Local Equilibrium Controllability of the Triaxial Attitude Control Testbed

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Triaxial Attitude Control Testbed (TACT)

- A rigid base body is supported at a fixed pivot point by a triaxial air bearing; thus the base body can rotate in three dimensions without friction.
- Control actuators, such as reaction wheels and proof mass actuators, are mounted on the base body; these actuators have additional degrees of freedom.
- Uniform constant gravity acts on the system.
Triaxial Attitude Control Testbed
Physical Facility
TACT Assumptions

• TACT configuration
  – Rotation matrix describes the base body attitude
  – Generalized shape coordinates $r = (r_1, \ldots, r_n) \in Q_S$
  – Configuration element $q = (R, r) \in Q = SO(3) \times Q_S$

• Underactuation assumptions
  – All degrees of freedom of the actuators are actuated
  – No base body rotational degrees of freedom are actuated

• Control objectives
  – Transfer between two equilibrium solutions
    • Specified terminal configuration in $SO(3) \times Q_S$
TACT Equations of Motion

- Lagrangian formulation
  - Reduced Lagrangian
    \[ L(a, \vec{\omega}, r, \dot{r}) = T(a, r, \dot{r}) \| V(a, r) \]
  - The kinetic energy is
    \[ T(a, r, \dot{r}) = \frac{1}{2} \left( a^T \dot{r}^T \right) M(r) \dot{\omega} \]
  - The potential energy is
    \[ V(a, r) = m_T a_g a^T \dot{a}_c(r) + V_s(r) \]

where \( \vec{\omega} \) is angular velocity of base body and the direction of gravity in the body frame is given by \( \vec{\omega} = R^T e_3 \)
TACT Equations of Motion

- Euler-Poincare-Lagrange equations
  - Base body dynamics
    \[
    \frac{d}{dt} \frac{\partial L}{\partial \dot{r}} - \frac{\partial L}{\partial r} = \dot{\mathbf{s}} + \frac{\partial L}{\partial \mathbf{I}}
    \]
  - Shape equations
    \[
    \frac{d}{dt} \frac{\partial L}{\partial \dot{\mathbf{r}}} \frac{\partial L}{\partial \mathbf{r}} = \dot{\mathbf{s}}
    \]
  - Base body kinematics
    \[
    \dot{\mathbf{I}} = \mathbf{I} \times \mathbf{I}
    \]
TACT Equations of Motion

- TACT equations of motion

\[
M(r) \ddot{r} = \frac{dM(r)}{dt} \dot{r} + \left[ \begin{array}{c} \frac{\partial V_s(r)}{\partial r} \end{array} \right] + m_T a_g \left[ \begin{array}{c} \frac{\partial \mathcal{O}_c(r)}{\partial r} \end{array} \right] + \left[ \begin{array}{c} 0 \end{array} \right]
\]

- Equilibrium conditions

\[
\mathcal{O}_e \cdot \mathcal{O}_c (r_e) = 0, \quad \frac{\partial V_s(r_e)}{\partial r} + m_T a_g \left[ \begin{array}{c} \frac{\partial \mathcal{O}_c(r_e)}{\partial r} \end{array} \right]^T \mathcal{O}_e = \mathcal{S}_e
\]

- Equilibrium manifold

\[
S_e = \{(R,r_e) \mid \mathcal{O}_e = R \mathcal{O}_e, R \in SO(3)\}
\]

- Conservation law

\[
\mathcal{O}^T (M_{11}(r) \mathcal{O} + M_{12}(r) \dot{r}) = 0
\]
Some Preliminaries

• The TACT has nontrivial symmetries and conservation laws
  – The TACT is not “state” controllable
  – The TACT is not “linearly” controllable
• The TACT may be controllable in a weaker sense
• Equilibrium controllability
  – An equilibrium configuration (zero velocity) can be transferred to any nearby equilibrium configuration (zero velocity)
• Development is based on configuration controllability results of Lewis and Murray
Equilibrium Controllability

- Expressed in terms of families of vector fields on the configuration manifold
- Basic vector fields
  - Potential vector fields
  - Control vector fields
- Iterated vector fields
  - Lie brackets
  - Symmetric products
- Configuration controllability requires checking
  - Configuration accessibility
  - Good and bad vector field properties for iterated symmetric products
Equilibrium Controllability for TACT

• Control vector fields
  – Let $Y^A$ denote the set of control vector fields on $Q$
    \[ Y_i^A = A_i(r) e_i \quad i = 1, \ldots, n \]
    where the mechanical connection is

  – Let $\text{grad} \ V$ denote the potential vector field on $Q$
    \[ \text{grad} \ V = M(r) 0 \]

TACT Configuration Accessibility

– Configuration accessibility

– There are general conditions for configuration accessibility that involve both the control vector fields and the potential vector field

– Conditions that involve only the control vector fields are
  
  • If \( \text{rank} \left\{ \text{Lie}(Y^A) \right\} = \text{dim}(SO(3)) + \text{dim}(Q_S) \)

  then the TACT is configuration accessible

– Configuration accessibility is local property
TACT Equilibrium Controllability

• Equilibrium controllability
  – Assume the TACT is configuration accessible. If every bad symmetric product from \{Y^A\} is a linear combination of lower degree good symmetric products from \{Y^A, \text{grad } V\}, then the TACT is equilibrium controllable.

• Equilibrium controllability is a local property.

• Sufficient condition for controllability (Shen)
  – If the TACT is configuration accessible and
    \[
    \dim \langle \text{span} \left[ \nabla_c(r) \cdot A_1(r) + \frac{\partial \nabla_c(r)}{\partial r_1}, \ldots, \nabla_c(r) \cdot A_n(r) + \frac{\partial \nabla_c(r)}{\partial r_n} \right] \rangle = 3
    \]
    then the TACT is equilibrium controllable.
TACT Controlled by Two Proof Mass Actuators

• Assumptions
  – Two proof mass actuators
  
  \[ \Box_c(r) = \frac{m_p}{m_T} (r_1, r_2, 0)^T \]

• Let \( J_{01} = J_1 + 2m_p l^2 \), \( J_{02} = J_2 + 2m_p l^2 \), and \( J_{03} = J_3 \). If
  
  \[ \begin{align*}
  J_{03} \neq \frac{(J_2 \boxplus J_1)(J_{01} + 2J_{02})}{J_{01}}, & \quad \text{if } J_1 < J_2; \\
  J_{03} \neq \frac{(J_2 \boxplus J_1)(2J_{01} + J_{02})}{J_{01}}, & \quad \text{if } J_1 > J_2;
  \end{align*} \]

• then the TACT is configuration accessible and equilibrium controllable.
TACT Controlled by Two Reaction Wheels

• Assumptions
  – Two reaction wheel actuators
  – The center of mass vector $\mathbf{r}_c$ is constant and nonzero

• If $\{A_1,A_2,\mathbf{r}_c\}$ is linearly independent, then the TACT controlled by two reaction wheels is configuration accessible and equilibrium controllable.
TACT Controlled by One Reaction Wheel

• Assumptions
  – One reaction wheel actuator
  – The center of mass vector $\mathbf{c}$ is constant and nonzero

• Suppose
  – $\mathbf{c}^T A = 0$,
  – $\mathbf{c}^T (A + M_{11}^{-1} A) \neq 0$,
  – $(M_{11} \mathbf{c} \times A)^T (A \times M_{11}^{-1} A) \neq 0$.

• Then the TACT controlled by one reaction wheel is configuration accessible and equilibrium controllable.
Conclusions

• Study of TACT, a rotating rigid body with multibody attachments
  – Equilibrium controllability results have been obtained

• Major themes
  – The TACT is an inherently nonlinear control problem
  – Several theoretical results on underactuated TACT control problems have been summarized
  – Several inherently nonlinear control examples for the TACT have been introduced
Conclusions

- There are many open theoretical and applied research problems on control of the TACT (and similar multibody control systems)
  - How can coupling between the base body rotational dynamics and the shape dynamics be exploited to achieve control objectives?
    - Do not cancel the coupling by feedback
    - Exploit recently developed theoretical and computational tools from geometric mechanics and nonlinear control
  - What are the “minimal” actuation assumptions required to control a multibody system?
  - How can these features be used to design new classes of mechanical systems?
    - System design
    - Actuator design
    - Control design