Chapter 2
Solution Manual

Written by: Albert Liu

2.1:

We have the equations of motion from Newton's 2nd law:

$$m_1\ddot{x}_1 = -K_s(x_1 - x_2) \qquad m_2\ddot{x}_2 = -K_s(x_2 - x_1) \tag{1}$$

Defining the normal and relative coordinates:

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \qquad x_{rel} = x_2 - x_1 \tag{2}$$

we can express $x_1(t)$ and $x_2(t)$ in terms of x_{cm} and x_{rel}:

$$x_1 = x_{cm} - \frac{m_2}{m_1 + m_2}x_{rel} \qquad x_2 = x_{cm} + \frac{m_1}{m_1 + m_2}x_{rel} \tag{3}$$

which transforms the equations of motion as:

$$m_1\ddot{x}_{cm} - \frac{m_1m_2}{m_1 + m_2}\ddot{x}_{rel} = K_sx_{rel} \qquad m_2\ddot{x}_{cm} + \frac{m_1m_2}{m_1 + m_2}\ddot{x}_{rel} = -K_sx_{rel} \tag{4}$$

We're only interested in the relative motion of the two masses, so combining the two equations gives:

$$\begin{aligned}\ddot{x}_{rel} &= -K_s\left(\frac{1}{m_1} + \frac{1}{m_2}\right)x_{rel} \\ &= -\frac{K_s}{\mu}x_{rel}\end{aligned} \tag{5}$$

The solution for $x_{rel}(t)$ is thus of the form:

$$x_{rel}(t) = c_1cos\left(\sqrt{\frac{K_s}{\mu}}t\right) + c_2sin\left(\sqrt{\frac{K_s}{\mu}}t\right) \tag{6}$$

2.2:

We are concerned with the forced oscillation of the oscillator at ω, and we will ignore the resonant contribution at ω_0. Assuming a solution of the form $x(t) = c_3cos(\omega t) + c_4sin(\omega t)$, we plug into the equation of motion to find:

$$\left[\gamma\omega c_4 - \omega^2c_3 + \omega_0^2c_3\right]cos(\omega t) + \left[\omega_0^2c_4 - \omega^2c_4 - \gamma\omega c_3\right]sin(\omega t) = \frac{F_0}{m}cos(\omega t) \tag{7}$$

which gives:

$$c_3 = \frac{F_0(\omega_0^2 - \omega^2)}{m[(\gamma\omega)^2 + (\omega_0^2 - \omega^2)^2]} \qquad c_4 = \frac{F_0\gamma\omega}{m[(\gamma\omega)^2 + (\omega_0^2 - \omega^2)^2]} \tag{8}$$

and the solution:

$$\begin{aligned} x(t) &= \frac{F_0}{m[(\gamma\omega)^2 + (\omega_0^2 - \omega^2)^2]} \left[(\omega_0^2 - \omega^2)cos(\omega t) + \gamma\omega sin(\omega t)\right] \\ &= \frac{F_0}{m\sqrt{(\gamma\omega)^2 + (\omega_0^2 - \omega^2)^2}} \frac{(\omega_0^2 - \omega^2)cos(\omega t) + \gamma\omega sin(\omega t)}{\sqrt{(\gamma\omega)^2 + (\omega_0^2 - \omega^2)^2}} \\ &= \frac{F_0}{m\sqrt{(\gamma\omega)^2 + (\omega_0^2 - \omega^2)^2}} \left[cos(\phi)cos(\omega t) + sin(\phi)sin(\omega t)\right] \\ &= \frac{F_0}{m\sqrt{(\gamma\omega)^2 + (\omega_0^2 - \omega^2)^2}} cos\left(\omega t - \phi\right) \end{aligned} \tag{9}$$

where we've defined the phase between $x(t)$ and the driving force according to:

$$cos(\phi) = \frac{\omega_0^2 - \omega^2}{\sqrt{(\gamma\omega)^2 + (\omega_0^2 - \omega^2)^2}} \qquad sin(\phi) = \frac{\gamma\omega}{\sqrt{(\gamma\omega)^2 + (\omega_0^2 - \omega^2)^2}} \tag{10}$$