Chapter 1
Solution Manual

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1.1:

Since glass has negligible absorption, we can approximate the index as purely real and find for the reflectivity:

$$\begin{aligned} R &= \frac{(n-1)^2}{(n+1)^2} \\ &= .0413 \end{aligned} \tag{1}$$

The transmission of the interface is thus:

$$\begin{aligned} T &= 1 - R \\ &= .9587 \end{aligned} \tag{2}$$

For a glass window, there are two interfaces, which means the transmission of the window is:

$$T^{window} = T^2 = .9191 \tag{3}$$

1.2:

From table 1.4, we have the index for fused silica and dense flint glass:

$$n^{fs} = 1.460 \qquad n^{df} = 1.746 \tag{4}$$

The ratio of their reflectivities is thus:

$$\begin{aligned} \frac{R^{fs}}{R^{df}} &= \frac{(n^{fs}-1)^2(n^{df}+1)^2}{(n^{fs}+1)^2(n^{df}-1)^2} \\ &= .4738 \end{aligned} \tag{5}$$

1.3:

We are given the complex dielectric constant of CdTe at $\lambda = 500nm$:

$$\tilde{\epsilon}_r = \epsilon_1 + i\epsilon_2 = 8.92 + i2.29 \tag{6}$$

We can then find the real and imaginary parts of the index:

$$n = \frac{1}{\sqrt{2}}\sqrt{\epsilon_1 + \sqrt{\epsilon_1^2 + \epsilon_2^2}} = 2.96 \qquad \kappa = \frac{1}{\sqrt{2}}\sqrt{-\epsilon_1 + \sqrt{\epsilon_1^2 + \epsilon_2^2}} = 0.38 \tag{7}$$

The absorption coefficient and phase velocity in CdTe are thus:

$$\alpha = \frac{4\pi\kappa}{\lambda} = 9.55 \times 10^6 m^{-1} \qquad v = \frac{c}{n} = 1.014 \times 10^8 \frac{m}{s} \tag{8}$$

with a reflectivity:

$$R = \frac{(n-1)^2 + \kappa^2}{(n+1)^2 + \kappa^2} = 25.19\% \tag{9}$$

1.4:

For the absorption of 90% of the light through a thickness z, we have the condition:

$$\begin{aligned} 0.1 &= \frac{I(z)}{I(0)} = e^{-(1.3\times 10^5)z} \\ &\rightarrow z = 17.71 \mu m \end{aligned} \tag{10}$$

1.5:

We first find the imaginary part of the index:

$$\kappa = \frac{\alpha\lambda}{4\pi} = .083 \tag{11}$$

The reflectivity of each interface is thus:

$$R = \frac{(n-1)^2 + \kappa^2}{(n+1)^2 + \kappa^2} = 32.81\% \tag{12}$$

The transmission coefficient of the plate is thus:

$$T = \frac{(1-R)^2 e^{-\alpha\ell}}{1 - R^2 e^{-2\alpha\ell}} = 3.35\% \tag{13}$$

Noting that the optical density does not incorporate reflection losses, the optical density is then:

$$OD = 0.434\alpha\ell = 1.13 \tag{14}$$

1.6:

We can find the absorption coefficient of sea water at $700nm$ according to:

$$\begin{aligned} .002 &= e^{-\alpha(10)} \\ &\rightarrow \alpha = .621 m^{-1} \end{aligned} \tag{15}$$

The complex part of the index is thus:

$$\kappa = \frac{\alpha\lambda}{4\pi} = 3.462 \times 10^{-8} \tag{16}$$

The complex dielectric constant is thus:

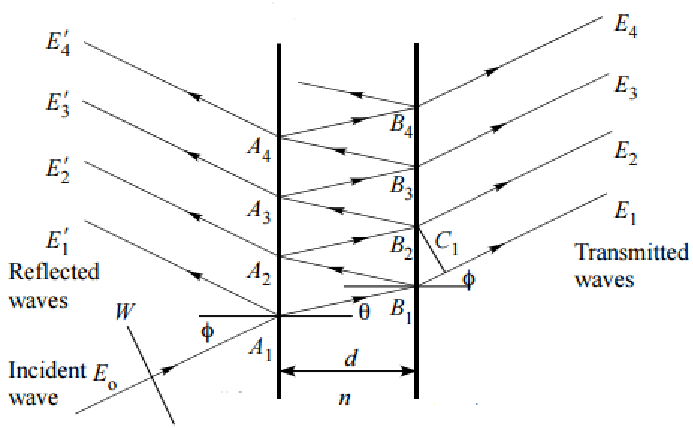
$$\begin{aligned}\tilde{\epsilon}_r &= (n^2 - \kappa^2) + i2n\kappa \\ &= 1.7689 + i(9.21 \times 10^{-8})\end{aligned} \tag{17}$$

1.7:

The color yellow can be formed by mixing red and green (no blue). We thus would expect a single absoprtion peak in the blue.

1.8:

(a)



The incident beam intensity transmitted into the slab is given by $I_0(1 - R_1)$. The different transmitted beams are given by:

$$\begin{aligned}I_1 &= I_0(1 - R_1)(1 - R_2)e^{-\alpha\ell} \\ I_2 &= I_0(1 - R_1)(1 - R_2)R_2R_1e^{-3\alpha\ell} \\ I_3 &= I_0(1 - R_1)(1 - R_2)R_2R_1R_2R_1e^{-5\alpha\ell} \\ &\vdots\end{aligned} \tag{18}$$

The total transmitted intensity is thus:

$$\begin{aligned}I(\ell) &= I_0(1 - R_1)(1 - R_2)e^{-\alpha\ell}\sum_{n=0}^{\infty}\left(R_1R_2e^{-2\alpha\ell}\right)^n \\ &= I_0\frac{(1 - R_1)(1 - R_2)e^{-\alpha\ell}}{1 - R_1R_2e^{-2\alpha\ell}}\end{aligned} \tag{19}$$

which gives the transmission coefficient:

$$T = \frac{I(\ell)}{I_0} = \frac{(1 - R_1)(1 - R_2)e^{-\alpha\ell}}{1 - R_1R_2e^{-2\alpha\ell}} \tag{20}$$

(b) The error between (1.8) and the equation incorporating the reflections for $R_1 = R_2 = R$ is:

$$\begin{aligned}\text{Error} &= \frac{(1-R)^2 e^{-\alpha\ell}}{1-R^2e^{-2\alpha\ell}} - (1-R)^2 e^{-\alpha\ell} \\ &= (1-R)^2 e^{-\alpha\ell}\left[\frac{1}{1-R^2e^{-2\alpha\ell}} - 1\right]\end{aligned} \tag{21}$$

In the approximation $n \gg \kappa$, the reflectivity for silicon is:

$$R = \frac{(3.44-1)^2}{(3.44+1)^2} = 29.75\% \tag{22}$$

(i) For $\alpha\ell = 0$, the error is:

$$\text{Error} = 4.79\% \tag{23}$$

(ii) For $\alpha\ell = 1$, the error is:

$$\text{Error} = 0.22\% \tag{24}$$

(iii) The reflectivity of the sapphire interface is:

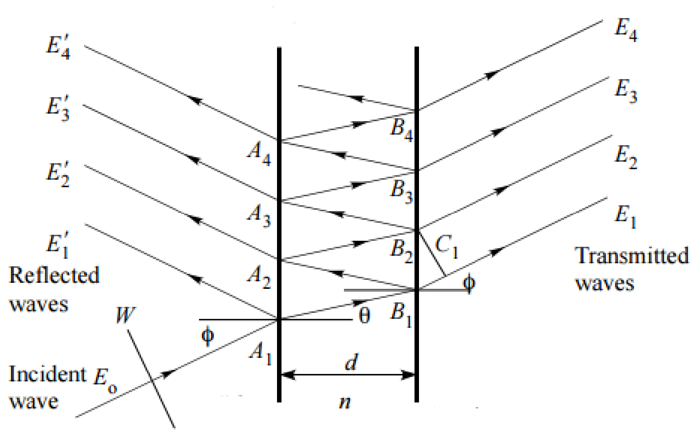
$$R = \frac{(1.77-1)^2}{(1.77+1)^2} = 7.73\% \tag{25}$$

which gives an error:

$$\text{Error} = .511\% \tag{26}$$

We thus see that the reflections can be neglected in two cases: (1) Low-index materials. (2) High-index materials with a high absorption coefficient.

1.9:



(a) To include interference effects, we must calculate the fields, not intensities. The optical path difference between E_1 and E_2 is given by:

$$\delta = \frac{2nd}{cos(\theta)} - 2dtan(\theta)sin(\phi) \tag{27}$$

which corresponds to a phase difference of:

$$\begin{aligned}
\Delta\phi &= \frac{2\pi}{\lambda}\delta = \frac{4\pi d}{\lambda cos(\theta)}\left[n - sin(\theta)sin(\phi)\right] \\
&= \frac{2\pi}{\lambda}\delta = \frac{4\pi d}{\lambda cos(\theta)}\left[n - sin(\theta)sin(\phi)\right] \\
&= \frac{4\pi nd}{\lambda cos(\theta)}\left[1 - sin^2(\theta)\right] \\
&= \frac{4\pi nd}{\lambda} cos(\theta) \qquad (28)
\end{aligned}$$

The transmitted fields are given by[1]:

$$\begin{aligned}
E_1 &= E_0(1-R)e^{-\frac{\alpha}{2}\ell} \\
E_2 &= E_0(1-R)^2 R e^{-3\frac{\alpha}{2}\ell} e^{-i\Delta\phi} \\
E_3 &= E_0(1-R)^3 R^2 e^{-5\frac{\alpha}{2}\ell} e^{-2i\Delta\phi} \\
&\vdots \qquad (29)
\end{aligned}$$

which gives the total transmitted field:

$$\begin{aligned}
E_T &= E_0(1-R)e^{-\frac{\alpha}{2}\ell}\sum_{n=0}^{\infty}\left(Re^{-\alpha\ell}e^{-i\Delta\phi}\right)^n \\
&= \frac{(1-R)e^{-\frac{\alpha}{2}\ell}}{1 - Re^{-\alpha\ell}e^{-i\Delta\phi}} \qquad (30)
\end{aligned}$$

The transmitted intensity is then:

$$I_T = |E_T|^2 = \frac{E_0^2(1-R)^2 e^{-\alpha\ell}}{1 - 2Re^{-\alpha\ell}cos(\Delta\phi) + R^2 e^{-2\alpha\ell}} \qquad (31)$$

(b) The reflected fields are then:

$$\begin{aligned}
E_1' &= E_0\sqrt{R} \\
E_2' &= E_0\sqrt{R}(1-R)e^{-\alpha\ell}e^{-i\Delta\phi}e^{-i\pi} \\
E_3' &= E_0\sqrt{R}(1-R)^2 e^{-2\alpha\ell}e^{-2i\Delta\phi}e^{-i2\pi} \\
&\vdots \qquad (32)
\end{aligned}$$

which gives the total reflected field:

$$\begin{aligned}
E_R &= E_0\sqrt{R}\sum_{n=0}^{\infty}\left[(1-R)e^{-\alpha\ell}e^{-i\Delta\phi}e^{-i\pi}\right]^n \\
&= \frac{E_0\sqrt{R}}{1-(1-R)e^{-\alpha\ell}e^{-i\Delta\phi}e^{-i\pi}} \\
&= \frac{E_0\sqrt{R}(1+e^{-\alpha\ell}e^{i\Delta\phi}e^{i\pi})}{1 + Re^{-\alpha\ell}e^{-i\Delta\phi}e^{-i\pi} + 2ie^{-\alpha\ell}sin(\Delta\phi+\pi) - (1-R)e^{-2\alpha\ell}} \qquad (33)
\end{aligned}$$

[1]Note that the field reflection coefficient is equal to $\sqrt{R}$, but the field transmission coefficient is equal to $\sqrt{1-R}$.

which gives the reflected intensity:

$$I_R = \frac{E_0^2 R}{1 - 2(1-R)e^{-\alpha\ell}cos(\Delta\phi + \pi) + (1-R)^2 e^{-2\alpha\ell}}$$

1.10:

Noting that the bandedge occurs at $870nm$, implying a bandgap of $1.428eV$, the imaginary part of the index is given by:

$$\kappa = \frac{\alpha\lambda}{4\pi} = \frac{\lambda}{4\pi}C\sqrt{\frac{2\pi\hbar c}{\lambda} - 1.428} \tag{34}$$

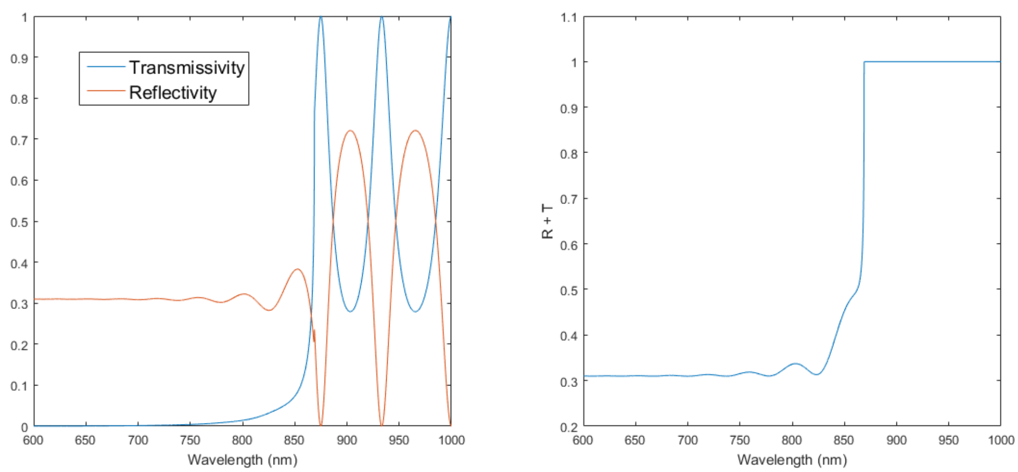
which gives the reflection coefficient:

$$R = \frac{2.5^2 + \kappa^2}{4.5^2 + \kappa^2} \tag{35}$$

Assuming normal incidence, the round trip phase difference through the platelet is:

$$\Delta\phi = \frac{4\pi nd}{\lambda} = \frac{8.796 \times 10^{-5}}{\lambda} \tag{36}$$

Plotting the reflectivity and transmissivity:



Note that below the bandedge ($\lambda > 870nm$), the platelet exhibits sharp etalon resonances and energy is conserved ($R + T = 1$). Above the bandedge ($\lambda < 870nm$), the absorption dominates, and the interference effects are washed out.

1.11:

For a transparent material, we make the approximation $\alpha \approx 0$, which gives the interface reflectivity:

$$R \approx \frac{(n-1)^2}{(n+1)^2} \tag{37}$$

We use the result from problem (1.8) to find the transmissivity in the incoherent limit:

$$\begin{aligned} T &\approx \frac{(1-R)^2}{1-R^2} \\ &= \frac{\left[\frac{(n+1)^2-(n-1)^2}{(n+1)^2}\right]^2}{1-\frac{(n-1)^4}{(n+1)^4}} \\ &= \frac{\left[(n+1)^2-(n-1)^2\right]^2}{(n+1)^4-(n-1)^4} \\ &= \frac{2n}{n^2+1} \end{aligned} \tag{38}$$

1.12:

The reflectivity of an interface between two different materials of is given by equation (A.55):

$$R = \left|\frac{\tilde{n}_2 - \tilde{n}_1}{\tilde{n}_2 + \tilde{n}_1}\right|^2 \tag{39}$$

Ignoring absorption, the reflectivities of each interface are given by:

$$R_1 = \left|\frac{2.5-1}{2.5+1}\right|^2 = 18.37\% \quad \text{(Air-Medium)} \tag{40}$$

$$R_2 = \left|\frac{1.5-2.5}{1.5+2.5}\right|^2 = 6.25\% \quad \text{(Medium-Substrate)} \tag{41}$$

$$R_3 = \left|\frac{1-1.5}{1+1.5}\right|^2 = 4\% \quad \text{(Substrate-Air)} \tag{42}$$

1.13:

We can write the optical density as:

$$\begin{aligned} \text{O.D.} &= log_{10}\left(\frac{I(\ell)}{I_0}\right) \\ &= log_{10}\left(T\right) \\ &= log_{10}\left(\frac{T^2}{T}\right) \\ &= 2log_{10}(T) - log_{10}(T) \\ &= -log_{10}(T) + 2log_{10}(1-R) \end{aligned} \tag{43}$$

1.14:

Assuming $\epsilon_2 \gg \epsilon_1$, we have at $\lambda = 100\mu m$:

$$\tilde{\epsilon}_r \approx i\frac{\sigma}{\epsilon_0\omega} = i \times 395460 \tag{44}$$

The complex index is thus:

$$\begin{aligned}\tilde{n} &= \sqrt{\tilde{\epsilon}_r} \\ &= \sqrt{i}\sqrt{395460} \\ &= \left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) \times 628.86 \\ &= 444.67(1+i) \end{aligned} \tag{45}$$

The reflectivity can thus be found by:

$$\begin{aligned} R &= \frac{(n-1)^2 + \kappa^2}{(n+1)^2 + \kappa^2} \\ &= 99.79\% \end{aligned} \tag{46}$$