Drude Conductivity and Resistivity

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Consider an electron with velocity $\mathbf{v_0}$ right after experiencing a collision inside a metal. When an electric field $\mathcal{E}$ is applied, the momentum the electron will gain in the t_0 seconds before the next collision is:

$$\Delta \mathbf{p} = -e\mathbf{E}t_0 \tag{1}$$

and a corresponding gain in velocity:

$$\Delta \mathbf{v} = -\frac{e\mathbf{E}t_0}{m} \tag{2}$$

Recall that in the Drude model, an electron will emerge from a collision with a completely random velocity. This means that the initial velocity of each electron will not contribute to the average velocity, since when all electrons in the metal are considered the initial velocity gained after each collision will average to zero. The average velocity is thus the change in velocity due to the field, with the average time before the next collision defined by $\langle t_0 \rangle = \tau$:

$$\mathbf{v}_{avg} = \Delta \mathbf{v}(\tau) = -\frac{e\mathbf{E}\tau}{m} \tag{3}$$

The average current density for electrons ($\mathbf{j}_{avg} = -ne\mathbf{v}_{avg}$), where n is the electron density, will thus be:

$$\mathbf{j} = \frac{ne^2\mathbf{E}\tau}{m} \tag{4}$$

By definition, the conductivity is:

$$\sigma_0 = \frac{\mathbf{j}}{\mathbf{E}} = \frac{ne^2\tau}{m} \tag{5}$$

where σ_0 is commonly referred to as the **Drude Conductivity**. Likewise, the **Drude Resistivity** is simply the inverse of σ_0:

$$\rho_0 = \frac{1}{\sigma_0} = \frac{m}{ne^2\tau} \tag{6}$$