Michigan-AFRL Collaborative Center in Control Science
Semi-Annual Review
Homing Guidance Using Binary Range-Rate Measurements

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Definitions

- **Homing Guidance Problem:** Cause an actively controlled pursuer to hit, or come close to, a pre-selected target despite unpredictable maneuvers of the target, disturbances, and navigation uncertainties.*

- **Binary range-rate** distinguishes whether range-rate is increasing or decreasing.
  - Similar to the sign of the range-rate
  - There is no possibility of a “stationary” reading

Motivation

Recovery of Beacons at Sea

- Autonomous Underwater Vehicles
- Beacons equipped with low-cost VHF receivers
- Dr. Antonio Pascoal, Lisboa Institute for Systems and Robotics, Portugal

Sensors

- Low cost
- Noisy

*Image: Bluefin Robotics Corp., 28 April 2006*
Applications

- Localization or recovery with VHF receivers:
  - Beacons at sea
  - Downed aircraft
  - Sinking ships
  - Wildlife tracking bands
  - Avalanche victims

- Guidance when GPS is unavailable

- Localization of an unknown pollution source with a mobile sensor
Air Force Relevance

- Using circumnavigation to cooperatively navigate an UGS network*
  - Range and range-rate are known
  - Estimating range-rate can cause instabilities.
- GPS-denied environments
- Autonomous surveillance of a moving target
- Single sensor, single beacon scenarios
  - Micro-UAVs

Other Approaches

Localization & Homing for VHF Beacons

- Directional antennas (Mech, 2002)(Nielsen, 2006)

AUV Guidance & Navigation

  - Long baseline
  - Short baseline
  - Ultra-short baseline
- Multiple receivers & directional techniques (Rountree, 2002)
- Optical and electromagnetic techniques (Cowen, 1997)(Feezor, 2001)
- Single beacon, single sensor
  - Range-only techniques (Baccou, 2002)(Vaganay, 2000)
  - Binary range-rate techniques (Wojcik, 1993)
Passive Localization Using Closest Point of Approach (CPA)

- Acoustic transmitting beacon
- Passive receiver
- Pulse interval is measured to determine sign of range-rate
- At the CPA, the beacon is located on a line of bearing perpendicular to the vehicle’s heading.

Demonstrates that a vehicle’s heading and binary range-rate information are sufficient for homing guidance.

1. Planar homing guidance law requiring only:
   - Vehicle’s heading
   - Binary range-rate measurements

2. Study of the response of this type of guidance law to:
   - Corrupted sensor readings
   - Initial estimates
   - Observer gain

These contributions provide a method of low-cost, autonomous planar homing-guidance using a single, omnidirectional transmitting beacon and a single, omnidirectional receiver.

Submitted to 2013 IEEE Conference on Decision and Control
Achievements

- Guidance law for nonlinear system with nonanalytic output
  - Sensors provide relay-type measurements with discontinuities
- Novel navigation problem
  - Fixes are regions instead of lines or conic sections

- Build-up of knowledge
  - Navigation & Guidance
  - Separation conditions for nonlinear control systems with observers
  - Design of nonlinear controllers and observers
  - Persistency of excitation

- Solid background for future work
  - Differential games
  - UAV aerial combat
    - UAV autonomous self-protection
Outline

- Motivation
- Contributions
- **Model**
  - Separation Criteria
- Problem Statement
- Design
  - Controller
  - Observer
- Results
  - Homing
  - Effect of corrupted measurements
  - Effect of observer gain
  - Moving Beacons
- Conclusions
- Future Work (Binary Range-Rate Problem)
- Future Research Plans
System Model

- Unicycle dynamics
- The origin is located on the vehicle.

**Symbols**

- \((r, \theta)\): Beacon’s location (polar)
- \((x_b, y_b)\): Beacon’s location (Cartesian)
- \(\psi\): Vehicle’s heading angle
- \(u_1\): Vehicle’s forward velocity
- \(u_2\): Vehicle’s turn rate
System Equations

Polar

\[
\begin{pmatrix}
\dot{r} \\
\dot{\theta} \\
\dot{\psi}
\end{pmatrix} = f(x, u, t) = \begin{pmatrix}
-u_1 \cos(\theta - \psi) \\
\frac{u_1}{r} \sin(\theta - \psi) \\
u_2
\end{pmatrix}
\]

Cartesian

\[
\begin{pmatrix}
\dot{x}_b \\
\dot{y}_b \\
\dot{\psi}
\end{pmatrix} = f_c(x_c, u, t) = \begin{pmatrix}
-u_1 \cos(\psi) \\
-u_1 \sin(\psi) \\
u_2
\end{pmatrix}
\]

Outputs

\[
y = \begin{pmatrix}
\psi \\
\text{sign}(\dot{r})
\end{pmatrix} = \begin{pmatrix}
\psi \\
\text{sign} \left( \frac{-u_1(x_b \cos(\psi) + y_b \sin(\psi))}{\sqrt{x_b^2 + y_b^2}} \right)
\end{pmatrix}
\]
Binary Range-Rate Measurement

Definition

\[ \rho = \text{sign} \left( \frac{-u_1(x_b \cos(\psi) + y_b \sin(\psi))}{\sqrt{x_b^2 + y_b^2}} \right) \]

- Binary sensor
  - Vehicle moving toward beacon: \( \rho = -1 \)
  - Vehicle moving away from beacon: \( \rho = 1 \)
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Separation Conditions

\[ \dot{x}(t) = f(t, x(t), u(t)) \]
\[ y(t) = g(t, x(t)) \]

Requirements:

1. \( f \) is continuously differentiable and vanishes when all of its arguments except \( t \) vanish; further, there are constants \( a \) and \( c \) such that:

\[ \|\nabla_x f(t, x, u)\| \leq a \]
\[ \|\nabla_u f(t, x, u)\| \leq a \]

\( \forall t \geq 0, \forall x \in B_c, \forall u \in B_c \)

2. \( g \) is continuous, and \( g(0, 0) = 0 \)

\((M. Vidyasagar, 1980)\)
Separation Conditions

If the system satisfies the two requirements, and if the system is stabilizable and weakly detectable, then \( x = 0, \ z = 0 \) is a uniformly asymptotically stable equilibrium point of the following system:

\[
\dot{x}(t) = f(t, x(t), h(t, z(t))) \\
\dot{z} = \gamma(t, z(t), g(t, x(t)), h(t, z(t)))
\]

That is, if the system is stabilized by the control law

\[
u(t) = h(t, x(t))
\]

then it is also stabilized by the control law

\[
u(t) = h(t, z(t))
\]

where \( z(t) \) is the output of a weak detector for \( x(t) \).

(M. Vidyasagar, 1980)
Requirement #1

\[ f_c(x_c, u, t) = \begin{pmatrix} -u_1 \cos(\psi) \\ -u_1 \sin(\psi) \\ u_2 \end{pmatrix} \]

\[ \nabla_x f_c = \begin{pmatrix} u_1 \psi \sin(\psi) \\ -u_1 \psi \cos(\psi) \\ 0 \end{pmatrix} \quad \nabla_u f_c = \begin{pmatrix} -u_1 \cos(\psi) \\ -u_1 \sin(\psi) \\ u_2 \end{pmatrix} \]

\[ \|\nabla_x f_c\|_2 = u_1 \psi \quad \|\nabla_u f_c\|_2 = \sqrt{u_1^2 + u_2^2} \]

**Conditions**

- If \( x \) and \( u \) are confined to a sphere of any finite size, \( \|\nabla_x f_c\| \) and \( \|\nabla_u f_c\| \) are bounded.
- \( f_c \) is continuously differentiable and \( f_c(0, 0, t) = 0 \)

\[ \Rightarrow \text{All of the conditions given in the first requirement hold.} \]
Requirement #2

\[ g \text{ is continuous, and } g(0, 0) = 0. \]

\[ y = \begin{pmatrix} \psi \\ \text{sign} \left( \frac{-u_1(x_b \cos(\psi) + y_b \sin(\psi))}{\sqrt{x_b^2 + y_b^2}} \right) \end{pmatrix} \]

Conditions

- The first output meets all conditions
- Problems with the second output:
  - Function of time, state, and input
  - Discontinuous at \( x_b = y_b = 0 \)
Requirement #2

\[ \rho = \text{sign} \left( \frac{-u_1(x_b \cos(\psi) + y_b \sin(\psi))}{\sqrt{x_b^2 + y_b^2}} \right) \]

- Restrict \( u_1 \) to be non-negative
- Remove \( u_1 \) and the denominator (the only loss of equality occurs at \( x_b = y_b = 0 \))

\[ \rho = \text{sign}(-x_b \cos(\psi) - y_b \sin(\psi)) \]

The only remaining condition is continuity.
Choose a small $\epsilon$ and approximate the function by:

$$
\rho \approx \frac{-x_b \cos(\psi) - y_b \sin(\psi)}{\sqrt{(-x_b \cos(\psi) - y_b \sin(\psi))^2 + \epsilon^2}}.
$$

- $\rho$ is equal to the original function for $\epsilon = 0$ and $x_b, y_b$ not both zero.
- Represents a smoothing of the original function with increasingly sharper corners as $\epsilon$ shrinks to zero.
- $x_b = y_b = 0 \implies \rho = 0$
Requirement #2

Conditions

- $\rho$ depends only upon the system state and time.
- $\rho$ is continuous for all states and times
- $\rho$ is equal to zero for the zero state for all $t$ (in particular, for $t = 0$)

$\Rightarrow$ All of the conditions given in the second requirement hold
$\Rightarrow$ We can use the output of a weak detector in place of the actual state as an input to our controller.
Problem Statement After Separation

1. Given a vehicle with a known heading angle $\psi$, and a beacon with a known location $(r, \theta)$, find a guidance law, i.e., a velocity function $u_1(t)$ and a turn-rate function $u_2(t)$ such that the vehicle will reach the beacon.

2. Given $\rho$, the sign of the vehicle’s range-rate to the beacon, find an estimate of the beacon’s location, $(\hat{r}, \hat{\theta})$, such that the control law is able to stabilize the system.
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Controller Design

- Assume a solution with a constant forward velocity $u_1$.
- Develop a controller using the polar form of the dynamics.
- Control strategy based on sliding mode control and integrator backstepping.
Choose first control surface to be:

\[ S_1 = r - r_{des} \]  

(1)

with the following desired dynamics:

\[ \dot{S}_1 = -\lambda_1 S_1 \]  

(2)

These dynamics guarantee that \( S_1 \) decays to zero exponentially at a rate given by \( \lambda_1 \), which is a design parameter.

Substitute (1) into (2) and equate the result with the derivative of (1).
First Sliding Surface

\[ \dot{r} - \dot{r}_{des} = -\lambda_1 (r - r_{des}) \]

Substituting the system dynamics for \( \dot{r} \) gives:

\[ -u_1 \cos(\theta - \psi) - \dot{r}_{des} = -\lambda_1 (r - r_{des}) \]

**Desired \( \psi \)**

\[ \theta - \psi_{des} = \arccos \left( \frac{\lambda_1 (r - r_{des}) - \dot{r}_{des}}{u_1} \right) \]
Choose the second control surface to be:

\[ S_2 = \alpha - \alpha_{des} \]  (3)

with the following desired dynamics:

\[ \dot{S}_2 = -\lambda_2 S_2 \]  (4)

Again, substitute (3) into (4) and equate the result with the derivative of (3).
Second Sliding Surface

\[
\dot{\alpha} - \dot{\alpha}_{des} = -\lambda_2 (\alpha - \alpha_{des})
\]

Substituting the system dynamics for \(\dot{\theta}\) and \(\dot{\psi}\) gives:

\[
\frac{u_1}{r} \sin(\theta - \psi) - u_2 - \dot{\alpha}_{des} = -\lambda_2 (\alpha - \alpha_{des})
\]

The control variable \(u_2\) appears in this equation, so we choose:

**Control Input**

\[
u_2 = \lambda_2 (\alpha - \alpha_{des}) + \frac{u_1}{r} \sin(\theta - \psi) - \dot{\alpha}_{des}\]
Controller Results

Figure: Vehicle’s Path for Known Beacon Location

Figure: Sliding Surfaces
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Observer Design

- Assume perfect knowledge of the heading angle, $\psi$
- Goal: develop an observer that estimates $x_b$ and $y_b$

Definition

$x_b, \ y_b, \ \hat{\psi}$: estimates of the individual states
$
\hat{\rho}$: estimate of the sensor output

$$\hat{\rho} = \frac{-x_b \cos(\psi) - y_b \sin(\psi)}{\sqrt{(-x_b \cos(\psi) - y_b \sin(\psi))^2 + \epsilon^2}}$$

$\tilde{\rho}$: error in the estimate of $\rho$

$$\tilde{\rho} = \rho - \hat{\rho}$$
Observer Design

Observer Dynamics

\[ \dot{x} = \begin{pmatrix} \dot{x}_b \\ \dot{y}_b \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} -u_1 \cos(\psi)(k_1 \rho + k_2 \tilde{\rho}) \\ -u_1 \sin(\psi)(k_1 \rho + k_2 \tilde{\rho}) \\ u_2 \end{pmatrix} \]

\[ k_1, k_2 > 0 \]
Parameter Selection

Estimation Error

\[ \tilde{x} = x - \hat{x} = \begin{pmatrix} \tilde{x}_b \\ \tilde{y}_b \\ \tilde{\psi} \end{pmatrix} = \begin{pmatrix} x_b - \hat{x}_b \\ y_b - \hat{y}_b \\ 0 \end{pmatrix} \]

Error Dynamics

\[ \dot{\tilde{x}} = \dot{x} - \dot{\hat{x}} = \begin{pmatrix} -u_1 \cos(\psi)(1 - k_1 \rho - k_2 \tilde{\rho}) \\ -u_1 \sin(\psi)(1 - k_1 \rho - k_2 \tilde{\rho}) \\ 0 \end{pmatrix} \]

- Goal: Set \( k_1 \) and \( k_2 \) such that the error magnitude always decreases.
- Note: \( \tilde{x}_b \) and \( \tilde{y}_b \) have the same form and evolve similarly.
Motion in One Dimension

Assume the vehicle has a heading of $\psi = 0$.

### $\tilde{x}_b > 0$

1. $0 < \hat{x}_b < x_b \Rightarrow \forall k_1, k_2 : \dot{\tilde{x}}_b < 0$
2. $\hat{x}_b < 0 < x_b \Rightarrow \forall k_1, k_2 : \dot{\tilde{x}}_b < 0$
3. $\hat{x}_b < x_b < 0 \Rightarrow \forall k_1 < 1, \forall k_2 : \dot{\tilde{x}}_b < 0$

### $\tilde{x}_b < 0$

4. $0 < x_b < \hat{x}_b \Rightarrow \forall k_1, k_2 : \dot{\tilde{x}}_b < 0$
5. $x_b < 0 < \hat{x}_b \Rightarrow \text{for } k_1 + 2k_2 > 1 : \dot{\tilde{x}}_b > 0$
6. $x_b < \hat{x}_b < 0 \Rightarrow \forall k_1 > 1, \forall k_2 : \dot{\tilde{x}}_b > 0$
Motion in One Dimension

Assume the vehicle has a heading of $\psi = 0$.

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<td>6. $x_b &lt; \hat{x}_b &lt; 0 \Rightarrow \forall k_1 &gt; 1, \forall k_2: \dot{x}_b &gt; 0$</td>
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Assume the vehicle has a heading of $\psi = 0$.

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Motion in One Dimension

Assume the vehicle has a heading of $\psi = 0$.

\[
\begin{array}{|c|}
\hline
\tilde{x}_b > 0 \\
\hline
4. 0 < \tilde{x}_b < x_b \Rightarrow \forall k_1, k_2 : \dot{x}_b < 0 \\
5. \tilde{x}_b < 0 < x_b \Rightarrow \forall k_1, k_2 : \dot{x}_b < 0 \\
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5. x_b < 0 < \tilde{x}_b \Rightarrow \text{for } k_1 + 2k_2 > 1 : \dot{x}_b > 0 \\
6. x_b < \tilde{x}_b < 0 \Rightarrow \forall k_1 > 1, \forall k_2 : \dot{x}_b > 0 \\
\hline
\end{array}
\]
Motion in One Dimension

Assume the vehicle has a heading of $\psi = 0$.

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No choice of $(k_1, k_2)$ can decrease the error in both cases.

Choose $k_1 = 1$ $\Rightarrow$ $\dot{\tilde{x}}_b = 0$
Motion in One Dimension

Assume the vehicle has a heading of $\psi = 0$.

\[
\tilde{x}_b > 0
\]

1. $0 < \hat{x}_b < x_b \Rightarrow \forall k_1, k_2 : \tilde{x}_b < 0$
2. $\hat{x}_b < 0 < x_b \Rightarrow \forall k_1, k_2 : \tilde{x}_b < 0$
3. $\hat{x}_b < x_b < 0 \Rightarrow \forall k_1 < 1, \forall k_2 : \hat{x}_b < 0$

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\]

4. $0 < x_b < \hat{x}_b \Rightarrow \forall k_1, k_2 : \tilde{x}_b < 0$
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6. $x_b < \hat{x}_b < 0 \Rightarrow \forall k_1 > 1, \forall k_2 : \tilde{x}_b > 0$
### Motion in One Dimension

Assume the vehicle has a heading of $\psi = 0$.

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Increasing Error

**Case #4**

\[ 0 < x_b < \hat{x}_b \Rightarrow \forall k_1, k_2 : \dot{x}_b < 0 \]

**Case #5**

\[ x_b < 0 < \hat{x}_b \Rightarrow for \ k_1 + 2k_2 > 1 : \dot{x}_b > 0 \]

- The error magnitude always increases in case 4.
  - The vehicle travels toward the beacon.
  - The vehicle eventually reaches the CPA and enters case 5.
- By choosing \( k_2 \) to be large enough, we can cause the error to decrease quickly after the transition from case 4 to case 5.
Motion in Two Dimensions

In some cases, $\rho$, $\hat{\rho}$, or both provide “incorrect” readings.

18 additional cases:

- 4 cases leave the error unchanged if $k_1 = 1$.
- With $k_1 = 1$, 6 cases decrease the error magnitude for any $k_2$.
- 8 cases always increase the error magnitude.
Motion in Two Dimensions

In some cases, $\rho$, $\hat{\rho}$, or both provide “incorrect” readings.

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Motion in Two Dimensions

In some cases, \( \rho, \hat{\rho}, \) or both provide “incorrect” readings.

18 additional cases:
- 4 cases leave the error unchanged if \( k_1 = 1. \)
- With \( k_1 = 1, \) 6 cases decrease the error magnitude for any \( k_2. \)
- 8 cases always increase the error magnitude.
Cases with Increasing Error

There is no way for the system to stay in these cases forever.

- The estimate moves to a location where the error remains constant.
- The vehicle heads toward the beacon and eventually passes the CPA.

By choosing $k_2$ to be large enough, the system transitions out of these cases quickly.

Large values of $k_2$ cause fast recovery of the error after the vehicle passes the CPA.

The controller constantly steers the vehicle toward the estimate.
Observer Behavior

Observer & System Dynamics:

\[
\begin{pmatrix}
\dot{x}_b \\
\dot{y}_b \\
\dot{\psi}
\end{pmatrix} =
\begin{pmatrix}
-u_1 \cos(\psi)(\rho + k_2 \tilde{\rho}) \\
-u_1 \sin(\psi)(\rho + k_2 \tilde{\rho}) \\
u_2
\end{pmatrix}
\]

\[
\begin{pmatrix}
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\begin{pmatrix}
-u_1 \cos(\psi) \\
-u_1 \sin(\psi) \\
u_2
\end{pmatrix}
\]

- When \( \tilde{\rho} = 0; \)
  - If the vehicle moves away from the beacon (\( \rho = 1 \)):
    - The estimate is constant (in a fixed frame)
  - If the vehicle moves toward the beacon (\( \rho = -1 \)):
    - The estimate moves at twice the vehicle’s speed (in a fixed frame)
    - If the estimate is farther away than the beacon (case 4), error accumulates

- When \( \tilde{\rho} \neq 0; \)
  - Fast recovery from accumulated error if \( k_2 \) overpowers the rate of error accumulation (\( k_2 >> k_1 \))
Outline

- Motivation
- Contributions
- Model
  - Separation Criteria
- Problem Statement
- Design
  - Controller
  - Observer
- Results
  - Homing
  - Effect of corrupted measurements
  - Effect of observer gain
  - Moving Beacons
- Conclusions
- Future Work (Binary Range-Rate Problem)
- Future Research Plans
Successful Homing

Good Initial Estimate with 5% Measurement Corruption and $k_2 = 10$

Figure: Vehicle Path (Overhead View)

Figure: Estimation Errors
Successful Homing

Bad Initial Estimate with 5% Measurement Errors and $k_2 = 10$

Figure: Vehicle Path

Figure: Estimation Errors
Effect of Corrupted Measurements ($k_2 = 10$)

For a perfect sensor, heading toward the beacon provides no new information until the CPA is reached.

The presence of measurement errors, whether real or artificial, can allow the estimation to improve prior to the vehicle reaching the CPA. (Persistency of Excitation)
Effect of Corrupted Measurements ($k_2 = 10$)

Figure: Good Initial Estimate

Figure: Bad Initial Estimate
The best choices of $k_2$ are those that are large enough to overpower $k_1$, but small enough to take advantage of good initial estimates.
Moving Beacons

Figure: Linear Motion

Figure: Circular Motion
Known heading angle, $\psi$, and binary range-rate measurement, $\rho$

Constant velocity input, $u_1$

Estimated beacon coordinates, $\hat{x}_b$ and $\hat{y}_b$, with the following dynamics:

$$\begin{pmatrix} \dot{\hat{x}}_b \\ \dot{\hat{y}}_b \end{pmatrix} = \begin{pmatrix} -u_1 \cos(\psi)(\rho + k_2 \tilde{\rho}) \\ -u_1 \sin(\psi)(\rho + k_2 \tilde{\rho}) \end{pmatrix}$$

The observer gain, $k_2 >> 1$, is a design parameter
Turn-rate calculated for the estimated beacon location, \((\hat{r}, \hat{\theta})\):

\[
u_2 = \lambda_2(\hat{\theta} - \psi - \hat{\alpha}_{des}) + \frac{u_1}{\hat{r}} \sin(\hat{\theta} - \psi) - \dot{\hat{\alpha}}_{des}
\]

with:

\[
\hat{\alpha}_{des} = \arccos\left(\frac{\lambda_1(\hat{r} - \hat{r}_{des}) - \dot{\hat{r}}_{des}}{u_1}\right)
\]

We choose \(\dot{r}_{des} = -0.99u_1\)

\[
\hat{r}_{des} = \hat{r} + \frac{\dot{\hat{r}} - \dot{r}_{des}}{\lambda_1}
\]

The controller gains, \(\lambda_1, \lambda_2 > 0\), are design parameters:
Conclusions

- Showed that knowledge of a vehicle’s heading and binary range-rate information are sufficient for planar homing guidance.
- Presented a guidance law consisting of a sliding mode controller and an observer.
- Demonstrated that this guidance law achieves successful homing.
- Characterized the response of the guidance law for different observer gains, proportions of measurement corruption, and initial estimates of the beacon’s location.

This guidance law provides a method of autonomous, planar homing-guidance that utilizes a single, omnidirectional receiver to guide a vehicle to a single, omnidirectional transmitting beacon.

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Future Work

Binary Range-Rate Sensors

- Enforcement of limited turn-rates will replace the unicycle model
- Planar restriction can be removed to generalize the guidance law to three dimensional space
- Extensions to navigation:
  - Localizing a vehicle given binary range-rate measurements to multiple beacons with known locations
  - Circumnavigation in an UGS network

Future Research Plans

- Air combat
  - Autonomous self-defense
- Differential games
Questions?

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References

- Y. Cao, et al., “Circumnavigation of an Unknown Target Using UAVs with Range-only Measurements,” submitted for publication.
References


References
