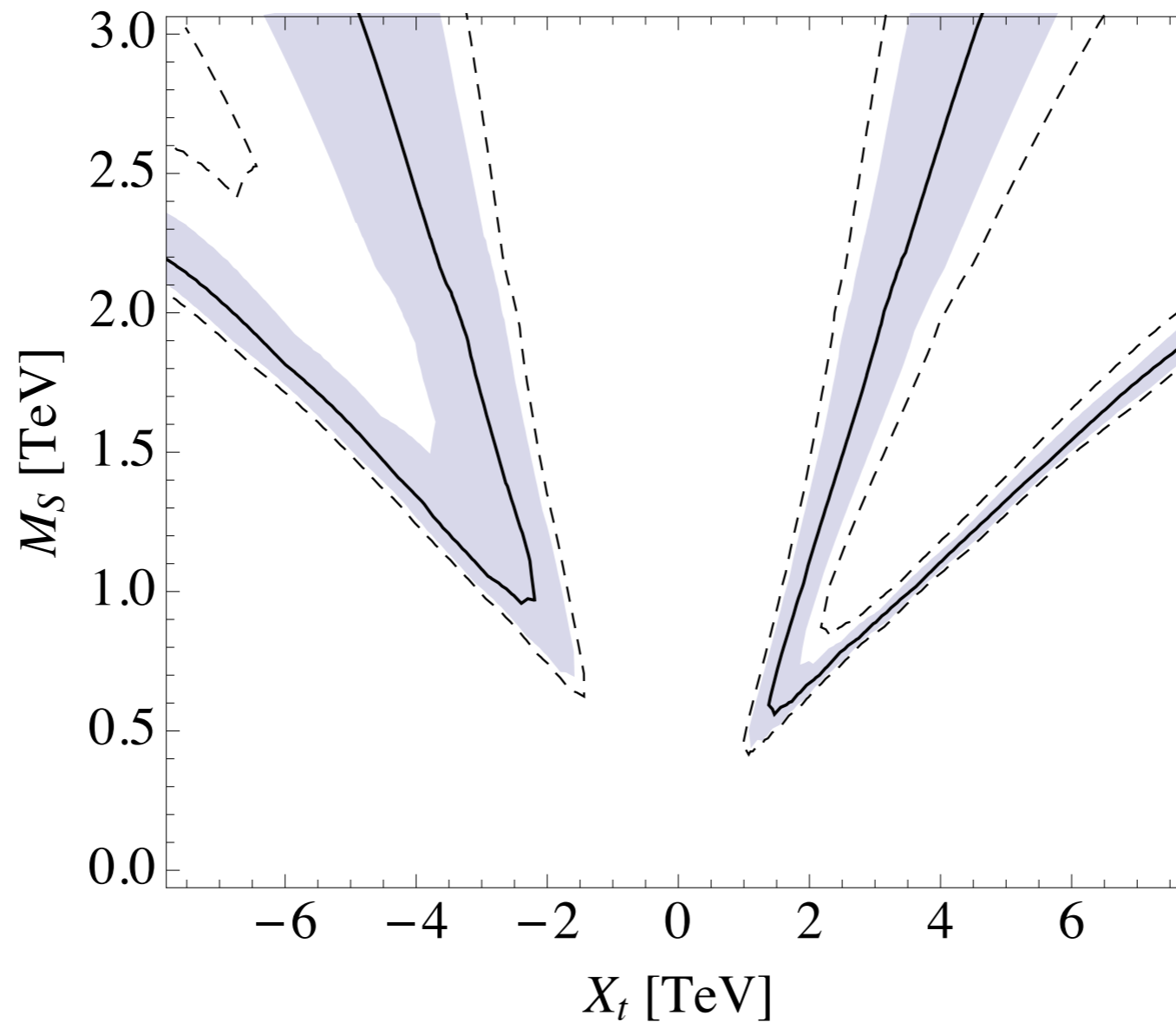
Carena, Haber, Heinemeyer, Hollik, Weiglein, C.W.'00



$$X_t = A_t - \mu / \tan \beta, \quad X_t = 0 : \text{No mixing}; \quad X_t = \sqrt{6} M_S : \text{Max. Mixing}$$



# Large Mixing in the Stop Sector Necessary



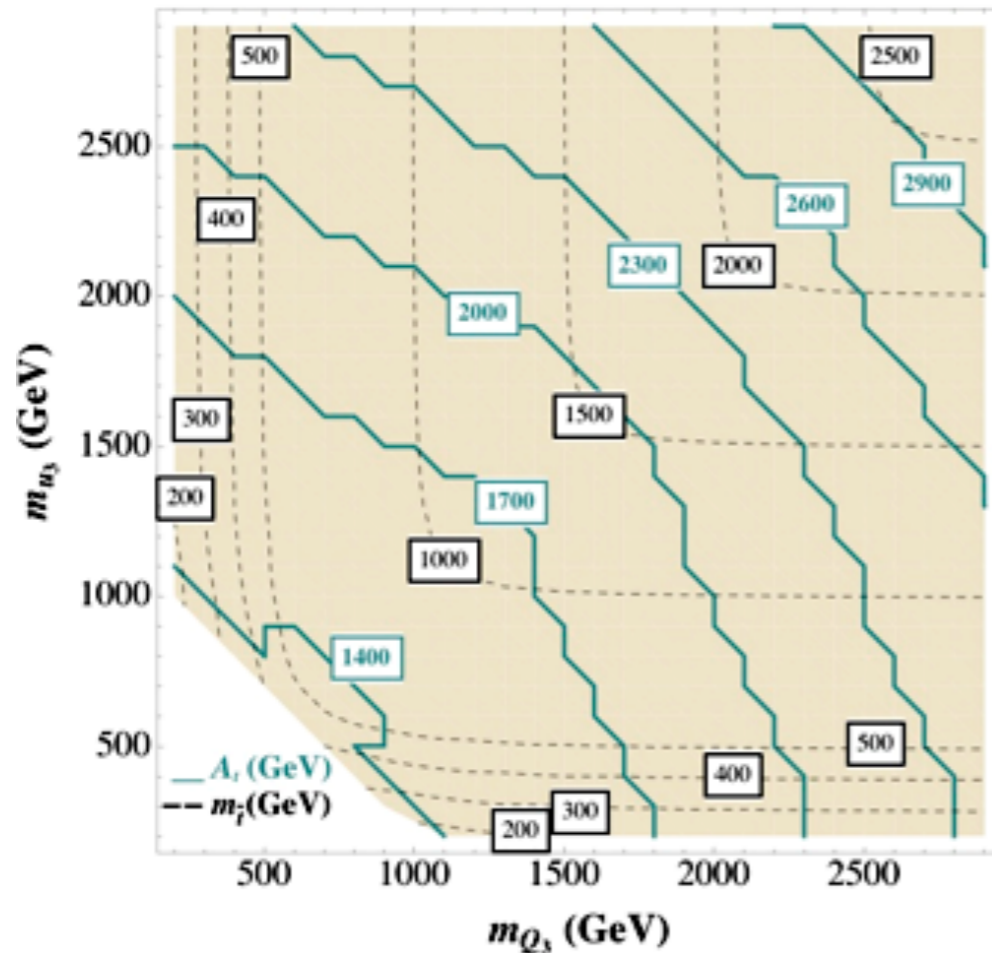
P. Draper, P. Meade, M. Reece, D. Shih' I I  
L. Hall, D. Pinner, J. Ruderman' I I  
M. Carena, S. Gori, N. Shah, C. Wagner' I I  
A. Arbey, M. Battaglia, A. Djouadi, F. Mahmoudi, J. Quevillon' I I  
S. Heinemeyer, O. Stal, G. Weiglein' I I  
U. Ellwanger' I I

...

# Soft supersymmetry Breaking Parameters

M. Carena, S. Gori, N. Shah, C. Wagner, arXiv:1112.336, +L.T.Wang, arXiv:1205.5842

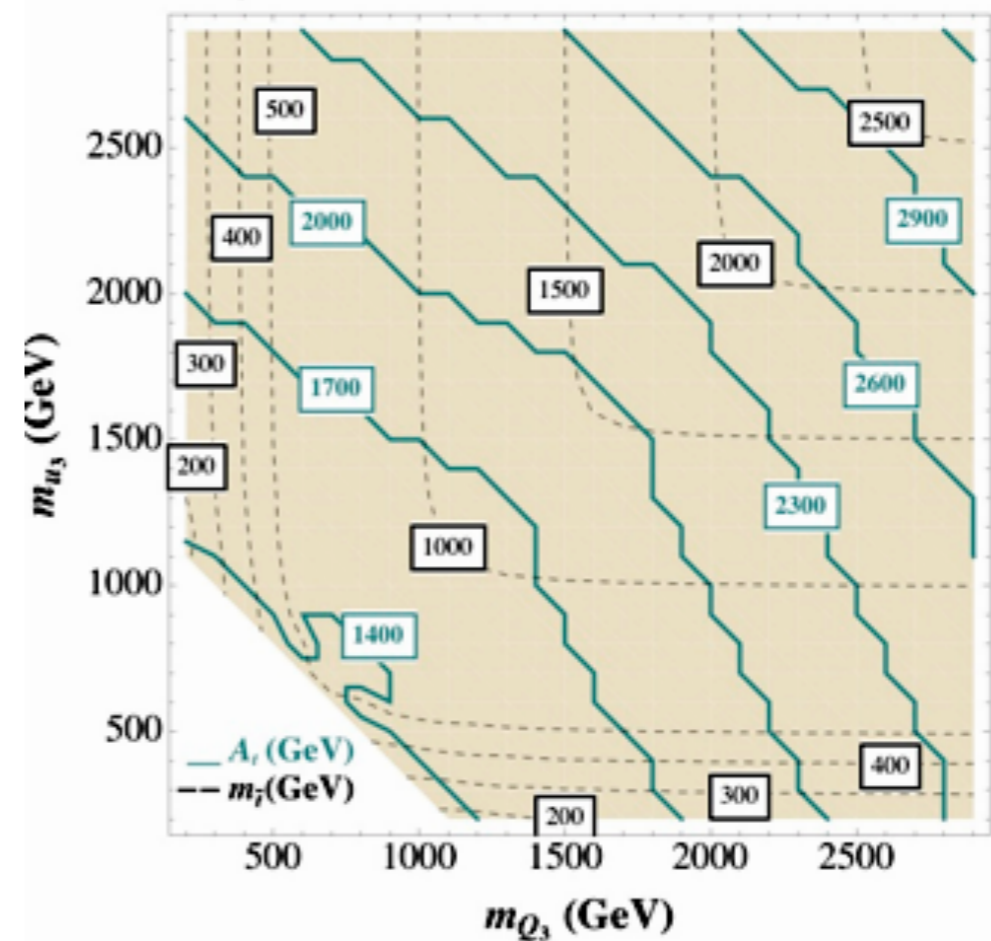
$A_t$  and  $m_{\tilde{t}}$  for  $124 \text{ GeV} < m_h < 126 \text{ GeV}$  and  $\tan \beta = 10$



Large stop sector mixing  
 $A_t > 1 \text{ TeV}$

No lower bound on the lightest stop  
 One stop can be light and the other heavy  
 or  
 in the case of similar stop soft masses.  
 both stops can be below 1 TeV

$A_t$  and  $m_{\tilde{t}}$  for  $124 \text{ GeV} < m_h < 126 \text{ GeV}$  and  $\tan \beta = 60$



Intermediate values of tan beta lead to  
 the largest values of  $m_h$  for the same values  
 of stop mass parameters

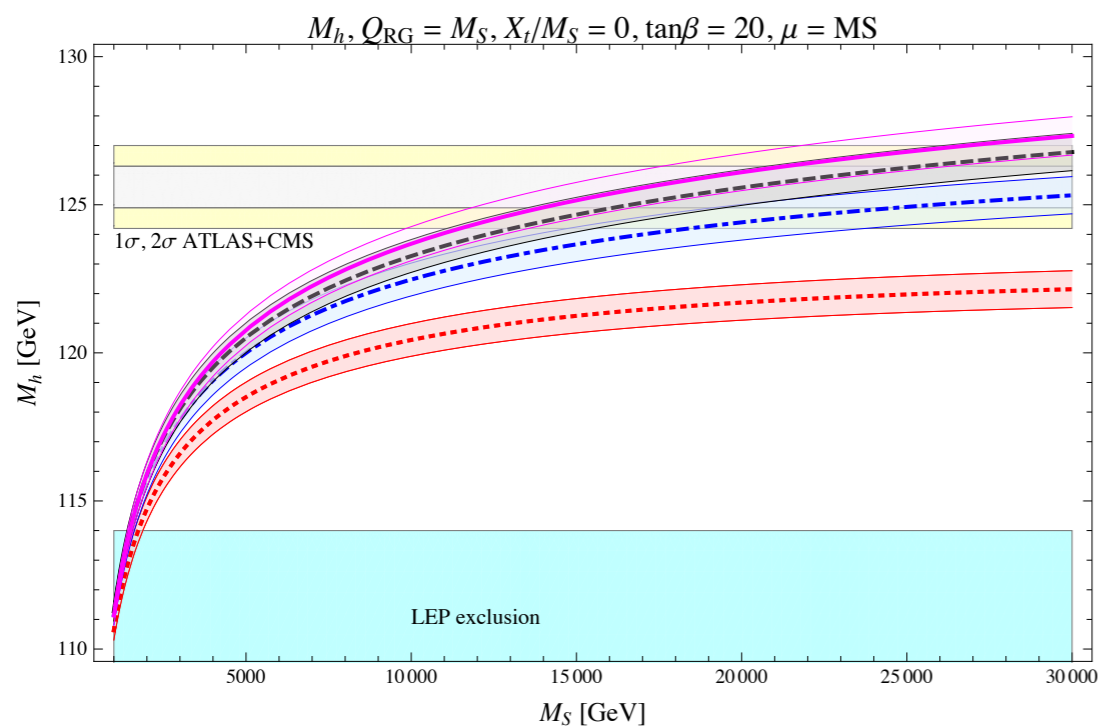
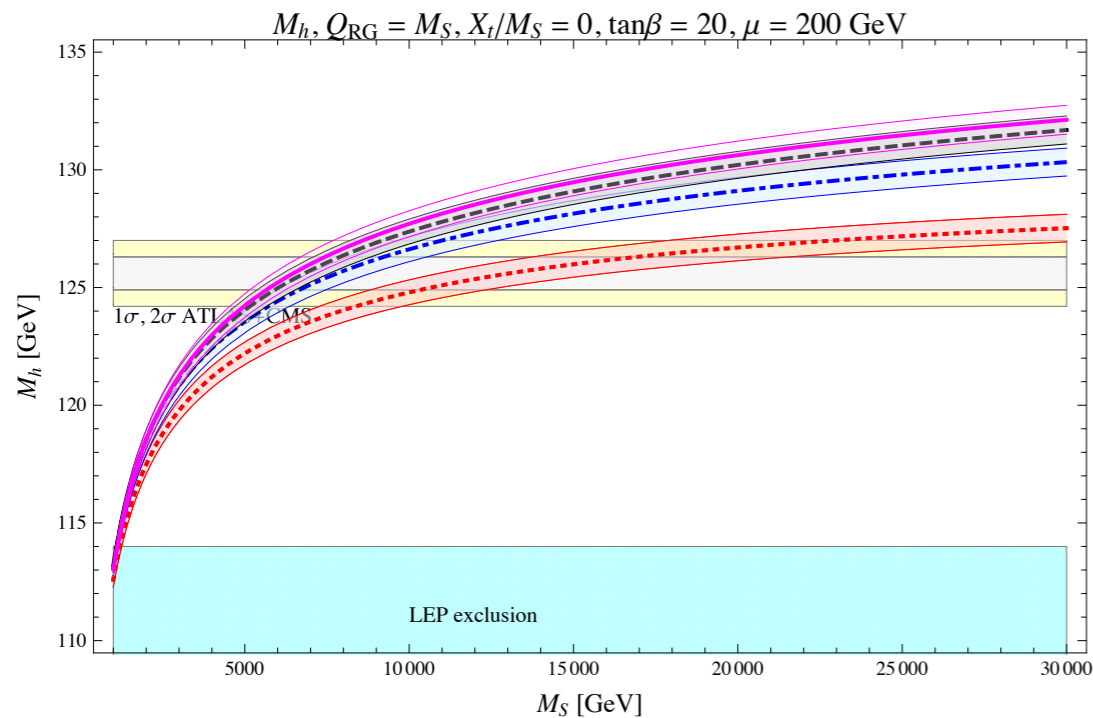
At large tan beta, light staus/sbottoms can decrease  
 $m_h$  by several GeV's via Higgs mixing effects  
 and compensate tan beta enhancement

# Case of heavy Stops

## Impact of higher loops

G. Lee, C.W'13

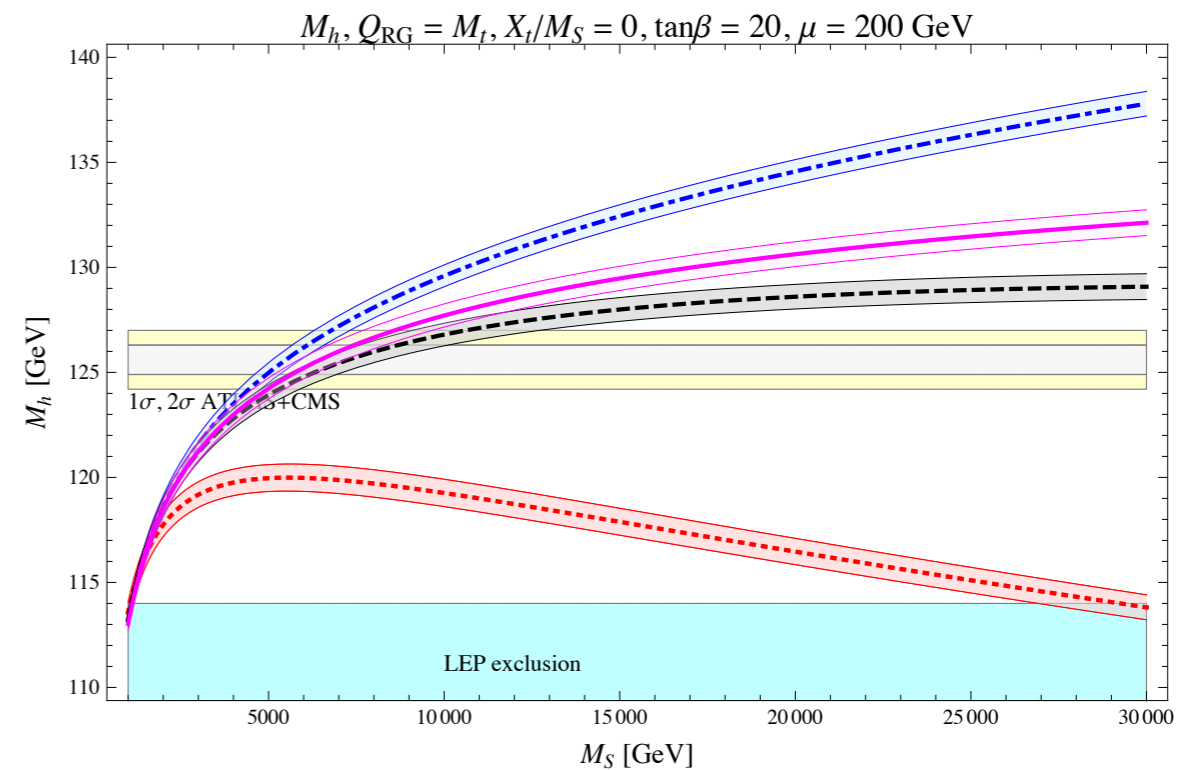
(See also S. Martin'07,  
P. Kant, R. Harlander, L. Mihalla, M. Steinhauser'10  
J. Feng, P. Kant, S. Profumo, D. Sanford'13, )



Recalculation of RG prediction including up to 4 loops in RG expansion.

Agreement with S. Martin'07 and Espinosa and Zhang'00, Carena, Espinosa, Quiros, C.W'00, Carena, Haber, Heinemeyer, Weiglein, Hollik and C.W'00, in corresponding limits.

Two loops results agree w FeynHiggs and CPsuperH results



In supersymmetric theories, there is one Higgs doublet that behaves like the SM one.

$$H_{SM} = H_d \cos \beta + H_u \sin \beta, \quad \tan \beta = v_u/v_d$$

The orthogonal combination may be parametrized as

$$H = \begin{pmatrix} H + iA \\ H^\pm \end{pmatrix}$$

where  $H$ ,  $H^\pm$  and  $A$  represent physical CP-even, charged and CP-odd scalars (non standard Higgs).

Strictly speaking, the CP-even Higgs modes mix and none behave exactly as the SM one.

$$h = -\sin \alpha \operatorname{Re}(H_d^0) + \cos \alpha \operatorname{Re}(H_u^0)$$

In the so-called decoupling limit, in which the non-standard Higgs bosons are heavy,  $\sin \alpha = -\cos \beta$  and one recovers the SM as an effective theory.

# CP-even Higgs Mixing Angle and Alignment

M. Carena, I. Low, N. Shah, C.W., arXiv:1310.2248

$$\sin \alpha = \frac{\mathcal{M}_{12}^2}{\sqrt{\mathcal{M}_{12}^4 + (\mathcal{M}_{11}^2 - m_h^2)^2}}$$

$$-\tan \beta \mathcal{M}_{12}^2 = (\mathcal{M}_{11}^2 - m_h^2) \longrightarrow \sin \alpha = -\cos \beta$$

Condition independent of the CP-odd Higgs mass.

$$\begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix} \begin{pmatrix} -s_\alpha \\ c_\alpha \end{pmatrix} = -\frac{v^2}{m_A^2} \begin{pmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{pmatrix} \begin{pmatrix} -s_\alpha \\ c_\alpha \end{pmatrix} + \frac{m_h^2}{m_A^2} \begin{pmatrix} -s_\alpha \\ c_\alpha \end{pmatrix}$$

## Alignment Conditions

$$(m_h^2 - \lambda_1 v^2) + (m_h^2 - \tilde{\lambda}_3 v^2) t_\beta^2 = v^2 (3\lambda_6 t_\beta + \lambda_7 t_\beta^3) ,$$

$$(m_h^2 - \lambda_2 v^2) + (m_h^2 - \tilde{\lambda}_3 v^2) t_\beta^{-2} = v^2 (3\lambda_7 t_\beta^{-1} + \lambda_6 t_\beta^{-3})$$

- If fulfilled not only alignment is obtained, but also the right Higgs mass,  $m_h^2 = \lambda_{\text{SM}} v^2$ , with  $\lambda_{\text{SM}} \simeq 0.26$  and  $\lambda_3 + \lambda_4 + \lambda_5 = \tilde{\lambda}_3$

$$\lambda_{\text{SM}} = \lambda_1 \cos^4 \beta + 4\lambda_6 \cos^3 \beta \sin \beta + 2\tilde{\lambda}_3 \sin^2 \beta \cos^2 \beta + 4\lambda_7 \sin^3 \beta \cos \beta + \lambda_2 \sin^4 \beta$$

- For  $\lambda_6 = \lambda_7 = 0$  the conditions simplify, but can only be fulfilled if

$$\lambda_1 \geq \lambda_{\text{SM}} \geq \tilde{\lambda}_3 \quad \text{and} \quad \lambda_2 \geq \lambda_{\text{SM}} \geq \tilde{\lambda}_3 ,$$

or

$$\lambda_1 \leq \lambda_{\text{SM}} \leq \tilde{\lambda}_3 \quad \text{and} \quad \lambda_2 \leq \lambda_{\text{SM}} \leq \tilde{\lambda}_3$$

- Conditions not fulfilled in the MSSM, where both  $\lambda_1, \tilde{\lambda}_3 < \lambda_{\text{SM}}$

# Details of the Calculation

Tree-level coupling, should be evaluated at the SUSY breaking scale :

$$\lambda_{\text{tree}} = \frac{1}{4}(g_Y^2 + g_2^2)c_{2\beta}^2$$

Simplified stop spectrum :

$$\mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 + m_t^2 + c_{2\beta}\left(\frac{1}{2} - \frac{2}{3}s_W^2\right)m_Z^2 & m_t X_t \\ m_t X_t & m_{\tilde{t}_R}^2 + m_t^2 + \frac{2}{3}c_{2\beta}s_W^2 m_Z^2 \end{pmatrix} \quad \mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} M_S^2 & m_t X_t \\ m_t X_t & M_S^2 \end{pmatrix}$$

$$m_{\tilde{t}_{1,2}}^2 = M_S^2 \mp |m_t X_t|.$$

This approximation is abandoned at the one-loop level, in the evaluation of the thresholds to the quartic coupling

# One loop thresholds to the quartic coupling and Yukawas in the MS scheme

$$\Delta_{\text{th}}^{(\alpha_t)} \lambda = 6\kappa h_t^4 s_\beta^4 \widehat{X}_t^2 \left(1 - \frac{\widehat{X}_t^2}{12}\right) + \frac{3}{4}\kappa h_t^2 s_\beta^2 (g_2^2 + g_Y^2) \widehat{X}_t^2 c_{2\beta},$$

$$\Delta_{\text{th}}^{(\alpha_b)} \lambda = -\frac{1}{2}\kappa h_b^4 s_\beta^4 \hat{\mu}^4,$$

$$\Delta_{\text{th}}^{(\alpha_\tau)} \lambda = -\frac{1}{6}\kappa h_\tau^4 s_\beta^4 \hat{\mu}^4,$$

$$y_t = h_t s_\beta, \quad y_b = h_b c_\beta, \quad y_\tau = h_\tau c_\beta;$$

$$h_t = \frac{y_t}{s_\beta} \frac{1}{1 - \kappa(\Delta h_t + \cot \beta \delta h_t)},$$

$$h_b = \frac{y_b}{c_\beta} \frac{1}{1 - \kappa(\Delta h_b + t_\beta \delta h_b)},$$

$$h_\tau = \frac{y_\tau}{c_\beta} \frac{1}{1 - \kappa t_\beta \delta h_\tau},$$



$$\Delta h_t = \frac{8}{3} g_3^2 m_{\tilde{g}} X_t I(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{g}}) - h_b^2 \mu \cot \beta X_b I(m_{\tilde{b}_1}, m_{\tilde{b}_2}, \mu), \quad (14)$$

$$\begin{aligned} \delta h_t = & g_2^2 M_2 \mu \left( [c_b^2 I(m_{\tilde{b}_1}, M_2, \mu) + s_b^2 I(m_{\tilde{b}_2}, M_2, \mu)] + \frac{1}{2} [c_t^2 I(m_{\tilde{t}_1}, M_2, \mu) + s_t^2 I(m_{\tilde{t}_2}, M_2, \mu)] \right) \\ & + \frac{1}{3} g_Y^2 M_1 \left( \frac{2}{3} X_t t_\beta I(m_{\tilde{t}_1}, m_{\tilde{t}_2}, M_1) - \frac{1}{2} \mu [c_t^2 I(m_{\tilde{t}_1}, M_1, \mu) + s_t^2 I(m_{\tilde{t}_2}, M_1, \mu)] \right. \\ & \left. + 2\mu [s_t^2 I(m_{\tilde{t}_1}, M_1, \mu) + c_t^2 I(m_{\tilde{t}_2}, M_1, \mu)] \right), \end{aligned} \quad (15)$$

$$\Delta h_b = \frac{8}{3} g_3^2 m_{\tilde{g}} X_b I(m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{g}}) - h_t^2 \mu t_\beta X_t I(m_{\tilde{t}_1}, m_{\tilde{t}_2}, \mu), \quad (16)$$

$$\begin{aligned} \delta h_b = & g_2^2 M_2 \mu \left( [c_t^2 I(m_{\tilde{t}_1}, M_2, \mu) + s_t^2 I(m_{\tilde{t}_2}, M_2, \mu)] + \frac{1}{2} [c_b^2 I(m_{\tilde{b}_1}, M_2, \mu) + s_b^2 I(m_{\tilde{b}_2}, M_2, \mu)] \right) \\ & + \frac{1}{3} g_Y^2 M_1 \left( -\frac{1}{3} X_b \cot \beta I(m_{\tilde{b}_1}, m_{\tilde{b}_2}, M_1) + \frac{1}{2} \mu [c_b^2 I(m_{\tilde{b}_1}, M_1, \mu) + s_b^2 I(m_{\tilde{b}_2}, M_1, \mu)] \right. \\ & \left. + \mu [s_b^2 I(m_{\tilde{b}_1}, M_1, \mu) + c_b^2 I(m_{\tilde{b}_2}, M_1, \mu)] \right), \end{aligned} \quad (17)$$

$$\begin{aligned} \delta h_\tau = & g_2^2 M_2 \mu \left( I(m_{\tilde{\nu}_\tau}, M_2, \mu) + \frac{1}{2} [c_\tau^2 I(m_{\tilde{\tau}_1}, M_2, \mu) + s_\tau^2 I(m_{\tilde{\tau}_2}, M_2, \mu)] \right) \\ & - g_Y^2 M_1 \left( X_\tau \cot \beta I(m_{\tilde{\tau}_1}, m_{\tilde{\tau}_2}, M_1) + \frac{1}{2} \mu [c_\tau^2 I(m_{\tilde{\tau}_1}, M_1, \mu) + s_\tau^2 I(m_{\tilde{\tau}_2}, M_1, \mu)] \right. \\ & \left. - \mu [s_\tau^2 I(m_{\tilde{\tau}_1}, M_1, \mu) + c_\tau^2 I(m_{\tilde{\tau}_2}, M_1, \mu)] \right), \end{aligned} \quad (18)$$

$$I(a, b, c) = \frac{ab \log(a/b) + bc \log(b/c) + ac \log(c/a)}{(a-b)(b-c)(a-c)}$$

$$I(1, 1, 1 - \delta) = \frac{1}{2} + \frac{\delta}{6} + \dots \quad (\mu \sim M_S),$$

$$I(1, 1, \delta) = 1 + \delta(1 + \log \delta) + \dots \quad (\mu \ll M_S).$$

## Two loop thresholds to the quartic Couplings in the SM scheme

$$\Delta_{\text{th}}^{(\alpha_s \alpha_t)} \lambda = 16\kappa^2 h_t^4 g_3^2 \left\{ -2\hat{X}_t^2 + \frac{1}{3}\hat{X}_t^3 - \frac{1}{12}\hat{X}_t^4 \right\},$$

$$\Delta_{\text{th}}^{(\alpha_t^2)} \lambda = 3\kappa^2 h_t^6 \left\{ -\frac{3}{2} + 6\hat{\mu}^2 - 2(4 + \hat{\mu}^2)f_1(\hat{\mu}) + 3\hat{\mu}^2 f_2(\hat{\mu}) + 4f_3(\hat{\mu}) - \frac{\pi^2}{3} \right.$$

$$+ \left[ -\frac{17}{2} - 6\hat{\mu}^2 - (4 + 3\hat{\mu}^2)f_2(\hat{\mu}) + (4 - 6\hat{\mu}^2)f_1(\hat{\mu}) \right] \hat{X}_t^2$$

$$+ \left[ 23 + 4s_\beta^2 + 4\hat{\mu}^2 + 2f_2(\hat{\mu}) - 2(1 - 2\hat{\mu}^2)f_1(\hat{\mu}) \right] \frac{\hat{X}_t^4}{4} - \frac{13}{24}\hat{X}_t^6 s_\beta^2$$

$$+ c_\beta^2 \left[ -\frac{9}{2} + 60K + \frac{4\pi^2}{3} + \left(\frac{27}{2} - 24k\right)\hat{X}_t^2 - 6\hat{X}_t^4 \right.$$

$$- (3 + 16K)(4\hat{X}_t + \hat{Y}_t)\hat{Y}_t + 4(1 + 4K)\hat{X}_t^3 \hat{Y}_t$$

$$\left. + \left(\frac{14}{3} + 24K\right)\hat{X}_t^2 \hat{Y}_t^2 - \left(\frac{19}{12} + 8K\right)\hat{X}_t^4 \hat{Y}_t^2 \right\}.$$

$$K = -\frac{1}{\sqrt{3}} \int_0^{\pi/6} dx \log(2 \cos x) \sim -0.1953256,$$

$$\hat{Y}_t = (A_t - \mu t_\beta)/M_S = \hat{X}_t + \frac{2\hat{\mu}}{\sin 2\beta},$$

$$f_1(\hat{\mu}) = \frac{\hat{\mu}^2}{1 - \hat{\mu}^2} \log \hat{\mu}^2,$$

$$f_2(\hat{\mu}) = \frac{1}{1 - \hat{\mu}^2} \left[ 1 + \frac{\hat{\mu}^2}{1 - \hat{\mu}^2} \log \hat{\mu}^2 \right],$$

$$f_3(\hat{\mu}) = \frac{-1 + 2\hat{\mu}^2 + 2\hat{\mu}^4}{(1 - \hat{\mu}^2)^2} \left[ \log \hat{\mu}^2 \log(1 - \hat{\mu}^2) + Li_2(\hat{\mu}^2) - \frac{\pi^2}{6} - \hat{\mu}^2 \log \hat{\mu}^2 \right]$$

# Evolution of the quartic Coupling

$$\kappa \equiv \frac{1}{16\pi^2}, \quad t \equiv \log Q, \quad \beta_\lambda^{(n,k)}(t) \equiv \frac{d^k \beta_\lambda^{(n)}}{dt^k}(t)$$

We want to evaluate the coupling at the weak scale ( $m_t$ ) starting from the stop mass scale. It can be done in two ways, depending on where the couplings are evaluated. Taking  $L \equiv \tilde{t} - t = \log(\tilde{Q}/Q) > 0$ ,  $\beta_\lambda^{(n)} \equiv \beta_\lambda^{(n,0)}$ .

$$\lambda(Q) = \lambda(\tilde{Q}) - \sum_{n=1}^{\infty} \kappa^n \sum_{k=0}^{\infty} (-1)^k \frac{\beta_\lambda^{(n,k)}(\tilde{t})}{(k+1)!} L^{k+1}$$

These two expressions are not equivalent, and represent a different reorganization of the perturbative expansion. The second one is implemented in CPsuperH. The first one leads to a faster convergence

$$\lambda(\tilde{Q}) = \lambda(Q) + \sum_{n=1}^{\infty} \kappa^n \sum_{k=0}^{\infty} \frac{\beta_\lambda^{(n,k)}(t)}{(k+1)!} L^{k+1}$$

$$\begin{aligned}
\delta_1 \lambda = & \left\{ -12\lambda^2 - \lambda \left[ 12y_t^2 + 12y_b^2 + 4y_\tau^2 - 9g_2^2 - \frac{9}{5}g_1^2 \right] + 12y_t^4 + 12y_b^4 + 4y_\tau^4 \right. \\
& \left. - \frac{9}{4}g_2^4 - \frac{9}{10}g_2^2g_1^2 - \frac{27}{100}g_1^4 \right\} L \\
& + \left\{ -6\lambda \left[ g_2^2 + \frac{1}{5}g_1^2 \right] + \left[ g_2^2 + \frac{3}{5}g_1^2 \right]^2 + 4g_2^4 \left[ 1 - 2s_\beta^2 c_\beta^2 \right] \right\} L_\mu, \tag{46}
\end{aligned}$$

$$\begin{aligned}
\delta_2 \lambda = & \left\{ 144\lambda^3 + \lambda^2 \left[ 216y_t^2 - 108g_2^2 - \frac{108}{5}g_1^2 \right] + \lambda \left[ -18y_t^4 + 27g_2^4 + \frac{54}{5}g_2^2g_1^2 + \frac{81}{25}g_1^4 \right] \right. \\
& + \lambda y_t^2 \left[ -96g_3^2 - 81g_2^2 - 21g_1^2 \right] + y_t^4 \left[ -180y_t^2 + 192g_3^2 + 54g_2^2 + \frac{102}{5}g_1^2 \right] \\
& \left. + y_t^2 \left[ \frac{27}{2}g_2^4 + \frac{27}{5}g_2^2g_1^2 + \frac{81}{50}g_1^4 \right] \right\} L^2 \\
& - \left\{ \left[ 24\lambda + 12y_t^2 - 9g_2^2 - \frac{9}{5}g_1^2 \right] \left[ 6\lambda \left[ g_2^2 + \frac{1}{5}g_1^2 \right]^2 - \left[ g_2^2 + \frac{3}{5}g_1^2 \right]^2 - 4g_2^4 \left[ 1 - 2s_\beta^2 c_\beta^2 \right] \right] \right\} LL_\mu \\
& + \left\{ 3 \left[ g_2^2 + \frac{1}{5}g_1^2 \right] \left[ 6\lambda \left[ g_2^2 + \frac{1}{5}g_1^2 \right]^2 - \left[ g_2^2 + \frac{3}{5}g_1^2 \right]^2 - 4g_2^4 \left[ 1 - 2s_\beta^2 c_\beta^2 \right] \right] \right\} L_\mu^2 \\
& + \left\{ 78\lambda^3 + 72\lambda^2 y_t^2 + \lambda y_t^2 (3y_t^2 - 80g_3^2) - 60y_t^6 + 64g_3^2 y_t^4 \right\} L, \tag{47}
\end{aligned}$$

$$\begin{aligned}
\delta_3 \lambda = & \left\{ -1728\lambda^4 - 3456\lambda^3 y_t^2 + \lambda^2 y_t^2 (-576y_t^2 + 1536g_3^2) \right. \\
& \left. + \lambda y_t^2 (1908y_t^4 + 480y_t^2 g_3^2 - 960g_3^4) + y_t^4 (1548y_t^4 - 4416y_t^2 g_3^2 + 2944g_3^4) \right\} L^3 \\
& + \left\{ -2340\lambda^4 - 3582\lambda^3 y_t^2 + \lambda^2 y_t^2 (-378y_t^2 + 2016g_3^2) \right. \\
& \left. + \lambda y_t^2 (1521y_t^4 + 1032y_t^2 g_3^2 - 2496g_3^4) + y_t^4 (1476y_t^4 - 3744y_t^2 g_3^2 + 4064g_3^4) \right\} L^2 \\
& + \left\{ -1502.84\lambda^4 - 436.5\lambda^3 y_t^2 - \lambda^2 y_t^2 (1768.26y_t^2 + 160.77g_3^2) \right. \\
& + \lambda y_t^2 (446.764\lambda y_t^4 + 1325.73y_t^2 g_3^2 - 713.936g_3^4) \\
& \left. + y_t^4 (972.596y_t^4 - 1001.98y_t^2 g_3^2 + 200.804g_3^4) \right\} L, \tag{48}
\end{aligned}$$

$$\begin{aligned}
\delta_4 \lambda = & \left\{ 20736\lambda^5 + 51840\lambda^4 y_t^2 + \lambda^3 y_t^2 (21600y_t^2 - 23040g_3^2) \right. \\
& + \lambda^2 y_t^2 (-30780y_t^4 - 18720g_3^2 y_t^2 + 14400g_3^4) \\
& + \lambda y_t^2 (-22059y_t^6 + 28512g_3^2 y_t^4 + 10560g_3^4 y_t^2 - 10560g_3^6) \\
& \left. + y_t^4 (-8208y_t^6 + 56016y_t^6 g_3^2 - 84576y_t^2 g_3^4 + 44160g_3^6) \right\} L^4 \\
& + \left\{ 48672\lambda^5 + 101808\lambda^4 y_t^2 + \lambda^3 y_t^2 (30546y_t^2 - 49152g_3^2 y_t^2) \right. \\
& + \lambda^2 y_t^2 (-50292y_t^4 - 40896y_t^2 g_3^2 + 45696g_3^4) \\
& + \lambda y_t^2 (-33903y_t^6 + 41376y_t^4 g_3^2 + 35440g_3^4 y_t^2 - 45184g_3^6) \\
& \left. + y_t^4 (-15588y_t^6 + 86880y_t^4 g_3^2 - 161632y_t^2 g_3^4 + 112256g_3^6) \right\} L^3 \\
& + \left\{ 63228.2\lambda^5 + 72058.1\lambda^4 y_t^2 + \lambda^3 y_t^2 (25004.6y_t^2 - 11993.5g_3^2) \right. \\
& + \lambda^2 y_t^2 (27483.8y_t^4 - 52858y_t^2 g_3^2 + 18215.3g_3^4) \\
& + \lambda y_t^2 (-51279y_t^6 - 5139.56y_t^4 g_3^2 + 50795.3y_t^2 g_3^4 - 33858.8g_3^6) \\
& \left. + y_t^4 (-24318.2y_t^6 + 72896y_t^4 g_3^2 - 73567.3y_t^2 g_3^4 + 36376.5g_3^6) \right\} L^2. \\
M_h^2 = & \lambda(M_t)v^2(M_t) + \kappa \left\{ 3y_t^2 (4\bar{m}_t^2 - m_h^2) B_0(\bar{m}_t, \bar{m}_t, m_h) - \frac{9}{2} \lambda m_h^2 \left[ 2 - \frac{\pi}{\sqrt{3}} - \log \frac{m_h^2}{Q^2} \right] \right. \\
& - \frac{v^2}{4} \left[ 3g_2^4 - 4\lambda g_2^2 + 4\lambda^2 \right] B_0(m_W, m_W, m_h) \\
& - \frac{v^2}{8} \left[ 3(g_2^2 + g_Y^2)^2 - 4\lambda(g_2^2 + g_Y^2) + 4\lambda^2 \right] B_0(m_Z, m_Z, m_h) \\
& \left. + \frac{1}{2} g_2^4 \left[ g_2^2 - \lambda \left( \log \frac{m_W^2}{Q^2} - 1 \right) \right] + \frac{1}{4} (g_2^2 + g_Y^2) \left[ (g_2^2 + g_Y^2) - \lambda \left( \log \frac{m_Z^2}{Q^2} - 1 \right) \right] \right\}
\end{aligned}$$

