The Energy Levels of a Rotating Particle

\[ E_n = \frac{n^2 \hbar^2}{2I} \quad \rightarrow \quad E_{m_l} = \frac{m_l^2 \hbar^2}{2I} \]

Angular momentum quantum number

\[ m_l = 0, \pm 1, \pm 2, \ldots \]

Doubly degenerate states (of same energy)

Also the angular momentum is quantized

\[ J = pr = \frac{hr}{\lambda} \quad \text{and} \quad \lambda = \frac{2\pi r}{m_l} \]

\[ \Rightarrow J = m_l \hbar \]

non-degenerate state (zero energy, but uncertain between 0° and 360°)
The Emission Spectrum of Hydrogen Atoms

1885:

In general: $n_1 = 1, 2,...; n_2 = n_1 + 1, n_1 + 2,....$

Transitions: photon emitted with

$$\Delta E = h\nu = hc\tilde{\nu}$$

(Bohr frequency condition)

Rydberg constant $R_H = 109,677 \text{ cm}^{-1}; \quad n = 3, 4,...$

$$\tilde{\nu} = \frac{1}{\lambda} = R_H \left( \frac{1}{2^2 - \frac{1}{n^2}} \right)$$
What are the Energy Levels of a Hydrogen Atom? Schrödinger can help!

\[-\frac{\hbar^2}{2m} \frac{d^2\Psi}{dx^2} + V\Psi = E\Psi\]

\[V = -\frac{Ze^2}{4\pi\varepsilon_0 r}\]

Nuclear atomic model; nuclear charge \(+Ze\), electronic charge \(-e\)

Boundary condition: \(\Rightarrow\) can be solved analytically:

\[E_n = -\frac{\mu e^4}{32\pi^2 \varepsilon_0^2 \hbar^2} \frac{Z^2}{n^2}; \quad \mu = \frac{m_e m_n}{m_e + m_n}\]
Quantum Mechanics explains Balmer’s Experimental Observations

\[ E_n = -\frac{\mu e^4}{32\pi^2 \varepsilon_0^2 h^2} \frac{Z^2}{n^2} = -\hbar c R_H \frac{Z^2}{n^2} \]

Principal quantum number

All energies are negative (Coulomb attraction)

\[ E_2 = -\frac{1}{4} \hbar c R_H \]

\[ \Delta E = \hbar c \nu = \hbar c R_H \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \]

Just as Balmer (Lyman) had measured!

Only certain transitions allowed:

for H atom: \[ E_1 = -\hbar c R_H \]

Rydberg constant!
The Resulting Quantum Numbers Define the Atomic Orbitals

1.) n = Principal quantum number = 1, 2, 3,…∞:
   - the only determinant of the energy of hydrogen-like atoms
   - defines the “size” of the orbital
   - K, L, M, N,… shells
   \[ E_n = -\frac{hcR_H Z^2}{n^2} \]

2.) l = Angular momentum quantum number = 0, 1, 2,…, n-1:
   - n different values
   - defines the “shape” of the orbital
   - s, p, d, f,… subshells, orbitals, electrons

3.) \( m_l \) = Magnetic quantum number = l, l-1, l-2,…, -l
   - 2l+1 different values
   - defines the “orientation” of the orbital

4.) \( m_s \) = Spin magnetic quantum number = +1/2, -1/2
   - electron spins clockwise (+1/2, \( \alpha \)) or counterclockwise (-1/2, \( \beta \))