Chapter 3
Solution Manual

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3.1:

Vegard's law states that for an alloy of two compounds XZ and YZ with concentrations defined by $X_xY_{1-x}Z$, the lattice parameter $a(x)$ as a function of alloy concentration x is defined by:

$$a(x) = xa(XZ) + (1-x)a(YZ) \tag{1}$$

where $a(XZ)$ and $a(YZ)$ are the lattice parameters of XZ and YZ respectively.

To lattice match $Ga_xIn_{1-x}P$ and $GaAs$, we have the condition:

$$5.6533 = x(5.4505) + (1-x)(5.8688) \rightarrow x = 0.5152 \tag{2}$$

To lattice match $Ga_xIn_{1-x}As$ and InP, we have the condition:

$$5.8688 = x(5.6533) + (1-x)(6.0584) \rightarrow x = 0.468 \tag{3}$$

To produce a compressive strain of $\frac{a(GaInAs)-a(InP)}{a(InP)} = 0.01$, we have the condition:

$$0.01 = \frac{x(5.6533) + (1-x)(6.0584) - 5.8688}{5.8688} \rightarrow x = 0.3232 \tag{4}$$

3.2:

Using the formula given, the bandgap of $Ga_{.5152}In_{.4848}P$ is given by:

$$E_g^{GaInP} = 1.35 + 0.73(.5152) + 0.7(.5152)^2 = 1.912eV \tag{5}$$

The band offset ratios for typical III-V compounds are:

$$\frac{\Delta E_c}{\Delta E_g} = 0.66 \quad \text{and} \quad \frac{\Delta E_v}{\Delta E_g} = 0.34 \tag{6}$$

Since the bandgap of $GaAs$ is $E_g^{GaAs} = 1.424eV$, the conduction and valence band offsets are given by:

$$\Delta E_c \approx 0.66(1.912 - 1.424) = 0.322eV \tag{7}$$
$$\Delta E_v \approx 0.34(1.912 - 1.424) = 0.166eV \tag{8}$$

3.3:

The lattice constants of Ge and $GaAs$ are 5.6579Åand 5.6533Årespectively. The strain of a Ge layer on a $GaAs$ substrate is thus:

$$\frac{a(Ge) - a(GaAs)}{a(GaAs)} = 8.1368 \times 10^{-4} \tag{9}$$

The lattice constants of Ge and $GaAs$ are matched very well, resulting in a very small strain in the Ge layer. So at least in terms of minimizing non-radiative recombination due to dislocations and defects, it is an excellent heterostructure for light-emission.

3.4:

We first note that the lattice constants for $In_{1-y}Al_yAs$ and $In_{1-x}Ga_xAs$ are given by:

$$a(In_{1-x}Ga_xAs) = x(5.6533) + (1-x)(6.0584)a(In_{1-y}Al_yAs) = y(5.66) + (1-y)(6.0584) \tag{10}$$

To lattice-match with InP, we have the condition:

$$5.8688 = za(InAlAs) + (1-z)a(InGaAs) \tag{11}$$

To lattice match for all values of z, we require:

$$a(InAlAs) = a(InGaAs) = 5.8688 \tag{12}$$

or explicitly in terms of the expressions above:

$$5.8688 = x(5.6533) + (1-x)(6.0584) \rightarrow x = 0.468 \tag{13}$$

$$5.8688 = y(5.66) + (1-y)(6.0584) \rightarrow y = 0.4759 \tag{14}$$

3.5:

The $InAlGaAs$ compound in problem (3.4) has a lattice constant of 5.8688. The relative concentration in lattice-matched $InGaAs$ is found to be:

$$5.8688 = xa(GaAs) + (1-x)a(InAs) \rightarrow x = 0.468 \tag{15}$$

Assuming the bandgap is a linear function of the lattice constant in $InGaAs$, the bandgap of the above compound is, according to Vegard's law:

$$E_g^{InGaAs} = (0.468)E_g^{GaAs} + (1-0.468)E_g^{InAs} = 0.85476eV \tag{16}$$

From the band offset ratio $\frac{\Delta E_c}{\Delta E_g} = 0.66$, we find:

$$\Delta E_g = \frac{0.2}{0.66} = 0.303eV \tag{17}$$

which gives a $InAlGaAs$ bandgap of $E_g^{InAlGaAs} = \Delta E_g + E_g^{InGaAs} = 1.1578eV$. This corresponds to a composition z of:

$$1.1578 = 0.76 + 0.49z + 0.20z^2 \rightarrow z = 0.643 \tag{18}$$