Dead-end and career jobs: skill-specific earnings profiles in an on-the-job search equilibrium with heterogeneous wage contracts.*

[DRAFT]

Cynthia L. Doniger†
Department of Economics - University of Michigan

May 21, 2014

Abstract

I include differential skill levels on the part of workers in an on-the-job search equilibrium which features heterogeneous wage contracts. Firms choose between posting a non-negotiable contract or hiring under contingent pay which matches the value of a workers best-to-date outside offer. I give conditions under which the market decomposes into parallel skill-specific sub-markets in each of which a separating equilibrium arises where only low productivity firms offer non-negotiable contracts. Even when skill-specific sub-markets are ex-ante identical, differences arise ex-post. More skilled workers are more likely to receive negotiable wage offers. Consequently, high skilled workers experience lower rates of unemployment, more wage dispersion, and higher returns to experience than low skilled workers. I explore the implications for evaluating job training which increases worker skill level while unemployed. The full effect of training is expressed in wages only over time; however, effects are expressed more rapidly in higher percentiles wage distributions. This suggests estimating the returns to training programs, at least in the shorter-run, based on impacts in the right tail of the wage distribution.

---

*The most recent version of this paper can be found at https://sites.google.com/site/donigerc/research

†Email: donigerc@umich.edu. This research was supported in part by an NIA training grant to the Population Studies Center at the University of Michigan (T32 AG000221). The content is solely the responsibility of the author and does not necessarily represent the official views of the National Institutes of Health. I thank Dmitriy Stolyarov, Jeff Smith, Matthew Shapiro, and Charlie Brown for advising and support. I also thank the insight and comments of the participants of the Second Joint Paris SaM Workshop. Finally, I thank Aditya Aladangady, Breno Braga, Péter Hudomiet, Isaac Sorkin, and the participants of the labor search reading group and informal macro and labor seminars at the University of Michigan. All remaining errors are my own.

1 Introduction

Some jobs feature negotiable wages, others do not. Some jobs feature on-the-job wage growth, others do not. How do these differences shape our understanding of firm, worker, and job heterogeneity? This paper includes differential skill levels on the part of workers in an equilibrium model of on-the-job search in which some employers offer jobs with uniform, non-negotiable pay while others offer targeted, negotiable pay with potential for on-the-job wage growth. The result is the ability to explicitly assess how contract type varies with worker skill; and how, as a result, employment opportunities, wage dispersion, and return to experience differ across workers with different levels of skill. This complements a developing literature which focuses on new hires' wages and wage negotiations.[1]

I allow firms to select between two wage setting mechanisms: wage posting (WP) and sequential auction (SA). Under WP, a firm sets a non-negotiable wage and commits to it for the duration of the employment relationship, as in Burdett and Mortensen (1998) and Bontemps et al. (2000). Under SA, pay is commensurate with the worker's best-to-date outside option and evolves as workers accrue new outside offers by a mechanism equivalent to a second price auction, as in Postel-Vinay and Robin (2002a,b). In the present paper, I allow wages set under both contract types to be conditioned on worker skill level.

Doniger (2014a) shows that, when workers are homogeneous, the arrival rate of job offers is exogenous, and firms must pay a per-firm fee in order to set wages by means of the SA mechanism, the market is characterized by a separating equilibrium in which only high productivity firms are interested in providing the flexible contract. Such separation is consistent with the intuition that dead-end jobs with inflexible wages and poor job-to-job transition prospects are likely to be offered by the least productive firms while more productive firms offer career jobs which offer the prospect of on-the-job wage gain and, often, significant job-to-job wage gain as well. This is also consistent with empirical evidence that

[1] For example Ellingsen and Rosén (2003) and Michelacci and Suarez (2006).

negotiable and non-negotiable wage contracts exist side-by-side in equilibrium (Hall and Krueger, 2008, 2010; Barron et al., 2006; Brenzel et al., 2013).

Here, I extend the model to admit workers who are heterogeneous with respect to their skill, modeling higher levels of skill as yielding greater effective labor. As is the usual construct, firms are modeled as producing via a constant returns to scale technology. Firms hire under skill-specific wage contracts under both wage contract types. Costs for the right to SA are skill specific: if firms hire some skill levels under WP and others under SA they pay costs *only* for the skill levels which are employed under flexible contracts. Under these conditions skill-specific sub-markets operate and can be analyzed in parallel. Within each sub-market contract choice follows the pattern described for a market with uniform labor: a separating equilibrium exists in which low productivity employers hire under WP and high productivity employees hire under SA.

I consider ex-ante identical sub-markets: identical distribution of productivity among potentially active firms, identical hazards of job offers and unemployment, and identical costs for the right to SA. Even when markets are ex-ante identical differences arise ex-post. Since costs for the right to SA are larger relative to output for low skilled workers, the share of SA firms is increasing with skill level and high skilled workers are more likely to be employed under a SA contract. The implications are striking: low skilled workers experience higher rates of unemployment, less log wage dispersion, and lower returns to experience than high skilled workers.

RELATE TO Hall and Krueger (2008, 2010); Barron et al. (2006); Brenzel et al. (2013). RELATE TO Lemieux (2008) AND OTHERS. RELATE TO Mincer (1974) and Braga (2014).

From a policy perspective, these results are informative about how job training programs may impact workers and how they may be best assessed. Typically evaluations find little or no effect on wages immediately following training but larger effects some years out: Jacobi and Kluve (2007) give a summary of evaluation of training programs in Germany; Card et al. (2010) provide a meta-analysis of active labor market policy assessments in the United States and Europe. A lag in the response of wages could be due to several sources: training may take time to sink in, firms may learn the value of workers' training only slowly, and/or firms may wait to reward training until required to do so by market pressures. The model described in this paper provides a structural mechanism of the third type.

I consider an experiment in which unemployed workers are provided training which is effective and increases skill level by X%. When costs of SA are identical for all skill levels, the effect of training is muted in re-employment wages due to differentials in the composition of contracts available to trained (high skill) and untrained (low skill) workers. Percent differences in average wages are compressed at low (continuous) experience, due to the predominance of WP for untrained (low skill) workers. As workers gain more (continuous) experience, however, they move to the competitive SA tail of employers and are paid wages proportionate to their training. This biases estimates of the impact of training based on differences in average wages toward zero on a short horizon. On the other hand, the full effect of training is realized in the right tail of the wage distribution more quickly since these workers, by chance, have climbed the job ladder to employment in an SA contract relatively quickly. This suggest quantile regression methods for evaluations of job training programs.

The remainder of this paper proceeds as follows. Section 2 lays out the labor market setting under consideration including the set of contracts available; describes worker and firm behavior in parallel sub-markets; and demonstrates that, under a set of costs for the right to SA in each sub-market, each sub-market exhibits a separating equilibrium in which low productivity firms WP while higher productivity SA. Section 3 provides comparative statics with respect to skill level for ex-ante identical markets. Section 4 applies results of Section 3 to evaluation of job training for the unemployed. Section 5 concludes. For the large part proofs and derivations are left to the appendix. Doniger (2014a) considers implications of the dual-contract model with respect to aggregate empirical regularities. Doniger (2014b) provides methods for identifying the composition of contracts using administrative data.

2 The model

I consider two wage setting strategies: wage posting (WP) and sequential auction (SA). The former is typified by a fixed wage for all workers of the same skill level for the duration of contracts. Specifically, each firm is constrained to select a single "posted" wage for each skill level and employ all workers of the same skill level at this wage regardless of tenure or alternate employment offers. Under the latter, firms set wages to match the value of the best-to-date outside option of each worker and retain workers' services by updating wages whenever profitable. Firms are fully informed about worker's outside option and offer wages equal to a worker's reservation given skill level and labor market history. When two firms employing under SA compete for an employee this results in a bidding war. The SA wage setting mechanism in this case is equivalent to a second price auction for the employee's services or a Bertrand competition over the offered value of employment: the worker is paid a wage which yields equal value as the highest profitable wage offer of the less productive competitor. As a result, wages under SA are dependent on skill insofar as the best-to-date alternate wage offer for a worker of given skill depends on skill.

Intuition of the separating equilibrium rests on noting that, all else equal, employing workers under SA results in a lighter wage bill than employing under WP. A cost for SA equivalent to the difference in wage bills for the threshold firm induces less productive firms to select WP and more productive to select SA.

2.1 Setting

I consider the steady state equilibrium of a search market in which firms possessing technology, p, and workers possessing skill, ε, are brought together by a sequential process of random matching: technology and skill are revealed to the other party only after workers and firms meet. ε distributes discretely, so that there is a positive mass, M^{ε}, of workers of each skill type. Meanwhile p distributes continuously. Skill yields effective labor, and production is constant returns to scale, meaning that technology of type p paired with a worker of skill ε produces output εp per period. Workers of type-ε receive flow εb when unemployed. This last assumption can be justified, as Postel-Vinay and Robin (2002b) point out, by presuming that more productive laborers are also more productive home producers. Each worker has linear utility and seeks to maximize the present discounted value of wages and unemployment.

Firms employ workers under to one of two contracts: wage posting (WP) or sequential auction (SA). If WP, the firm offers a non-negotiable wage for as long as the worker wishes or until exogenous separation. This wage may be conditioned on skill but is uniform within each skill type. If SA, the firm offers a wage chosen to match the value of the workers best-to-date outside offer. The SA wage updates as the outside offer evolves and, in addition the SA firm commits to bid up the workers wage in the event of a job-to-job transition.[2] SA wages are thus described by the wage setting strategy and productivity of the incumbent firm as well as the wage setting strategy and productivity of the best-to-date outside option and (through these) the worker's skill level. Meanwhile, WP wages depend only on the current incumbent's productivity and the skill level of the employee.

If the firm chooses the SA contract for workers of skill type ε it must pay a flow fee of c^{ε}. The fee is independent of firm size. However, if a firm employs some, but not all, skill levels under SA it pays costs only for the skill levels for which SA is implemented. The

[2] Note that this "commitment" is sub-game perfect.

micro-foundations of this cost are left for another paper; however, one possible story is that there are legal and/or administrative fees associated with posting a SA vacancy for ε-skill level workers. These fees must be paid whether or not the vacancy is filled.[3] Since output and costs associates with employing one skill level are independent of employing any other, the market for labor can be decomposed into parallel, but independent, sub-markets for each skill.

In each sub-market, measure N^ε of firms operate technologies which produce output that distributes according to a continuous distribution $\Gamma(p|\varepsilon)$ which may be different in each skill-specific sub-market. Each productivity distribution has support $[\underline{p}^\varepsilon, \bar{p}^\varepsilon]$ where the maximal productivity $\bar{p}^\varepsilon$ may be infinite. Workers search for contracts both off- and on-the-job using uniform sampling, meaning the probability of a ε-type worker sampling a firm of productivity p or less is $\Gamma(p|\varepsilon)$. Job offers arrive at exogenous Poisson arrival rates λ_0^ε when unemployed and λ_1^ε when employed. Workers are exogenously separated from employment contracts at Poisson arrival rate δ^ε and discount the future at rate μ^ε.

2.2 Equilibrium in each sub-market

Doniger (2014a) demonstrates that, when workers are homogeneous, a separating equilibrium arises when firms face a per-firm cost for implementing the flexible SA contract. The result applies here to each of the ε-skill sub-markets.

Claim 1. *For every set of skill-specific costs, $\{c^\varepsilon\}$, there exists a set of skill-specific thresholds, $\{\check{p}^\varepsilon\}$, such that within every skill-specific sub-market an equilibrium exits in which firms with productivity less than the skill-specific threshold all prefer WP while more productive firms all strictly prefer SA. Threshold productivity firm is indifferent.*

Proof is in the appendix. The main steps are first to show that the flow of labor between

[3]Note that, although firms differ in size in equilibrium, each offers an identical number of vacancies. Size is determined ex-post by the rate of vacancy filling and the duration for which contracts persist.

firms is constrained efficient, second that a SA firm can hire from a WP firm at wage cut, and third that the magnitude of this wage cut is increasing in the SA firm's productivity.

The first, requires that if an employee receives a job offer from a firm more productive than her incumbent employer there is always a wage offer which is profitable to the approaching firm and acceptable to the worker. Further, such a wage offer is always available in the sense that equilibrium wage setting strategies do not constrain the firm and prevent it from making said wage offer. That this holds in the proposed equilibrium is shown in the appendix. From this we can infer that $V^P(w^1, p^1, \varepsilon) < V^A(w^2, p^2, \varepsilon) < V^A(w^3, p^3, \varepsilon)$ if $w^1 < w^2 < w^3$ and $p^1 < p^2 < p^3$, where $V^i(w, p, \varepsilon)$ is the value of employment in a WP ($i = P$) or SA ($i = A$) contract in a p-productivity firm at wage w for a ε-skilled worker.

Second, the optimal wage strategy of a p-productivity SA firm when hiring from a q-productivity WP firm is to select a wage, $w_{PA}(q, p, \varepsilon)$, such that the value of employment in each contract is identical:

$$\underbrace{V^A(w_{PA}(q,p,\varepsilon),p,\varepsilon)}_{\text{value in the SA firm}} = \underbrace{V^P(w_{PP}(q,\varepsilon),q,\varepsilon)}_{\text{value in the WP firm}}$$

where $w_{PP}(q, \varepsilon)$ is the optimal posted wage choice of the WP firm. The appendix provides value functions and shows that the optimal wage offer is

$$w_{PA}(q,p,\varepsilon) = w_{PP}(q,\varepsilon) + \tag{2.1}$$
$$\underbrace{\lambda_1\left\{\bar{\Gamma}(\check{p}^{\varepsilon}|\varepsilon)V^P(w_{PP}(q,\varepsilon),q,\varepsilon) - [\Gamma(p|\varepsilon) - \Gamma(\check{p}^{\varepsilon}|\varepsilon)]\mathbb{E}[V^A(\varepsilon x,x,\varepsilon)|\check{p} < x < p] - \bar{\Gamma}(p|\varepsilon)V^A(\varepsilon p,p,\varepsilon)\right\}}_{\text{difference in option values in the SA contract and best-to-date WP outside option}}.$$

where $V^P(w_{PP}(q, \varepsilon), q, \varepsilon)$ is the value of employment in a q-productivity WP firm at wage $w_{PP}(q, \varepsilon)$, and $V^A(\varepsilon x, x, \varepsilon)$ is the value of employment in a x-productivity SA firm at wage εx. WP firms do not alter their wage offers when facing competition. Meanwhile, competition between two SA firms results in the less productive firm offering a wage equal to the worker's output. The more productive firm then hires the worker under a wage contract which provides exactly the same value.

Note that the difference in option values in equation 2.1 is negative, since $q < x < p$ and therefore $w_{PP}(q, \varepsilon) < \varepsilon x < \varepsilon p$, when $w_{PP}(q, \varepsilon)$ is affordable to the q-productivity WP firm, so $V^P(w_{PP}(q, \varepsilon), q, \varepsilon) < V^A(\varepsilon x, x, \varepsilon) < V^A(\varepsilon p, p, \varepsilon)$. This shows that, within every sub-market, otherwise identical firms are able to operate lighter wage bills under SA than under WP. This arises for two reasons: 1) the SA firm hires each worker at a wage no greater than is required to outbid the best-to-date outside option, and 2) the SA firm partially compensates each worker with a commitment to aggressively bid up wages upon future job-to-job transition. The second means that an SA firm can hire a worker from every WP firm at a *wage cut*.

This wage cut, and therefore the wedge between wage bills, is larger for more productive SA firms since the option value associated with the firms' commitment to bid up wages is increasing in productivity: $V^A(w, x, \varepsilon) \leq V^A(w, y, \varepsilon)$ whenever $x \leq y$. This means that for any worker history the optimal wages offered by a more productive SA firm is lower. Observe that the difference in option values in equation 2.1 is increasing in absolute value as p increases. This is sufficient to show that if the cost of countering is equal to the willingness to pay for the right to SA for the threshold productivity firm:

$$c^{\varepsilon} = \{\mathbb{E}[w|\check{p}, P, \varepsilon] - \mathbb{E}[w|\check{p}, A, \varepsilon]\} \, \ell(\check{p}|\varepsilon) \tag{2.2}$$

then separation is an equilibrium.

When costs of SA are null or subsidized all firms select SA and under sufficiently high costs all firms select WP. Existence of a separating equilibrium for any choice of costs the follows from the intermediate value theorem. Equilibria are unique if the distribution of firms is sufficiently sparse in the right tail. Conditions for uniqueness are not restrictive and are given in the appendix.

2.3 Skill-specific value and wage functions

With the equilibrium characterized, I derive skill-contingent wage functions for each skill type. It is instructive to begin by considering the value function of a worker employed in a WP firm. The value of employment at a p-productivity WP firm can be written as:

$$
\begin{aligned}
\mu^{\varepsilon} V^{P}(w_{PP}(p,\varepsilon),\varepsilon) = {} & w_{PP}(p,\varepsilon) \\
& + \underbrace{\lambda_1^{\varepsilon}[\Gamma(\check{p}^{\varepsilon}|\varepsilon) - \Gamma(p|\varepsilon)]\{\mathbb{E}[V^{P}(w_{PP}(x,\varepsilon),x,\varepsilon)|q < x < \check{p}^{\varepsilon}] - V^{P}(w_{PP}(p,\varepsilon),\varepsilon)\}}_{\text{job-to-job transition to a more productive WP firm}} \\
& + \underbrace{\lambda_1^{\varepsilon}[\bar{\Gamma}(\check{p}^{\varepsilon}|\varepsilon)]\{\mathbb{E}[V^{A}(w_{UA}(x,\varepsilon),\varepsilon)|\check{p} < x] - V^{P}(w_{PP}(p,\varepsilon),\varepsilon)\}]}_{\text{job-to-job transition to a more productive SA firm}} \\
& + \underbrace{\delta^{\varepsilon}\{\mathbb{E}[V^{U}(\varepsilon) - V^{P}(w_{PP}(p,\varepsilon),\varepsilon)\}]}_{\text{unemployment shock}}, \qquad (2.3)
\end{aligned}
$$

where $w_{PP}(p,\varepsilon)$ is the optimal wage choice of the p-productivity WP firm.

The option value of receiving an employment offer from a more productive WP firm is positive since posted wages are not contingent on labor market history of workers and are, therefore, typically in excess of what would be required to induce transition: e.g. they are typically larger than the reservation wage conditional on labor market history. The option value of receiving an employment offer from a SA firm behaves quite differently. The fully flexible, informed, and rent extracting wage setting policy of SA firms coupled with the passive wage setting strategy of WP firms has interesting implications. In particular, since SA firms yield only enough rent to new employees to best the best wage offer of the workers' best-to-date outside option and since WP firms do not update their wage offers in the face of competition, the gain in value when transitioning from employment in a WP firm to a SA firm is exactly zero!

The value of employment in a WP firm is, surprisingly, *independent* of the distribution of firms in the tail of the productivity distribution which comprises the SA sector. Assuming that $w_{PP}(x, \varepsilon)$ is a differentiable function (I will verify that it is momentarily), the value of employment in a p-productivity WP firm is:

$$V^P(w_{PP}(p,\varepsilon),\varepsilon) = \frac{w_{PP}(p,\varepsilon)}{\mu^\varepsilon + \delta^\varepsilon} + \frac{\lambda_1^\varepsilon}{\mu^\varepsilon + \delta^\varepsilon} \int_p^{\check{p}^\varepsilon} \frac{\Gamma(\check{p}^\varepsilon|\varepsilon) - \Gamma(x|\varepsilon)}{\mu^\varepsilon + \delta^\varepsilon + \lambda_1^\varepsilon[\Gamma(\check{p}^\varepsilon|\varepsilon) - \Gamma(x|\varepsilon)]} \frac{dw_{PP}(x,\varepsilon)}{dx} + \frac{\delta^\varepsilon V^U(\varepsilon)}{\mu^\varepsilon + \delta^\varepsilon}. \tag{2.4}$$

Similarly, the value of unemployment for each skill level is:

$$V^U(\varepsilon) = \frac{\varepsilon b}{\mu^\varepsilon} + \frac{\lambda_0^\varepsilon}{\mu^\varepsilon} \int_{\underline{p}^\varepsilon}^{\check{p}^\varepsilon} \frac{\Gamma(\check{p}^\varepsilon|\varepsilon) - \Gamma(x|\varepsilon)}{\mu^\varepsilon + \delta^\varepsilon + \lambda_1^\varepsilon[\Gamma(\check{p}^\varepsilon|\varepsilon) - \Gamma(x|\varepsilon)]} \frac{dw_{PP}(x,\varepsilon)}{dx} \tag{2.5}$$

It is easy to verify that the value in both states is larger when more firms operate the WP contract: $\frac{dV^P(w_{PP}(p,\varepsilon)}{d\check{p}^\varepsilon} > 0$ and $\frac{dV^U(\varepsilon)}{d\check{p}^\varepsilon} > 0$. Section 3 shows that the relation between threshold productivity and the value of unemployment and employment in a WP contract impacts wage distributions and wage-experience profiles in an interesting and intuitive way.

The reservation wage for employment in a WP firm in the ε-skill sub-market, $w_{UP}(\varepsilon)$, equates the value of unemployment with the value of employment in the least productive active firm in the ε-skill sub-market:

$$V^U(\varepsilon) = V^P(w_{PP}(\underline{p}^\varepsilon), \underline{p}^\varepsilon, \varepsilon).$$

So, the reservation wage for employment in a WP firm is

$$w_{UP}(\varepsilon) = \varepsilon b + (k_0^\varepsilon - k_1^\varepsilon) \int_{w_{UP}(\varepsilon)/\varepsilon}^{\check{p}^\varepsilon} \frac{[\Gamma(\check{p}^\varepsilon|\varepsilon) - \Gamma(x|\varepsilon)]}{1 + k_1^\varepsilon[\Gamma(\check{p}|\varepsilon) - \Gamma(x|\varepsilon)]} \frac{dw_{PP}(x,\varepsilon)}{dx}, \tag{2.6}$$

where $k_0 = \lambda_0^\varepsilon/(\delta^\varepsilon + \mu^\varepsilon)$ and $k_1 = \lambda_1^\varepsilon/(\delta^\varepsilon + \mu^\varepsilon)$. Assuming the fixed cost of entry for firms into each sub-market is zero the lowest WP wage and the output of a ε-skill worker matched with the least productive active firm in each sub-market should be identical. This allows me to substitute $w_{UP}(\varepsilon)/\varepsilon$ for $\underline{p}^\varepsilon$ as the lower bound of integration.

Now, I derive the optimal wage schedule for WP firms and check that it is, indeed, differentiable. In the equilibrium, if the p-productivity firm selects WP then it must be the case that all less productive firms also select WP. The problem facing the firm is thus very similar to the problem considered by Bontemps et al. (2000). The optimal posted wage offer, $w_{PP}(p)$, maximizes the expected profit from the posted contract:

$$\pi^P(p|\varepsilon) = \underbrace{[\varepsilon p - w_{PP}(p,\varepsilon)]}_{\text{rent per worker}} \underbrace{\ell(p|\varepsilon)}_{\text{labor supply}} , \tag{2.7}$$

Bontemps et al. (2000) show that when all firms WP, make optimal wage choices, and productivity distributes according to a continuous distribution labor supply is pinned down by firms' productivity type via mass-balance. More productive firms prefer to post higher wages and continuity of $\Gamma(p)$ yields the required one-to-one mapping between $w_{PP}(p,\varepsilon)$ and p.[4] The result extends to the separating equilibrium, since the mapping remains one-to-one in the presence of (more productive) SA firms. Every SA firm can outbid the highest profitable posted wage of every WP firm and that the measure of $\check{p}$ posting firms is zero since $\Gamma(p|\varepsilon)$ is continuous in every sub-market. Labor supply is thus:

$$\ell(p|\varepsilon) = \frac{1+k_1^\varepsilon}{[1+k_1^\varepsilon \bar{\Gamma}(p|\varepsilon)]^2} \frac{M^\varepsilon - U^\varepsilon}{N^\varepsilon}. \tag{2.8}$$

Details are provided in the appendix.

The solution to posting firms' maximization problem follows from applying the envelope theorem and solving the implied differential equation:

$$w_{PP}(p,\varepsilon) = \varepsilon p - [1+k_1^\varepsilon \bar{\Gamma}(p|\varepsilon)]^2 \int_{w_{UP}/\varepsilon}^{p} [1+k_1^\varepsilon \bar{\Gamma}(x|\varepsilon)]^{-2} dx \qquad \text{for } p < \check{p}, \tag{2.9}$$

See Bontemps et al. (2000, pg. 315-316) for explicit details. Now, one can easily check that

[4] See Burdett and Mortensen (1998) and Bontemps et al. (2000) for details and proofs of these results.

$w_{PP}(p, \varepsilon)$ is indeed differentiable in p.

Turning to the value of employment in a SA firm. Full ex-post rent extraction implies that an employee in an SA firm is receives the value of the best-to-date outside option. The value of employment in a SA firm is either $V^P(w_{PP}(q), \varepsilon)$ if the best-to-date outside option is WP ($q < \check{p}$),

$$V^A(w_{PA}(q, p, \varepsilon), p, \varepsilon) = V^P(w_{PP}(q), \varepsilon) \qquad \text{if } q < \check{p} \tag{2.10}$$

as was previously noted in the derivation of equation 2.1, or $V^A(\varepsilon q, q, \varepsilon)$ if the best-to-date outside option is SA, $\check{p} \leq q$,

$$V^A(w_{AA}(q, p, \varepsilon), p, \varepsilon) = V^A(\varepsilon q, q, \varepsilon) \qquad \text{if } \check{p} \leq q. \tag{2.11}$$

The appendix provides the value function for employment in a SA firm and shows that the value of employment in a q-productivity SA firm at the largest profitable wage, εq, has a surprisingly simple expression:

$$V^A(\varepsilon q, q, \varepsilon) = \frac{\varepsilon q + \delta V^U(\varepsilon)}{\mu^\varepsilon + \delta^\varepsilon},$$

as shown by Postel-Vinay and Robin (2002b).

Note also that, the value of employment with sufficiently robust labor market history (best-to-date outside option a SA firm) is independent of the distribution of WP firms! This is perhaps less surprising than independence of the value of employment in a WP firm from the distribution of SA firms, since typically the value of employment does not depend on the distribution of firms which are unable to make attractive wage offers. However, employment in a SA firm alone is not sufficient for independence, since when the best-to-date outside offer is a WP firm the wages is set to match value of employment in the WP firm.

Wages set under SA contracts as a function of labor market history - best-to-date outside offer, q, and incumbent employer, p - and worker skill level are:

$$w_{PA}(q,p,\varepsilon) = w_{PP}(q,\varepsilon) \tag{2.12}$$
$$-k_1^\varepsilon \bar{\Gamma}(\check{p}^\varepsilon|\varepsilon)\big[\check{p}^\varepsilon - w_{PP}(q,\varepsilon) - \int_q^{\check{p}^\varepsilon} \tfrac{k_1^\varepsilon[\Gamma(\check{p}|\varepsilon)-\Gamma(x|\varepsilon)]}{1+k_1^\varepsilon[\Gamma(\check{p}|\varepsilon)-\Gamma(x|\varepsilon)]}\tfrac{dw_{PP}(x,\varepsilon)}{dx}\big] - k_1^\varepsilon \int_{\check{p}^\varepsilon}^{p} \bar{\Gamma}(x|\varepsilon)dx \quad \text{for } q \le \check{p} \le p$$

and

$$w_{AA}(q,p,\varepsilon) = \varepsilon\{q - k_1^\varepsilon \int_q^p \bar{\Gamma}(x|\varepsilon)dx\} \qquad \text{for } \check{p}^\varepsilon < q \le p. \tag{2.13}$$

The first simplifies wage equation 2.1. The second solves equation 2.11 as shown by Postel-Vinay and Robin (2002b, pg. 2339-2340).

Finally, when entering employment in a SA firm, reservation wages depend on both ε and the SA firm's productivity p. However, since the value of unemployment equals the value of employment in the least productive wage posting firm, $V^U(\varepsilon) = V^P(\underline{p},\varepsilon)$, and the countering firm selects the wage to match the value of the best-to-date outside option, the schedule of reservation wages for each skill type is equal to the wage required to hire that skill-type from the least productive firm: $\underline{w}_{UC}(p,\varepsilon) = w_{PC}(\underline{p}^\varepsilon, p, \varepsilon)$.

Thus all wages are pinned down as a function of worker type and labor market history. All equations have closed form solutions for the case when $\lambda_0 = \lambda_1$, since in this case $\underline{w}_{UP} = \varepsilon b$.

2.4 Steady state wage distribution

Within each sub-market, the distribution of wages can be expressed as a function of the wage schedules just derived and the distribution of workers across firms and across wage offers within firms.

Since workers never reject an employment offer from a more productive firm the method of mass balance pins down the distribution of labor across productivity as:

$$L(p|\varepsilon) = \frac{\Gamma(p|\varepsilon)}{1 + \kappa_1^\varepsilon \bar{\Gamma}(p|\varepsilon)}. \tag{2.14}$$

The mass of workers employed in a firm at wage less than or equal to w can also be derived by the method of mass balance. For WP firms the mass is null everywhere except the posted wage where it is equal to the employment of ε-skilled workers in the firm. This follows since WP firms offer only a single wage for each skill level. Within SA firms, however, workers willing to accept wage $w_{iA}(q, p, \varepsilon)$ or less must have best-to-date outside option q or less (where $i = P$ if $q < \check{p}$ and $i = A$ if $q \geq \check{p}$). The mass flowing into such contracts must be $U\lambda_0\Gamma(q)$ and the mass flowing out must be $[\delta + \lambda_1\bar{\Gamma}(q)](M - U)L(q)$. This yields $\ell(w(q,p)|p) = \ell(q)$.[5] Within each firm we have that wages are distributed according to:

$$G(w|p) = \begin{cases} \mathbb{1}_{w \geq w_{PP}(p)}, & \text{if } p < \check{p} \\ \frac{\ell(q(w,p))}{\ell(p)} = \left[\frac{1+\kappa_1[1-\Gamma(p)]}{1+\kappa_1[1-\Gamma(q(w,p))]}\right]^2, & \text{if } \check{p} \leq p \end{cases}$$

Aggregating requires summing the mass employed at wages less than or equal to w in each firm weighted by that firms fraction of aggregate employment. So we have

$$G(w) = \int_{w(\underline{p},\bar{p})}^{w} G(w|p)dL(p) = \int_{w(\underline{p},\bar{p})}^{w} \ell(q(w,p))d\Gamma(p).$$

Noting that if p is a WP firm then $q(w,p) = p$ at the posted wage. The distribution of wages for a single skill-level is plotted in figure 1. The bottom panel decomposes this distribution by labor market history.

The wage distribution exhibits three modes. The left most are entry wages in SA firms: wages for workers with best-to-date outside option a WP firm or unemployment. SA firms

[5]Note, these results can also be found in Postel-Vinay and Robin (2002a).

are able to employee these workers at low wages by promising future on-the-job wage growth and lucrative job-to-job transition prospects. This is a direct consequence of the difference in option value supplied by SA and WP firms. Recall that the separating equilibrium is induced by difference in expected wage bills for the threshold WP and SA firm in equilibrium.

Wages in the middle mode are offered by WP firms. Typically these firms yield rent when hiring, since they are constrained to offer a single wage to all employees. Additionally, wages are high since WP firms commit not to attempt to retain workers when they transition to alternate employment and, as a result, offer poor job-to-job transition prospects. As a result wags in WP firms are higher than entry wages in SA firms.

Finally, wages in the right tail are set by competition between two SA firms. Workers in this tail earn wages that are (sometimes very) near the output produced in their employment match.

3 Comparative statics with respect to skill-type

Turning to the relation between worker skill, contract type, and wages, I show that differences in workers' opportunities arise even in ex-ante identical sub-markets. Ex-post differences in the composition of contracts play out in the distribution of wages, returns to experience, and employment opportunities of workers with different skill levels. Before proceeding I define ex-ante identical sub-markets.

The cost of SA is also identical in each sub-market. The distribution of potential firms in every sub-market is identical: $\Gamma(p)$. Vacancy posting is uniform, meaning that the probability of posting a vacancy in the ε-sub-market is equal to the portion of workers who are ε-type. This results in equal market tightness in all sub-markets: $M^{\varepsilon}/N^{\varepsilon} = M/N$ in all sub-markets. The hazards and discount rate; λ_0, λ_1, and δ, and μ; are identical across sub-markets, consistent with uniform market tightness.

It is possible that some firms are not active in all skill markets. Denote $\underline{p}^{\varepsilon}$ as the least productive firm which is able to profitably hire a ε-skilled worker. Firms of p-productivity less than $\underline{p}^{\varepsilon}$ are inactive in the ε-skill sub-market. The mass of active firms in the ε-skill sub-market is then $N^{\varepsilon} = \bar{\Gamma}(\underline{p}^{\varepsilon})N$. Similarly the arrival rates of acceptable wage offers are $\bar{\Gamma}(\underline{p}^{\varepsilon})\lambda_0$ and $\bar{\Gamma}(\underline{p}^{\varepsilon})\lambda_1$. Finally, the distribution of active firms is described by $\Gamma(p|\varepsilon) = [\Gamma(p) - \Gamma(\underline{p}^{\varepsilon})]/\bar{\Gamma}(\underline{p}^{\varepsilon})$.[6] Without loss of generality define $\varepsilon = 1$ as the maximal skill type and $\Gamma(p|1) = \Gamma(p)$.

Before proceeding I establish the following useful benchmark:

Claim 2. *If sub-markets are* ***nearly*** *ex-ante identical but the cost of SA is* ***proportional to skill level****,* $c^{\varepsilon} = \varepsilon c^1$*, then:*

- *threshold productivity and the sent of active firms is identical in all sub-markets:*

$$\check{p}^{\varepsilon} = \check{p} \text{ and } \Gamma(p|\varepsilon) = \Gamma(p) \; \forall \; \varepsilon,$$

- *wages decompose into the product of skill and labor market history:*

$$w(q, p, \varepsilon) = \varepsilon w(q, p, 1) \; \forall \; \varepsilon.$$

Proof of Claim 2 is mildly tedious and presented in the appendix. Direct implications are that the proportion of firms that hire under SA, hire under WP, or are inactive is identical in all sub-markets.[7]

[6] Note that these imply somewhat restrictive assumptions on the un-modeled matching function.

[7] An interesting additional consequence is that when thresholds are identical log wages decompose into a return to skill and a return to labor market history.

$$ln(w(q, p, \varepsilon)) = ln(\varepsilon) + \ln(w(q, p, 1)). \tag{3.1}$$

The implication is that a regression of log wages on skill, labor market history, and the interaction should yield an insignificant coefficient on the interaction term(s). This is consistent with the classic results of Mincer (1974) but not more recent results, for example Braga (2014).

Empirically, as Braga (2014) points out, the discrepancy in results stems from (miss)measurement of labor market history. Braga (2014) shows that, in U.S. data, results are significantly different if history is measured as actual versus potential experience. The specification suggested by this and other job-ladder models would be to measure labor market history, $\ln(w(q, p, 1))$, as *continuous employment experience* (eg. the time since the *most recent* spell of unemployment) or as the *number and quality of wage offers in the current employment spell*. The measure of Braga (2014) comes much nearer these definitions.

Observe that if the cost of SA are identical in all sub-markets then they are larger relative to output in lower skilled sub-markets. The direct implication is the following foundational result:

Claim 3. *If sub-markets are ex-ante identical then threshold productivity is decreasing in skill type:* $\varepsilon < 1 \Leftrightarrow \check{p}^1 < \check{p}^\varepsilon$.

This follows from the monotonicity of $\check{p}$ in c in each sub-market.

Juxtaposition of the case of constant costs (Claim 3) and with that of proportional costs (Claim 2) is central to the interpretation of the remaining results of this section. I seek comparative statics for differentials in worker skill. Claim 3 informs that lower skill markets feature higher threshold productivity. Claim 2 allows me to compare these to counter factual sub-markets for low-skill workers in which threshold productivity is constrained to be identical to the threshold for the high skilled. As a result a comparative static with respect to threshold productivity can be reinterpreted as a comparative static with respect to skill level.

When comparative statics regard wages a notion of proportionality is required. I call a statistic, $X(\varepsilon)$, proportional to skill if $X(\varepsilon) = \varepsilon X(1)$. If $X(\varepsilon) > \varepsilon X(1)$, I call this larger than proportionate or disproportionately large and visa versa. Claim 2 describes the case when wages are proportional to skill. Note that the special cases of pure-SA and pure-WP equilibria simultaneously satisfy Claim 3 and Claim 2: null costs of SA are consistent with pure-SA while infinite costs are consistent with pure-WP. All intermediate cases, however, reveal interesting ex-post disparities in the composition of contract types, composition of employer types, employment, wage levels and dispersion, and entry wages and return to experience.

For each outcome, I begin by considering the simplified case in which the chance of receiving a wage offer is identical on- and off-the job: $\lambda_1 = \lambda_0$. This case is simplest because identical offer arrival rates pin down the reservation wage for employment in a posting firm as the flow value of leisure, shutting down equilibrium feedback from the threshold productivity, $\check{p}^{\varepsilon}$, to the minimum productive productivity, $\underline{p}^{\varepsilon} = w_{UP}(\varepsilon)/\varepsilon$. The results of in the simplified case isolate the effect of the *composition* of contract types when the set of active firms is identical in all sub-markets.

I then consider the total effect when offer arrival is more likely for the unemployed, $\lambda_1 < \lambda_0$, as is likely the case in reality. The situation becomes both more interesting, more realistic, and a great deal more complex when the arrival rate of offers off-the-job exceeds the arrival rate on-the-job. Complexity arises since differentials in threshold productivity induced by skill differentials now feed back into the distribution of active firms. This produces interesting comparative statics with respect to unemployment and utilization. To maintain tractability, I consider the case where offer arrival rates are sufficiently similar off- and on-the-job. In this case feedback through the distribution of active firms is minor.

3.1 Entry wages and return to experience

I begin by considering entry wages.

Claim 4. *Entry wages for low-skill workers are disproportionately large,* $\varepsilon w_{PA}(q,p,1) < w_{PA}(q,p,\varepsilon)$ *and* $\varepsilon w_{PP}(p,1) = w_{PP}(p,\varepsilon)$*.*

To see this refer first to the wage schedule for employees of SA firms with best-to-date outside option a WP firm, equation 2.6, which under ex-ante identical sub-markets, simplifies to:

$$\begin{aligned} w_{PA}(q,p,\varepsilon) &= w_{PP}(q,\varepsilon) \\ &\quad -k_1\bar{\Gamma}(\check{p}^{\varepsilon})\big[\check{p}^{\varepsilon} - w_{PP}(q,\varepsilon) - \int_q^{\check{p}^{\varepsilon}} \tfrac{k_1[\Gamma(\check{p})-\Gamma(x)]}{1+k_1[\Gamma(\check{p})-\Gamma(x)]}\tfrac{dw_{PP}(x,\varepsilon)}{dx}\big] - k_1\int_{\check{p}^{\varepsilon}}^{p}\bar{\Gamma}(x)dx \quad \text{for } q \le \check{p} \le p \end{aligned} \tag{3.2}$$

It is straightforward to show that the wage schedule each SA firm offers when hiring workers with best-to-date outside options WP firms is increasing in the threshold productivity:

$\frac{dw_{PA}(q,p,\varepsilon)}{d\check{p}^{\varepsilon}} > 0$. From Claim 3 we know that if the threshold is identical in the sub-markets for two skill levels 1 and ε then $w_{PA}(q,p,\varepsilon) = \varepsilon w_{PA}(q,p,1)$. From this we have that if the threshold is higher in the ε sub-market then wages in SA firms when the best-to-date outside offer is a WP firm are disproportionately large for low-skilled workers: $w_{PA}(q,p,\varepsilon) > \varepsilon w_{PA}(q,p,1)$.

Now refer to the schedule of wages offered to employees of WP firms, equation 2.9. When the arrival of wage offers is constant across employment states the minimal active productivity is identical in all sub-markets. Inspection of the WP wage schedule, equation 2.9, then immediately reveals that with ex-ante identical sub-markets and identical sets of active firms wages set under WP are proportionate to skill: $w_{PP}(q,p,\varepsilon) = \varepsilon w_{PP}(q,p,1)$. However, if when offer arrival is more likely off-the-job than on-the-job it is easy to show that $w_{UP}(\varepsilon) > \varepsilon w_{UP}(1)$ and as a result $w_{PP}(p,\varepsilon) > \varepsilon w_{PP}(p,1)$ for $p < \check{p}^1$.

So, entry wages are disproportionately large regardless of initial employer (strictly when $\lambda_0 > \lambda_1$. Finally, when offer arrival rates are sufficiently similar, wages set under WP are more prevalent in low skilled sub-markets since the threshold productivity is higher and feedback to the set of active firms is minimal, as is shown later in this section. This means that newly employed low-skilled workers are more likely to be employed in WP contracts. Since entry wages in WP firms are always larger than entry wages in SA firms this reinforces the result.

I now consider wages of workers with sufficient experience to have best-to-date outside option a SA firm.

Claim 5. *For workers with sufficient labor market experience wages are proportionate to skill, and* $\varepsilon w_{AA}(q,p,1) = w_{AA}(q,p,\varepsilon)$.

Wages set by Bernard competition between two SA firms are proportionate to skill. To see this refer to the wage schedule set by SA firms when hiring workers with best-to-date outside options SA firms, equation 2.13. Quick inspection reveals that this is independent of threshold productivity: $w_{AA}(q,p,\varepsilon) = \varepsilon w_{AA}(q,p,1)$. With sufficient labor market experience a worker will be employed in a SA firm with best-to-date outside option a SA firm and be employed at wages set under wage schedule 2.13. So, sufficiently experienced workers are compensated proportionately to their skill level.

The implication for returns to continuous employment are striking.

Claim 6. *Low-skilled workers experience lower returns to continuous employment experience.*

This is a direct consequence of inflated entry wages coupled with proportionate long term prospects. The result is reinforced since the share of job offers originating from SA firms is smaller for low-skilled workers and, thus, it takes on average longer for low-skilled workers to climb to the lucrative right tail of the wage distribution characterized by Bertrand competition between two SA employers.

Figure 2 plots the wage-experience profile (top) and return to experience (bottom) for workers of type .5, .75, and 1, when the job offer arrival rate is constant on- and off-the-job. Solid lines plot simulated histories while hatched lines plot counterfactual histories for a labor market in which the threshold productivity is identical in all sub-markets. Consistent with Claim 4 entry wages are disproportionately high for low skilled workers. Wages are more out of proportion the larger the difference in skill. As continuous experience increases, however, workers of all skill levels climb the job ladder toward the SA sector. As a result, the premium low skill workers earn early in their careers dies out with experience. In the longer run, low skill workers wages fall behind and receive wages that are disproportionately low. This occurs since it takes low skill workers longer to climb the job ladder to the smaller SA sector in the low skilled sub-market. As a result of disproportionately high entry wages and greater average experience required achieve employment with best-to-date outside option a SA firm, low skill workers experience flatter wage-experience profiles compared to high skilled workers.

Figure 3 plots the wage-experience profile (top) and return to experience (bottom) for workers of type .5, .75, and 1, when the job offer arrival rates are $\lambda_0 = 2$ and $\lambda_1 = 0.2$

per year. These are roughly consistent with labor market histories of Germans 2006-2008 documented in Doniger (2014b). Solid lines plot simulated histories while hatched lines now plot counterfactual histories for a labor market in which the set of active firms is identical for all skill levels (the previous case). Consistent entry wages are, as before, disproportionately high for low skilled workers and wage-experience profiles follow a similar patter. Over all differences are extremely moderate. The overall features of differential return to experience are moderately accentuated.

Table 1 records differentials in the proportion of contract types with respect to skill level. As discussed, the likelihood of receiving an offer from a WP firm is decreasing in skill level. Meanwhile the value of unemployment, average entry wages and average wages are decreasing disproportionately slowly with respect to skill level.

Differentials in the distributions of employment for different skill levels complicate proof of the remaining. Clearly these are analogous to the simpler case and their proof is maintained for sufficiently small differences in offer arrival rates. This is treated in the appendix.

Meanwhile, table 2 records statistics analogous to table 1 as well as the unemployment rate and fraction of firms which are inactive in each sub-market. A substantial fraction of firms do not operate in the low skill labor market. The result are unemployment rates which are increasing as skill declines.

3.2 Average wages and wage dispersion

Next I turn to average wages.

Claim 7. *Average wages for low-skill workers are disproportionately large:* $\varepsilon\mathbb{E}[w|1] < \mathbb{E}[w|\varepsilon]$.

The result not straightforward to obtain. On the one hand, for the vast majority of labor market histories wages weakly rise when threshold productivity rises. The logic above implies that wages fall disproportionately slowly with respect to a decline in skill for these histories. On the other hand, for labor market histories for which the best-to-date outside option changes strategy from SA to WP when the threshold rises the wage *falls dramatically*. Again, this implies that for a small difference in skill workers with identical labor market histories but slightly larger skill will have *dramatically larger wages*. The second occurs since the less skilled individuals wages are *not* set by Bertrand competition between two SA firms while the more skilled individuals wages are. The claim is proved if in a market with homogeneous workers every firms' wage bill weakly rises when threshold productivity rises. This will imply that for each firm the percent change in average wages falls (weakly) more slowly than skill level. Proof that this is indeed the case when offer arrival is constant on- and off- the job is tedious and presented in the appendix. The result is maintained if offer arrivals are sufficiently similar; since, feedback from threshold productivity to minimum productivity is minor.

Claim 4 and claim 6 have additional implications for wage dispersion. Figure 4 plots the distribution of log wages implied by the model for a worker of skill level 1, .75 and .5 in ex-ante identical sub-markets. 10^{th}, 50^{th}, and 90^{th} percentiles are indicated by the vertical bars. Distributions of wages for less skilled individuals are more compressed. Compression is particularly pronounced for wages in SA employers with best-to-date outside option unemployment or a WP employer (the left mass). This portion of the distribution is shifted disproportionately toward larger wages for low skilled individuals, since the value of unemployment and employment in a WP contract are disproportionately high for these individuals. Starting wages are more generous due to this shift in the left tail of wags and the increased odds of becoming employed in a WP firm out of unemployment (the middle mass). Notably, however, distribution in the right tail, where wages are set by SA firms facing best-to-date outside option a SA firm, is proportional to skill.

Discuss in relation to Lemieux (2008) and others.

3.3 Employment

Turning to employment, the following result follows from slower than proportionately declining reservation wage for employment in the WP sector as skill falls.

Claim 8. *Low-skilled workers experience higher rates of unemployment.*

Since reservation wages are not proportionately low offers from the least productive firms are rejected (equivalently, these firms exit the low skilled market) resulting in inflated unemployment for low skilled workers.

3.4 Composition of contract types and employers

Claim 9. *Low-skilled workers are less likely to be employed in SA contracts.*

This is a direct implication of higher threshold productivity. When on- and off-the-job arrival rates are identical the corollary is clearly that low skilled workers are more likely to be employed in WP contracts. Allowing for differences in on- and off-the-job arrival rates requires considering a third category, unemployment. In this case low-skilled workers are more likely to be unemployed, as just noted, or employed in a WP contract than their high-skilled counterparts. Conditioning on employment, low skilled workers less likely to be employed in a SA contract so long as the on- and off-the-job arrival rates are sufficiently similar.

Finally turning to the composition of employers.

Claim 10.

1. *Some low productivity firms employ no low-skilled workers.*
2. *The fraction of workers employed at firms of productivity less than p is increasing in skill level.*
3. *Firms that employ both high-skilled and low-skilled workers employ a larger mass of low-skilled workers.*

Firms inability to direct vacancies toward high skill workers motivates these. Now consider that in the low skilled sub-market the value of unemployment is disproportionately high relative to the high skilled sub-market. Since the arrival rate of offers is less frequent on-the-job this induces a disproportionately large reservation wage for employment in a WP firm in the low skill sub-market. As a result, some of the least productive firms are effectively "priced out" of the low skilled sub-markets and these firms employ no low-skilled workers. This is the corollary of claim 8.

The second and third result are proved in the appendix. Intuition rests on noting that low skill workers reject the least productive employers and, thus, the distribution of their employment is skewed toward more productive firms.

4 Application: evaluating job training

Many countries offer, and sometimes require, job training as an activation policy for the unemployed. Evaluation of these policies has been extensive but is complicated by considerable econometric difficulty. In particular training programs an not typically designed with pseudo-experimental evaluation in mind. This results in difficulty obtaining data on suitable "control" individuals to serve as counterfactual in empirical assessment. In addition, even when policy is designed or implemented in a way where a control group can be identified, estimates of treatment effects may be attenuated due to spill-overs from the behavior of trained individuals to the behavior of untrained individuals (Heckman et al., 1999).

Despite econometric difficulties, job training programs are extensively studied in part because many training programs are accompanied with a mandate for cost-benefit analysis. Studies typically find little or no impact of training on re-employment wages in the short run but often find sizeable positive impact in the medium and long run (Card et al., 2010).

Differences in results between the short and the longer run are not well explained by a lack of suitable or suitably matched control group. Why should a more suitable control group appear over time? Differences in results with respect to time horizon are also not easily explained by spill-overs since typically one suspects spill-overs attenuate estimates and spill-over is likely to be more prevalent in the longer term.

A lag in the impact of training, however, can be explained by structural models of learning or labor market behavior in which training takes time to sink in for workers, be recognized by firms, or be rewarded by firms even if it is immediately recognized. The results of this paper provide a mechanism of the third type: firms recognize training immediately, however, differences in labor markets for low skilled (untrained) versus high skilled (trained) workers results in attenuated expression of training in wages for short durations of continuous employment experience.

This suggests that the policy evaluator must defer evaluation some time in order to observe the full effect of training in average wages, from which the effect of training on productivity can be inferred. This helps explains the relative plethora of negative results obtained from evaluations with short time horizons.

However, if the mechanism described contributes significantly to patterns in the data, there is another way. As discussed, as workers move into the SA sector, and in particular when their best-to-date outside option is a SA firm, wages are proportional to skill level (training). This make take a long time and even in steady state average wages are disproportionately high for low skilled (untrained) workers. However, the policy evaluator can selectively observe workers who have climbed the job ladder by targeting workers whose wages are in the right tail of the distribution.

Under the hypothesis that the model presented here is the true model, statistics obtained from the right tail of the wage distribution are less attenuated at all time horizons and reveal the *true* value of training at reasonably short horizons.

4.1 Example

Consider training unemployed individuals of skill type .75. Suppose that training is effective and after training 100% of trained individuals are of skill type 1: an increase in skill of 33.3 percent. Suppose also that the policy maker has an experimental design. Finally suppose that there are no general equilibrium effects of training. The last assumption imposes that both before and after training the effective labor produced by a worker of a given skill level is identical and that the arrival rate of job offers and distribution of productivity are also invariant to the presence of the training program.

I simulate such an experiment for 5000 trained and 5000 untrained workers. I set hazards to be $k_0 = 2$ and $k_1 = .2$ per year, consistent with estimates from German workers 2006-2008 obtained in Doniger (2014b). I model productivity as having a Pareto distribution and let the shape and scale be 3.73 and 113 respectively. These imply an unemployment rate of 9.38 and mean and standard deviation of the wage distribution 114.88 and 51.9773 for trained workers 87.39 and 37.6402 for untrained workers. These are broadly consistent with labor market histories of West German males 2005-2008. I then simulate continuous spells of employment experience for each individual for 15 years.

Estimates of the percent increase in skill from a regression at 1, 2, and 3 years of continuous experience, the typical horizons tested in the literature, are 27.6, 28.1, and 29.7. These are under estimates of 3-5 percentage points of the true effect. The percent difference in average wages between the trained and untrained group are plotted in the top panel of figure 5. Indeed it takes more than 10 years for the percentage differences in wages to reflect the increase in skill from training.

The policy maker may have more success at short horizons by considering statistics which focus on the right tail of the wage distribution, where wages are set competitively through competition between SA employers. This suggests analyzing percentiles of the distribution. The difficulty is to be sure of when the X percentile individual is sure to have best-to-date outside option a SA firm. For the simulation conducted here this occurs within 2 years for the 95th percentile and 4 years for the 90th percentile. After this point the percent difference in wages at the Xth percentile reveal the true value of training after this point.

An unfortunate difficulty is that time until the X percentile individual has best-to-date outside option a SA firm is longer for the untrained control group since more firms hire the untrained under WP. As a result the X percentile diverges radically from the true estimate just before becoming informative. Fortunately, the time horizon is larger for lower percentiles and the research can compare estimates based on percentiles and on average wages to rule out gross overstatements.

5 Conclusion

This paper considers an on the job search model in which heterogenous firms and workers form employment matches under one of two contract types: wages posting (WP) or sequential auction (SA). Workers are characterized by their skill and firms by their productivity. In equilibrium, more productive firms select the SA contract for all worker types while less productive firms select the WP contract when employing under the SA contract is costly. The model nests pure-contract equilibria which feature only WP, Bontemps et al. (2000), and only SA, Postel-Vinay and Robin (2002b). Intermediate cases, which feature both contract types simultaneously, deliver a new look at differences in employment, wage dispersion, and returns to experience for workers of different skill levels.

References

Barron, J. M., M. C. Berger, and D. A. Black (2006, July). Selective counteroffers. *Journal of Labor Economics 24*(3), 385–410.

Bontemps, C., J.-M. Robin, and G. J. van den Berg (2000, May). Equilibrium search with continuous productivity dispersion: Theory and nonparametric estimation. *International Economic Review 41*(2), 305–58.

Braga, B. (2014). *Schooling, Experience, Career Interruptions, and Earnings*. Ph. D. thesis, University of Michigan.

Brenzel, H., H. Gartner, and C. Schnabel (2013, September). Wage Posting or Wage Bargaining? Evidence from the Employers' Side. IZA Discussion Papers 7624, Institute for the Study of Labor (IZA).

Burdett, K. and D. T. Mortensen (1998, May). Wage differentials, employer size, and unemployment. *International Economic Review 39*(2), 257–273.

Card, D., J. Kluve, and A. Weber (2010, November). Active labour market policy evaluations: A meta-analysis. *Economic Journal 120*(548), F452–F477.

Doniger, C. L. (2014a). *Chapter 1: Posted and negotiated wages in markets with equilibrium on-the-job search: Theory*. Ph. D. thesis, University of Michigan.

Doniger, C. L. (2014b). *Chapter 3: Empirical identification of the composition of contracts in an on-the-job search equilibrium with heterogeneous wage contracts using administrative data*. Ph. D. thesis, University of Michigan.

Ellingsen, T. and A. Rosén (2003, May). Fixed or flexible? wage-setting in search equilibrium. *Economica 70*(278), pp. 233–250.

Hall, R. E. and A. B. Krueger (2008, September). Wage formation between newly hired workers and employers: Survey evidence. Working Paper 14329, National Bureau of Economic Research.

Hall, R. E. and A. B. Krueger (2010, May). Evidence on the determinants of the choice between wage posting and wage bargaining. Working Paper 16033, National Bureau of Economic Research.

Heckman, J. J., R. J. Lalonde, and J. A. Smith (1999, October). The economics and econometrics of active labor market programs. In O. Ashenfelter and D. Card (Eds.), *Handbook of Labor Economics*, Volume 3 of *Handbook of Labor Economics*, Chapter 31, pp. 1865–2097. Elsevier.

Jacobi, L. and J. Kluve (2007). Before and after the hartz reforms: The performance of active labour market policy in germany. *Zeitschrift fr ArbeitsmarktForschung - Journal for Labour Market Research 40*(1), 45–64.

Lemieux, T. (2008, January). The changing nature of wage inequality. *Journal of Population Economics 21*(1), 21–48.

Michelacci, C. and J. Suarez (2006). Incomplete wage posting. *Journal of Political Economy 114*(6), pp. 1098–1123.

Mincer, J. A. (1974, October). *Schooling, Experience, and Earnings*. Number minc74-1 in NBER Books. National Bureau of Economic Research, Inc.

Postel-Vinay, F. and J.-M. Robin (2002a, November). The distribution of earnings in an equilibrium search model with state-dependent offers and counteroffers. *International Economic Review 43*(4), 989–1016.

Postel-Vinay, F. and J.-M. Robin (2002b, November). Equilibrium wage dispersion with worker and employer heterogeneity. *Econometrica 70*(6), 2295–2350.

A Figures

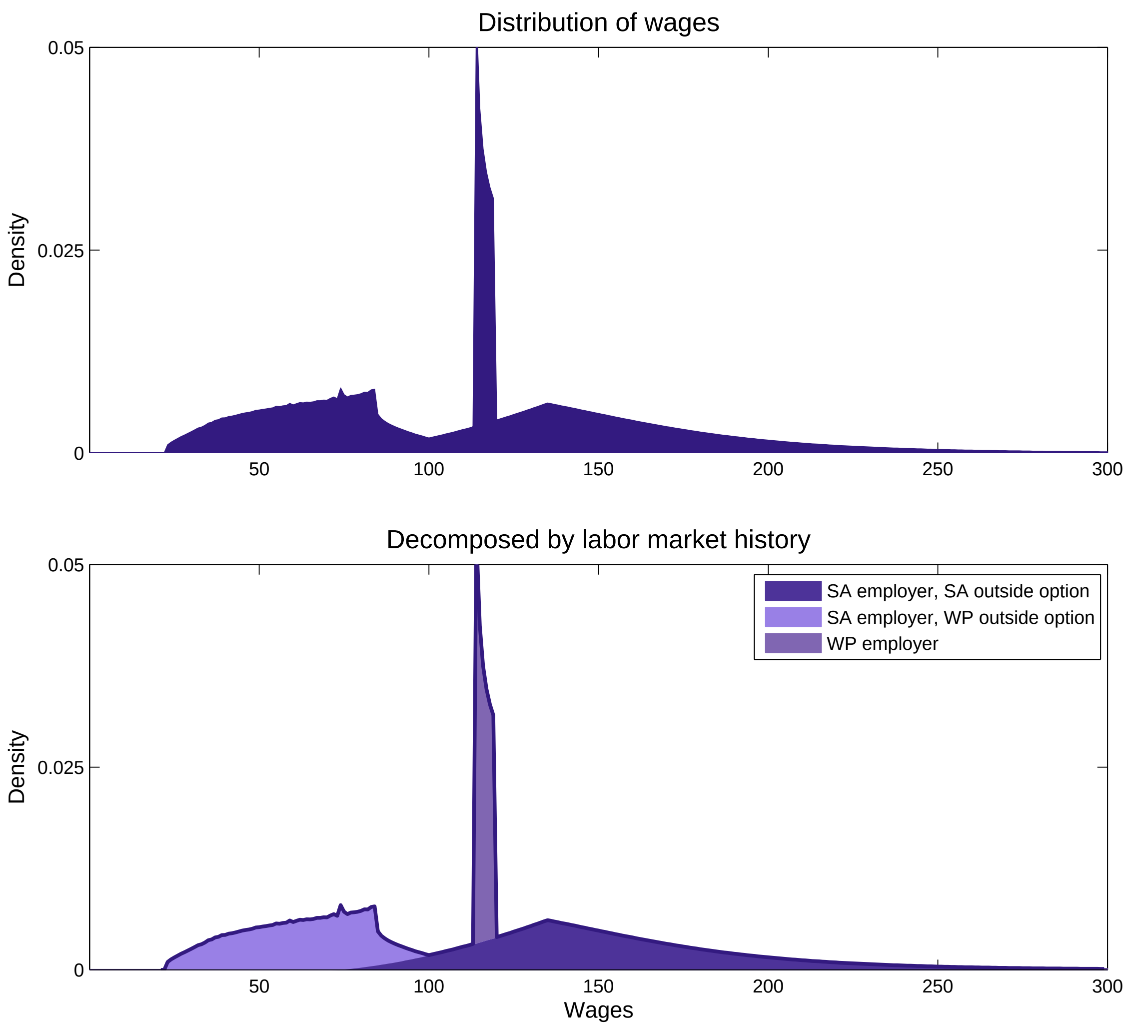

Figure 1: **Distribution of wages (top) for a single skill type. Distribution decomposed by labor market history (bottom).** The wage distribution exhibits three modes. The left most are wages in SA firms for workers with best-to-date outside option a WP firm or unemployment. SA firms are able to employee these workers at low wages by promising future on-the-job wage growth and lucrative job-to-job transition prospects. Wages in the middle mode are offered by WP firms. Typically these firms yield rent when hiring, since they are constrained to offer a single wage to all employees. Additionally, wages are high since WP firms commit not to attempt to retain workers when they transition to alternate employment and, as a result, offer poor job-to-job transition prospects. Finally, wages in the right tail are set by competition between two SA firms. Workers in this tail occasionally earn wages that are very near the output produced in their employment match. This occurs when the productivity of the current employer and best-to-date outside option are similar.

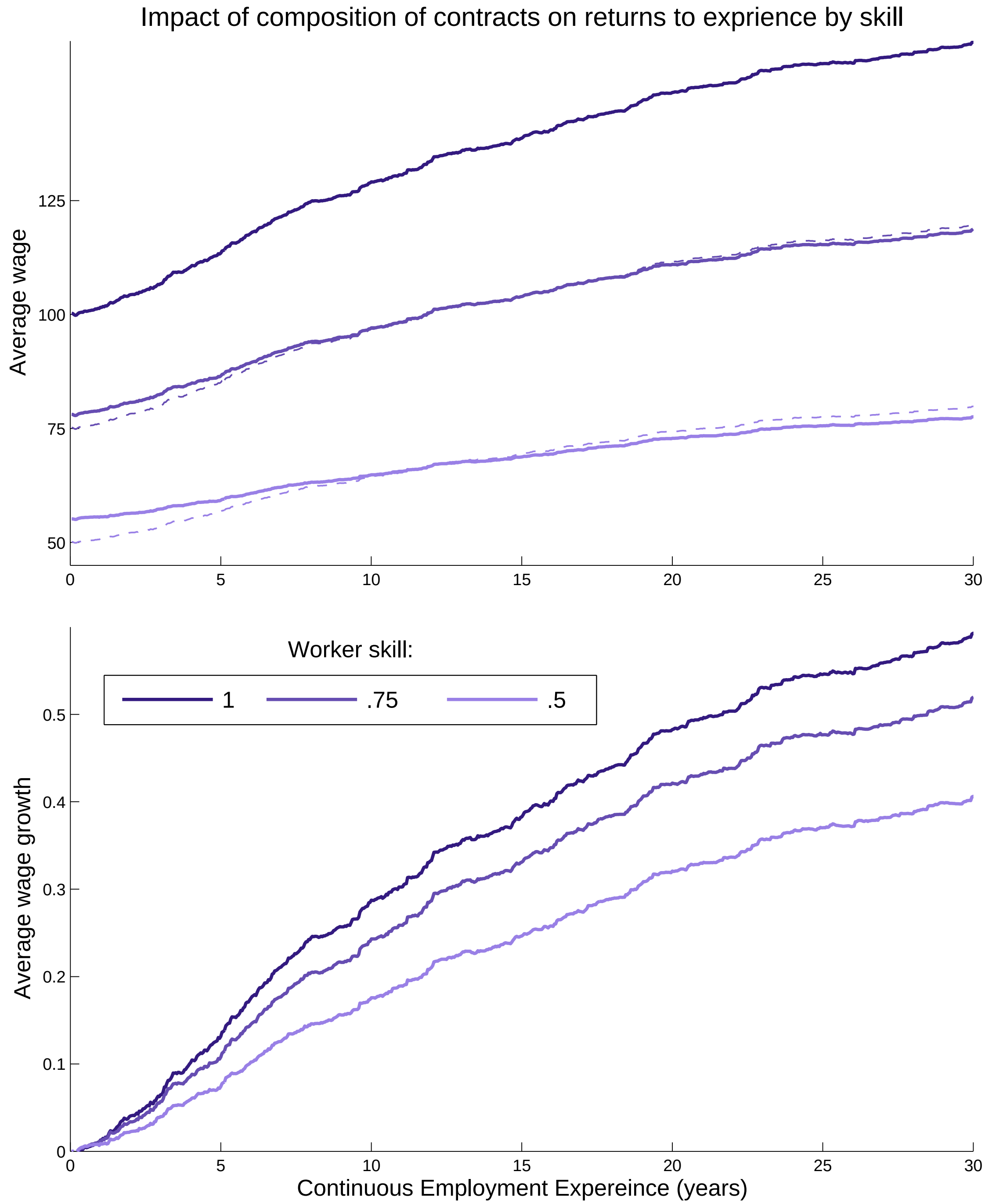

Figure 2: Average wages and average wage growth by skill level and continuous employment experience. Compression in the left tail of the wage distribution coupled with greater odds of initial employment in a WP firm result in higher than proportional starting wages for low skilled workers. As workers climb the job lader the odds of employment in a SA firms facing best-to-date outside option a SA firm increase. Wages for such a history *are* proportionate to skill. The over all effect is that more skilled workers experience greater wage growth.

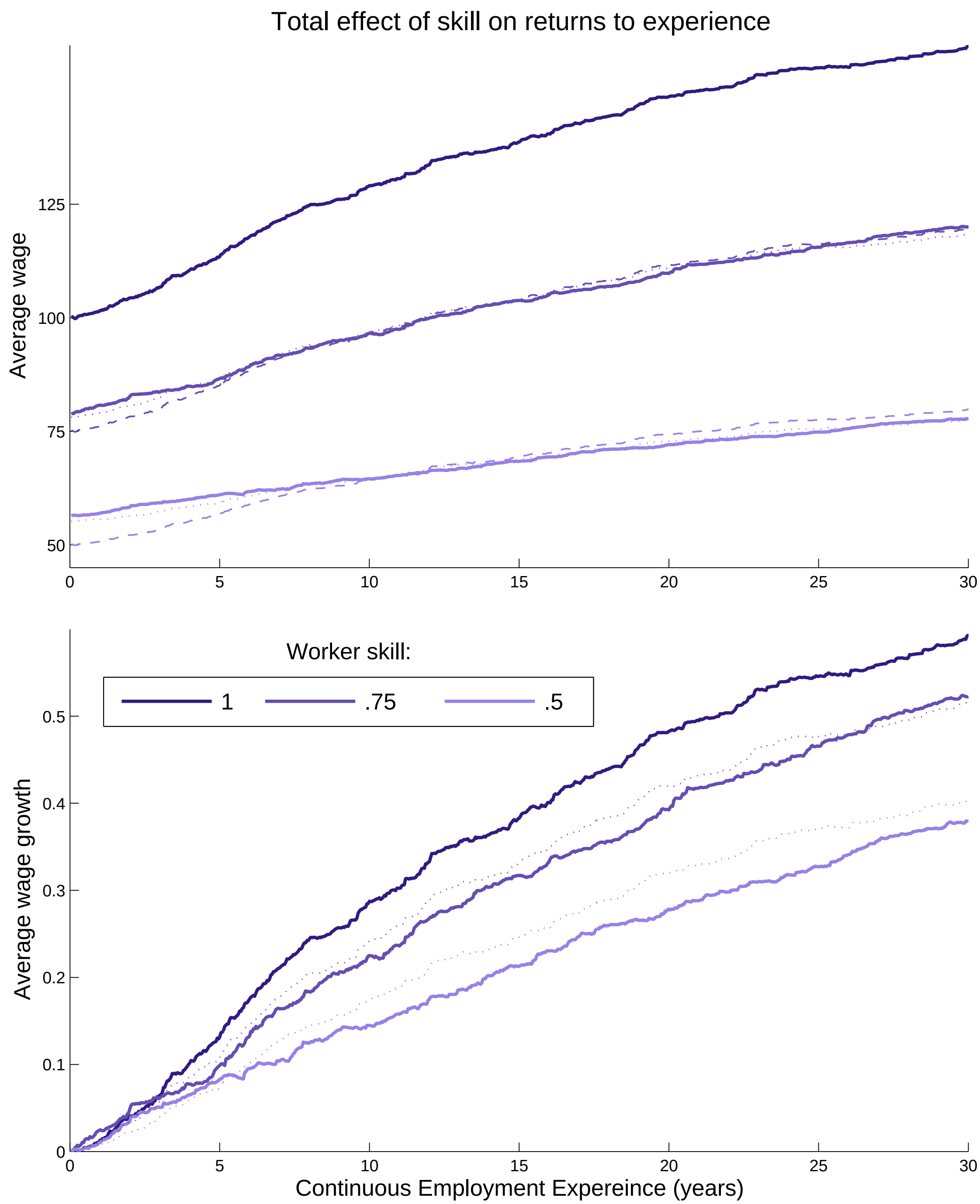

Figure 3: Average wages and average wage growth by skill level and continuous employment experience. Compression in the left tail of the wage distribution coupled with greater odds of initial employment in a WP firm result in higher than proportional starting wages for low skilled workers. As workers climb the job lader the odds of employment in a SA firms facing best-to-date outside option a SA firm increase. Wages for such a history *are* proportionate to skill. The over all effect is that more skilled workers experience greater wage growth.

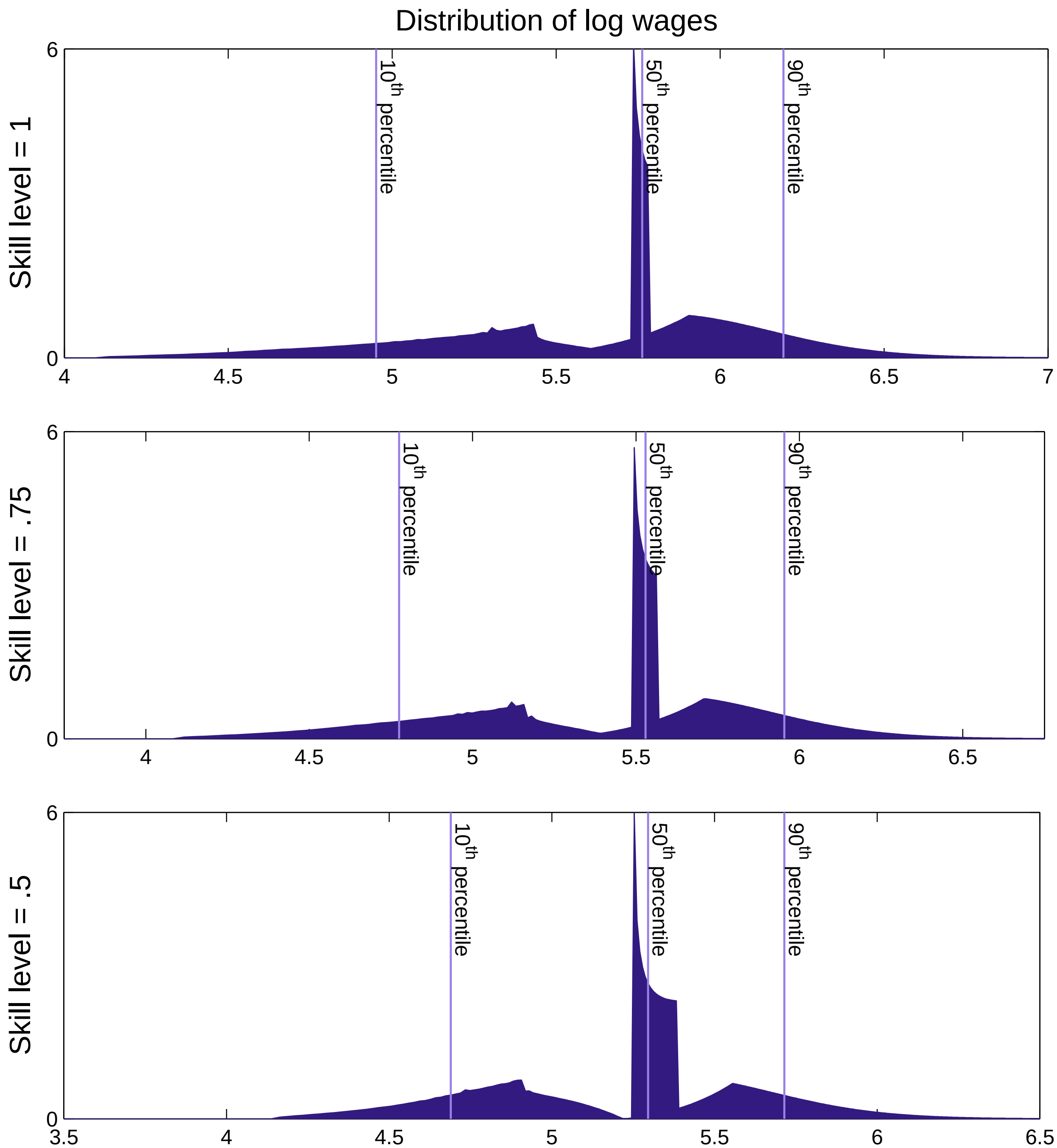

Figure 4: **Distributions of wages skill levels 1, .75, and .5.** 10^{th}, 50^{th}, and 90^{th} percentiles are indicated by the vertical bars. Distributions of wages for less skilled individuals are more compressed. Compression is particularly pronounced in starting wages in SA firms which is shifted disproportionately toward larger wages for low skilled individuals. Starting wages are again more generous due to the increased odds of becoming employed in a WP firm out of unemployment. Notably, however, mass in the right tail, where wages are set by SA firms facing best-to-date outside option a SA firm, is not affected by skill.

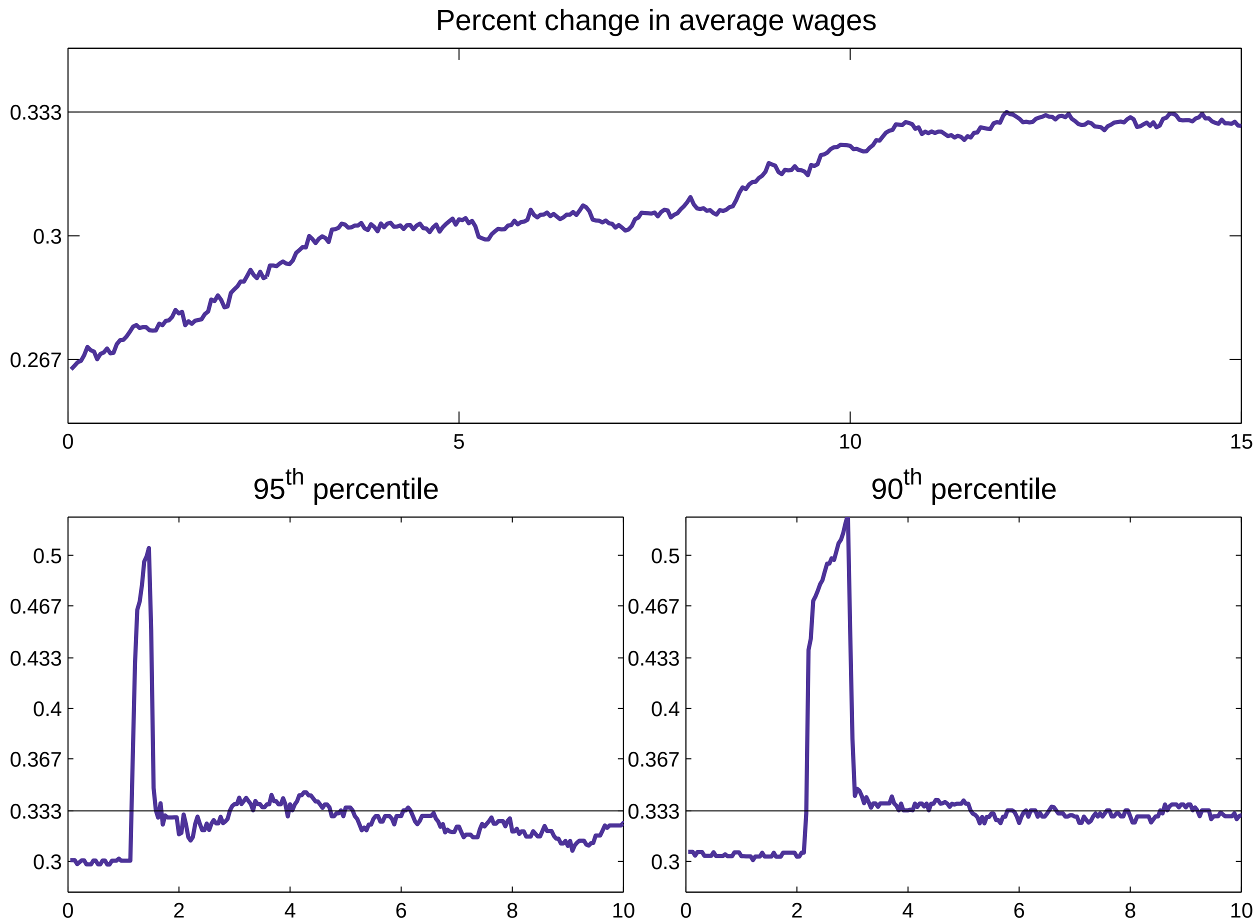

Figure 5: The difference in average wages understates the impact of training even at high experience levels. Differences in high percentiles reveal the value of training on much shorter horizons.

B Tables

Table 1: Market composition average wages and value of unemployment: equally efficient search off- and on-the-job

	Worker skill:		
	.5	.75	1
Average wage	59.57	87.17	114.88
(fraction of full skill)	*51.86%*	*75.88%*	*100%*
Average entry wage	55.17	78.10	100.29
(fraction of full skill)	*55.01%*	*77.88%*	*100%*
Average entry — 10 years continuous experience	64.55	96.58	129.11
(fraction of full skill)	*50.00%*	*74.8%*	*100%*
μ(Val. Nonemp.)	59.57	85.51	113.62
(fraction of full skill)	*50.53%*	*75.26%*	*100%*
Firms			
Inactive	0.00%	0.00%	0.00%
Posting	68.08%	56.18%	47.76%
Countering	31.92%	43.82%	52.24%
Workers			
Unemployed	9.38%	9.38%	9.38%
Posting	37.90%	26.84%	20.73%
Countering	62.10%	73.16%	79.27%

Table 2: Market composition average wages and value of unemployment: more efficient search off-the-job

	Worker skill:		
	.5	.75	1
Average wage	57.77	87.39	114.88
(fraction of full skill)	*50.29%*	*76.07%*	*100%*
Average entry wage	56.39	78.82	100.29
(fraction of full skill)	*56.22%*	*78.60%*	*100%*
Average entry — 10 years continuous experience	64.87	97.10	129.11
(fraction of full skill)	*50.24%*	*75.19%*	*100%*
μ(Val. Nonemp.)	58.53	86.18	113.62
(fraction of full skill)	*51.51%*	*75.85%*	*100%*
Firms			
Inactive	6.33%	1.63%	0.00%
Posting	62.13%	54.55%	47.76%
Countering	31.54%	43.82%	52.24%
Workers			
Unemployed	9.95%	9.52%	9.38%
Posting	38.19%	26.72%	20.73%
Countering	61.81%	73.28%	79.27%

C Appendix

Proof of Claim 1

I begin by proving that for every threshold a cost exists such that separation is consistent with a Nash equilibrium. I then observe that the mapping is continuous, that null costs are consistent with all firms selecting SA and all firms select WP for sufficiently large cots. The claim then follows from the intermediate value theorem.

Labor supply

Workers seek to maximize the value of their current employment contract. In order to pin down labor supply to a firm of type p I must show how workers' value maximization results in labor flow between firms.

As a starting point, in the pure-WP and pure-SA equilibria Burdett and Mortensen (1998) and Postel-Vinay and Robin (2002a), respectively, show that higher productivity firms are able and prefer to offer higher value contracts in equilibrium.

Now note that the largest possible WP wage is at most equal to the marginal product of the threshold firm. Next, the least productive SA firm has larger marginal product since it is, in the proposed separating equilibrium, more productive. So, each SA firm can profitably pay a wage at least as large as the largest possible WP wage and, therefore, must hire employees of WP firms (when it meets them) in equilibrium.[8] Therefore, the flow between sectors is efficient.

Within the WP sector firms behave as if the whole economy posted wages, and so flows are efficient. This follows from noting that the presence of SA firms does not alter the solution to the posting firms' maximization problem since 1) profitable wage choices are

[8]We will see that, in equilibrium, the countering firm is actually able to hire these workers for less than their best-to-date posted wage. The reason is that the countering firm compensates its workers partially through the option value of contingent pay.

only competitive against other WP firms and 2) since $\Gamma(p)$ is continuous there is no mass of WP firms at the threshold productivity. The proof due to Burdett and Mortensen (1998) applies: more productive firms can employ workers of less productive firms a trivially greater wages and at greater profits. Finally, within the SA sector higher productivity firms are still able to outbid lesser productivity firms and so the flows are efficient. This shows that, in total, flows of employed workers in the separating equilibrium are efficient.

Since workers accept any wage offer originating from a more productive firm, labor supply to a p-type firm can be pinned down by the method of mass balance. In steady state, the mass of workers flowing into firms of p-type or less must be equal to the mass flowing out:

$$\underbrace{U^{\varepsilon}\lambda_0^{\varepsilon}\Gamma(p|\varepsilon)}_{\text{in}} = \underbrace{[\delta^{\varepsilon} + \lambda_1^{\varepsilon}\bar{\Gamma}(p|\varepsilon))](M^{\varepsilon} - U^{\varepsilon})L(p|\varepsilon)}_{\text{out}}, \tag{C.1}$$

where $\bar{\Gamma}(p|\varepsilon) = 1 - \Gamma(p|\varepsilon)$ is the fraction of firms with productivity greater than p, U denotes the mass of unemployed workers, and $L(p|\varepsilon)$ is the mass of workers employed in a firm of productivity no greater than p.

Evaluating equation (C.1) at the supremum of productivity types yields steady state unemployment rate: $u = U^{\varepsilon}/M^{\varepsilon} = 1/(1+\kappa_0^{\varepsilon})$, where $\kappa_0^{\varepsilon} = \lambda_0^{\varepsilon}/\delta^{\varepsilon}$. Also, the fraction of workers working for a firm with technology p or less is

$$L(p|\varepsilon) = \frac{\Gamma(p|\varepsilon)}{1 + \kappa_1^{\varepsilon}\bar{\Gamma}(p|\varepsilon)}, \tag{C.2}$$

where $\kappa_1^{\varepsilon} = \lambda_1^{\varepsilon}/\delta^{\varepsilon}$ is the expected number of job offers per employment spell.

The supply of labor from ε-type workers to a firm of type p can then be expressed as:

$$\ell(p|\varepsilon) = \overbrace{\frac{\lambda_0^{\varepsilon}U^{\varepsilon} + \lambda_1^{\varepsilon}(M^{\varepsilon} - U^{\varepsilon})L(p|\varepsilon)}{N^{\varepsilon}}}^{\text{hiring}} \overbrace{\frac{1}{\delta^{\varepsilon} + \lambda_1^{\varepsilon}\bar{\Gamma}(p|\varepsilon)}}^{\text{ex-post expected duration}} = \frac{1+\kappa_1^{\varepsilon}}{[1+\kappa_1^{\varepsilon}\bar{\Gamma}(p|\varepsilon)]^2}\frac{M^{\varepsilon} - U^{\varepsilon}}{N^{\varepsilon}}. \tag{C.3}$$

Note that the flow of workers between employers is constrained efficient.

Wage choice

The SA firms's optimal wage choice equates the value of employment in the SA firm at the optimal wage with the value of employment at the best-to-date outside option at that competitors optimal wage choice. The optimal wage offer equates the value of employment in the SA firm with the value of employment in the WP firm at the optimal posted wage.

$$V^P(w_{PA}(q,p,\varepsilon),p,\varepsilon) = V^P(w_{PP}(q,\varepsilon),q,\varepsilon) \tag{C.4}$$

The value of employment at a SA firm at some wage $w_{PA}(q,p)$ is consistent with best-to-date outside offer originating from a q-productivity WP competitor is:

$$\begin{aligned}
\mu^\varepsilon V^A(w_{PA}(q,p,\varepsilon),p,\varepsilon) = w_{PA}(q,p,\varepsilon)& \\
&+ \underbrace{\lambda_1^\varepsilon[\Gamma(\check{p}|\varepsilon) - \Gamma(q|\varepsilon))][\mathbb{E}[V^P(w_{PP}(x,\varepsilon),x,\varepsilon)|q < x < \check{p}] - V^A(w_{PA}(q,p,\varepsilon),p,\varepsilon)]}_{\text{on-the-job wage gain due to a credible threat from a WP competitor}} \\
&+ \underbrace{\lambda_1^\varepsilon[\Gamma(p|\varepsilon) - \Gamma(\check{p}|\varepsilon)][\mathbb{E}[V^A(\varepsilon x,x,\varepsilon)|\check{p} < x < p] - V^A(w_{PA}(q,p,\varepsilon),p,\varepsilon)]}_{\text{on-the-job wage gain due to a credible threat from a SA competitor}} \\
&+ \underbrace{\lambda_1^\varepsilon[\bar{\Gamma}(p|\varepsilon)][V^A(\varepsilon p,p,\varepsilon) - V^A(w_{PA}(q,p,\varepsilon),p,\varepsilon)]}_{\text{job-to-job transition to a SA competitor}} \\
&+ \underbrace{\delta^\varepsilon[V^U(\varepsilon) - V^A(w_{PA}(q,p,\varepsilon),p,\varepsilon)]}_{\text{unemployment shock}},
\end{aligned} \tag{C.5}$$

where $w_{PP}(x)$ and x are the optimal competing wage offers of x-type WP and SA firms with productivity less than p, respectively.

Meanwhile, the value of employment in the employees best-to-date outside option, a

q-productivity WP firm, is:

$$\begin{aligned}
\mu^{\varepsilon} V^P(w_{PP}(q,\varepsilon),q,\varepsilon) = {} & w_{PP}(q,\varepsilon) \\
& + \underbrace{\lambda_1^{\varepsilon}[\Gamma(\check{p}|\varepsilon)-\Gamma(q|\varepsilon)][\mathbb{E}[V^P(w_{PP}(x,\varepsilon),x,\varepsilon)|q<x<\check{p}]-V^P(w_{PP}(q,\varepsilon),q,\varepsilon)]}_{\text{job-to-job transition to a WP competitor}} \\
& + \underbrace{\lambda_1^{\varepsilon}[\bar{\Gamma}(\check{p}|\varepsilon)][\mathbb{E}[V^A(w_{PA}(q,x,\varepsilon),x,\varepsilon)|\check{p}<x]-V^P(w_{PP}(q,\varepsilon),q,\varepsilon)]}_{\text{job-to-job transition to a SA competitor}} \\
& + \underbrace{\delta^{\varepsilon}[V^U(\varepsilon)-V^P(w_{PP}(q,\varepsilon),q,\varepsilon)]}_{\text{unemployment shock}}. \qquad \text{(C.6)}
\end{aligned}$$

The optimal wage choice can thus be expressed as the posted wage at the best-to-date outside offer and the difference in the option values of the two employment contracts:

$$\begin{aligned}
& w_{PA}(q,p,\varepsilon) = w_{PP}(q,\varepsilon) + \qquad \text{(C.7)} \\
& \underbrace{\lambda_1\left\{\bar{\Gamma}(\check{p}^{\varepsilon}|\varepsilon)V^P(w_{PP}(q,\varepsilon),q,\varepsilon)-[\Gamma(p|\varepsilon)-\Gamma(\check{p}^{\varepsilon}|\varepsilon)]\mathbb{E}[V^A(\varepsilon x,x,\varepsilon)|\check{p}<x<p]-\bar{\Gamma}(p|\varepsilon)V^A(\varepsilon p,p,\varepsilon)\right\}}_{\text{difference in option values in the SA contract and best-to-date WP outside option}}.
\end{aligned}$$

Contract choice

Current operating surplus for WP firms is pinned down simply as rent per worker times labor supply: $[p - w_{PP}(p)]\ell(p)$. Deriving current operating surplus for SA firms requires deriving the fraction of their employees earning each wage. Following the usual solution strategy, the mass flowing in and out of such wages must balance. Note that workers willing to accept wage $w_{PA}(q,p)$ (or $w_{AA}(q,p)$ if q is a SA firm) or less must have best-to-date outside option q or less. The mass flowing into such contracts will be $U\lambda_0\Gamma(q)$ and the mass flowing out must be $[\delta + \lambda_1\bar{\Gamma}(q)](M-U)L(q)$. This yields $\ell(w(q,p)|p) = \ell(q)$.[9] The current operating

[9] Note, these results can also be found in Postel-Vinay and Robin (2002a).

surplus (exclusive of the cost of countering) for a firm of type p offering the SA contract is

$$\pi^A(p|\varepsilon) = \int_{\underline{p}^\varepsilon}^{\check{p}^\varepsilon} (p - w_{PA}(q, p, \varepsilon))d\ell(q|\varepsilon) + \int_{\check{p}^{\varepsilon+}}^{p} (p - w_{AA}(q, p, \varepsilon))d\ell(q|\varepsilon). \tag{C.8}$$

For threshold productivity in the interior of the support of the productivity distribution, the threshold productivity firms willingness to pay for the right to SA is:

$$\begin{aligned} c^\varepsilon &= \pi^A(\check{p}^\varepsilon|\varepsilon) - \pi^P(\check{p}^\varepsilon|\varepsilon) \\ &= \int_{\underline{p}^\varepsilon}^{\check{p}^\varepsilon} [w_{PP}(\check{p}^\varepsilon, \varepsilon) - w_{PA}(q, \check{p}^\varepsilon, \varepsilon)]d\ell(q|\varepsilon) \\ &= \{\mathbb{E}[w|\check{p}, P, \varepsilon] - \mathbb{E}[w|\check{p}, A, \varepsilon]\}\, \ell(\check{p}|\varepsilon) \end{aligned}$$

Separating Nash equilibrium for appropriate $\{c^\varepsilon\}$ and $\{\check{p}^\varepsilon\}$

The proposed separating equilibrium is a Nash equilibrium of the labor market if each firm prefers the prescribed wage contract and wage schedule conditional on all other firms playing the assigned contract and wage schedule and labor flowing toward more productive firms. In order to prove this I must show that current operating surplus from the proposed strategies exceed current operating surplus from each firms best deviation.

Suppose WP is prescribed: A firm for which WP is prescribed must have $p < \check{p}^\varepsilon$. For the p-productivity firm, current operating surplus from playing optimal wage under the prescribed wage contract, WP, and the best deviation to SA can be written as

$$\pi^P(p|\varepsilon) = [p - w_{PP}(p, \varepsilon)]\ell(p|\varepsilon)$$

and

$$\pi^{BD}(p|\varepsilon) = \int_{\underline{p}}^{\check{p}} [p - w_{PA}(q, p, \varepsilon)]d\ell(q|\varepsilon) - c^\varepsilon$$

where $\grave{p}$ is the productivity of the most productive firm which offers a posted wage less than p (e.g. the most productive firm which the p-type firm can outbid by switching to SA). Simplifying and suppressing worker skill type for notational convenience

$$\begin{aligned}\pi^{BD}(p) =& \int_{\underline{p}}^{\grave{p}} [p - w_{PP}(\check{p}) + \underbrace{w_{PA}(q,\check{p}) - w_{PA}(q,p)}_{<0, \text{ since } \frac{dw_{PA}(q,p)}{dp}<0}] d\ell(q) - \int_{p}^{\check{p}} [\underbrace{w_{PP}(\check{p}) - w_{PA}(q,\check{p})}_{\geq 0}] d\ell(q) \\ <& [p - w_{PP}(\check{p})]\ell(\grave{p}) \\ \leq& \pi^{P}(p).\end{aligned}$$

The last line follows from noting $w_{PP}(p,\varepsilon)$ was the unique profit maximizing posted wage choice for the p-type firm when hiring skill level ε workers.

In other words, the WP firm could increase its labor supply by deviating to SA. However, the firm could also increase its labor supply by the same amount by deviating to a larger posted wage. Willingness to pay for the right to SA is then strictly less than the difference between the wage bill under the deviation to SA and the deviation to a higher posted wage. This in turn is strictly less than the cost of SA. Figure 7 depicts wages schedules under WP and SA in the threshold firm (left) and for a less productive firm (right). The cost of SA and bound on the willingness to pay for the right to SA are represented by the shaded regions. The cost or willingness to pay are calculated as the mass in these regions weighted by the supply of labor to the firm with each possible best-to-date outside option.

Suppose SA is prescribed: A firm for which SA is prescribed must have $\check{p}^{\varepsilon} \leq p$. For the p-productivity firm, current operating surplus from playing the prescribed SA wage schedule

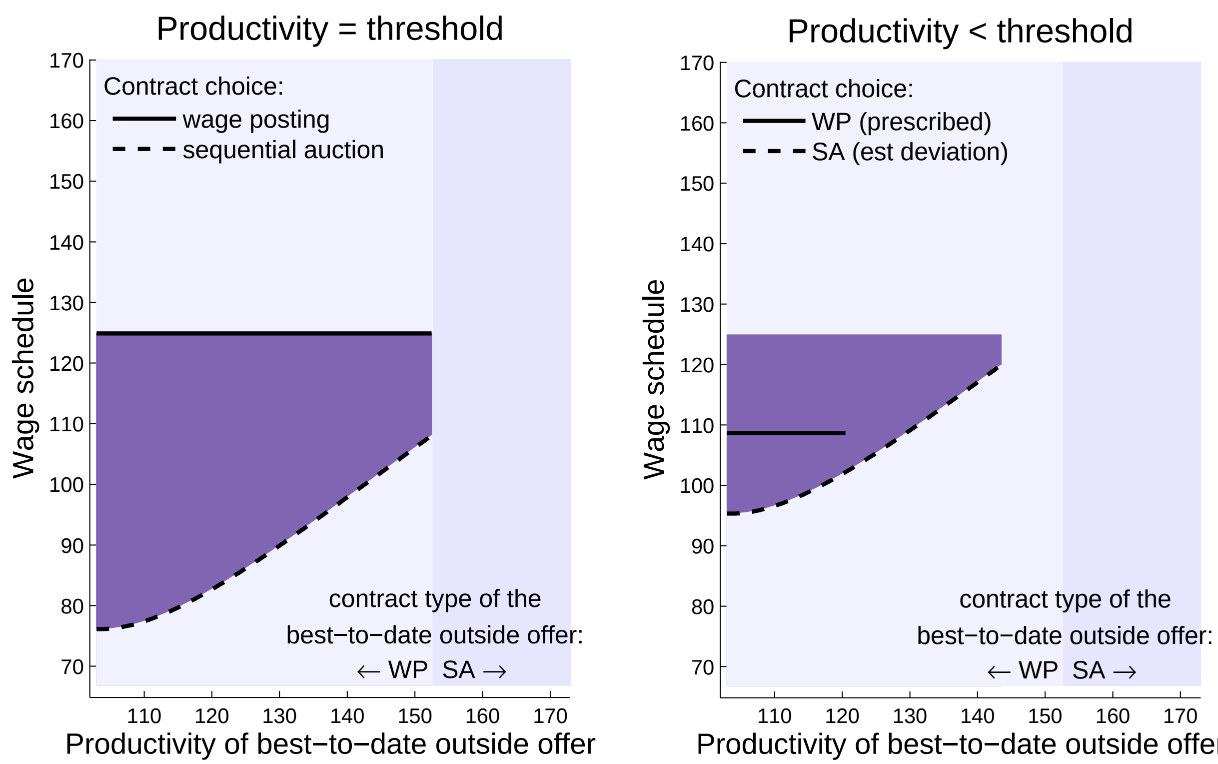

Figure 6: Regions over which the cost of SA and bound on the willingness to pay to SA are integrated for the threshold productivity firm and a firm of lesser productivity.

and deviating to the best posted wage.

$$\pi^A(p|\varepsilon) = \int_{\underline{p}}^{\check{p}^\varepsilon} [p - w_{PA}(q,p,\varepsilon)] d\ell(q|\varepsilon) + \int_{\check{p}}^{p} [p - w_{AA}(q,p,\varepsilon)] d\ell(q|\varepsilon) - c^\varepsilon$$

and

$$\pi^{BD}(p) = [p - \grave{w}]\ell(\grave{w}|\varepsilon)$$

Note that $\grave{w} \geq w_{PP}(\check{p}^{\varepsilon})$ since $p \geq \check{p}$. Simplifying and suppressing skill level,

$$\begin{aligned}
\pi^{BD}(p) &= \int_{\underline{p}}^{\check{p}} [p - \grave{w}] d\ell(q) + \int_{\check{p}}^{\grave{w}} [p - \grave{w}] d\ell(q) \\
&< \int_{\underline{p}}^{\check{p}} \underbrace{[p - w_{PP}(\check{p})]}_{<(p - w_{PA}(q,p) - c), \text{ since } \frac{dw_{PA}(q,p)}{dp} < 0} d\ell(q) + \int_{\check{p}}^{\grave{w}} \underbrace{[p - \grave{w}]}_{<p - w_{AA}(q,p)} d\ell(q) \\
&< \pi^{A}(p).
\end{aligned}$$

The best deviation to WP involves a reduction in the SA firm's labor supply. I can find a bound on the minimum willingness to pay for the right to SA by considering only the labor supply which would arise under the *smallest possible* best deviation the SA firm might select: $w_{PP}(\check{p}, \varepsilon)$. Willingness to pay for the right to SA is then larger less than the difference between the wage bill under the deviation to WP and the wage bill for these employees under the prescribed SA contract. This in turn is strictly greater than the cost of SA. Figure **??** depicts wages schedules under WP and SA in the threshold firm (left) and for a more productive firm (right). The cost of SA and bound on the willingness to pay for the right to SA are represented by the shaded regions. The cost or willingness to pay are calculated as the mass in these regions weighted by the supply of labor to the firm with each possible best-to-date outside option.

Since no firm wishes to unilaterally deviate the pair $\{c^{\varepsilon}, \check{p}^{\varepsilon}\}$ form a Nash equilibrium. ■

Existence of $\check{p}$ for any c.

Again, suppressing skill level:

Since $\check{p}$ was chosen arbitrarily c is defined for any possible threshold in the support of Γ.

First consider $\Gamma(p)$ with finite support $[\underline{p}, \bar{p}]$. To show that for every cost, c, there exists a threshold, $\check{p}$, I must first extend the definition of c to include the boundaries of the support

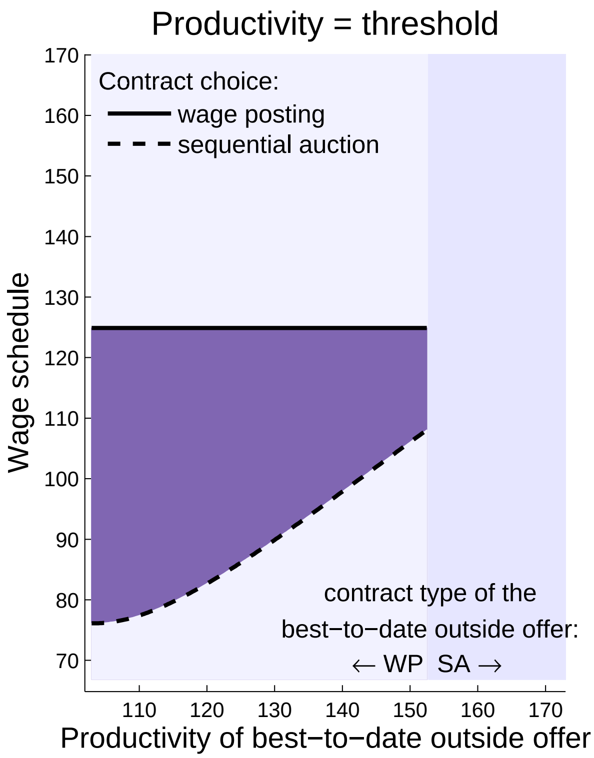

Figure 7: Regions over which the cost of SA and bound on the willingness to pay to SA are integrated for the threshold productivity firm and a firm of lesser productivity.

of $\Gamma(p)$.

- $c(\underline{p}) = (-\infty, 0]$ (when SA is subsidized or free all firms select SA).
- $c(\bar{p}) \supset [\ell(\bar{p})\bar{p}, \infty)$ (if the cost of SA exceeds the output of the most productive firm, $\ell(\bar{p})\bar{p}$, then no firm selects SA).

So, c is upper hemicontinuous on support $[\underline{p}, \bar{p}]$ and continuous on support $(\underline{p}, \bar{p})$. The intermediate value theorem implies that there exists at least one threshold for every cost.

The result can be generalized to Γ with infinite upper support by considering the limit as $\bar{p} \to \infty$: for every $\bar{p}$ there exists a $c = \ell(\bar{p})\bar{p}$ such that all firms SA. ■

Uniqueness of equilibria

Equilibrium is unique if $[(w_{PP}(\check{p})-\underline{p})k_1]^{-1} \geq d\Gamma(\check{p})$ for all $\check{p}$. This condition requires that the distribution of productivity be "thin enough" everywhere in the tail that the shift $\frac{dw_{PA}(q,\check{p})}{d\check{p}}$ due to indirect upward pressure on schedules $w_{PA}(q,p)$ from the now larger WP sector is dominated by the direct downward pressure on the schedule in the marginal firm due to the now larger productivity of the marginal firm. Proof, which stems from differentiating the marginal wage schedule, is available upon request.

This guarantees that $\frac{dc}{d\check{p}}$ is increasing for all $\check{p}$ in the interior of the support of $\Gamma(p)$ and the mapping from $\check{p}$ to c is one-to-one.

Proof of Claim 7

The claim is proved if every firms wage bill weakly rises. Consider a small increase in the costs of SA. Firms are of three types: always WP, switch from SA to WP, always SA. Wage bills for always WP firms are clearly unaffected by the change in threshold productivity induced by the change in cost. This follows from noting that I am considering an identical set of active firms. Switching firms strictly increase their wage bill, this follows from noting that before the increase in costs they paid for the right to SA but after they prefer to pay larger WP wage bills in order to evade the higher cost of SA. The third category of firms requires heavier lifting.

First note that the wages of workers with best-to-date outside option a WP are set under schedule $w_{PA}(q,p)$ in each p-productivity SA firm and that the mass of such workers in each firm is $\ell(\check{p})$. Also note that $\frac{d^2 w_{PA}(q,p)}{dq d\check{p}} < 0$.[10] So the change in the wage bill associated with these workers when $\check{p}$ rises is at least

$$\frac{dw_{PA}(\check{p},p)}{d\check{p}} = [1 + k_1\bar{\Gamma}(\check{p})]\frac{dw_{PP}(\check{p})}{d\check{p}} + k_1 d\Gamma(\check{p})[\check{p} - w_{PP}(\check{p})] > 0.$$

[10]Proof, which follows from performing the derivatives, is available on request.

Meanwhile, each p-productivity firm can lower the wage paid to employees whose best-to-date outside option was a SA firm before the cost increase and is now a WP firm. This reduces the p-productivity firms wage bill by $d\ell(\check{p})[\check{p} - w_{PP}(\check{p})]$.

The increase in the schedule of wages for workers with best-to-date outside option a WP firm dominates. This is made explicit by noting that $\frac{dw_{PP}(\check{p})}{d\check{p}} = \frac{2k_1 d\Gamma(\check{p})}{1+k_1\Gamma(\check{p})}[\check{p} - w_{PP}(\check{p})]$ and $d\ell(\check{p}) = \frac{2k_1 d\Gamma(\check{p})}{1+k_1\bar{\Gamma}(\check{p})}\ell(\check{p})$.

Since the wage bill for every productivity firm weakly rises the total wage bill rises. ■

Proof of Claims 4-10 when $\lambda_0 > \lambda_1$

The mass of firms posting acceptable job offers is $N^\varepsilon = \bar{\Gamma}(\underline{p}^\varepsilon)N^1$. The result is that the arrival rate of *acceptable* job offers are diminished proportionally to the fraction of firms which are priced out of the market: $\lambda_0^\varepsilon = \bar{\Gamma}(\underline{p}^\varepsilon)\lambda_0^1$ and $\lambda_1^\varepsilon = \bar{\Gamma}(\underline{p}^\varepsilon)\lambda_1^1$ off- and on-the-job respectively.[11]

Decrease in the arrival of acceptable offers yields the result that unemployment rates are higher in low skilled sub-markets: mass balance implies that

$$u_\varepsilon = \frac{1}{1 + k_0\bar{\Gamma}(\underline{p}^\varepsilon)} = u_1 \frac{1 + k_0}{1 + k_0\bar{\Gamma}(\underline{p}^\varepsilon)} > u_1. \tag{C.9}$$

This gives the third result. Further, the distribution of employment across productivity in markets with $\varepsilon < 1$ is:

$$L(p|\varepsilon) = \frac{\Gamma(p) - \Gamma(\underline{p}^\varepsilon)}{\bar{\Gamma}(\underline{p}^\varepsilon)[1 + k_1\bar{\Gamma}(p)]} < L(p|1). \tag{C.10}$$

Equation C.10 gives a somewhat non-intuitive result which must be discussed. Low skilled workers spend additional time in unemployment because value of search is relatively higher. This results in low skilled workers obtaining starting jobs which are on average higher up

[11]NOTE: these are strong assumptions on the matching function. Deviations from these are left to future work.

the job ladder. Producing a first order stochastically dominant distribution of employment as described by equation C.10. However, due to additional unemployment the mass of low skilled workers employed in firms with productivity less than or equal to p exceeds the analogous mass for high-skilled workers: $(1-u^{\varepsilon})L(p|\varepsilon) > (1-u^{1})L(p|1)$.

And finally the mass of workers employed in a firm of type p:

$$\begin{aligned}\ell(p|\varepsilon) &= \frac{1+k_1\bar{\Gamma}(\underline{p}^{\varepsilon})}{[1+k_1\bar{\Gamma}(p)]^2}\frac{M-U^{\varepsilon}}{N^{\varepsilon}} \\ &= \frac{1+k_1}{[1+k_1\bar{\Gamma}(p)]^2}\frac{M-U}{N}\left[\left(\frac{1+k_0}{1+k_1}\right)\left(\frac{1+k_1\bar{\Gamma}(\underline{p}^{\varepsilon})}{1+k_0\bar{\Gamma}(\underline{p}^{\varepsilon})}\right)\right] > \ell(p|1). \qquad \text{(C.11)}\end{aligned}$$

These give the forth result. Intuitively this arises from the larger pool of unemployed workers which each firm can hire from in the low-skilled sub-market.

Somewhat surprisingly, these differences in steady state distributions have a relatively benign impact on firm's wage choices. This can be seen clearly by applying the notion of acceptable job offer to equations 2.6, 2.9, 2.1, and 2.13. First note that, whenever the distribution of productivity occurs so does the hazard of a job offer. The result is that the impact of some firms being priced out of the market is offset. Furniture, shifts in labor supply do not depend on firm productivity (for active firms) resulting in a similar wage choice for posting firms:

$$\underline{w}_{UP}(\varepsilon) = \varepsilon b + (k_0 - k_1)\int_{\underline{w}_{UP}(\varepsilon)}^{\check{p}^{\varepsilon}} \frac{\Gamma(\check{p}^{\varepsilon}) - \Gamma(x)}{1 + k_1[\Gamma(\check{p}^{\varepsilon}) - \Gamma(x)]} dw_{PP}(x,\varepsilon)$$

$$w_{PP}(p,\varepsilon) = \varepsilon\left\{p - [1+k_1\bar{\Gamma}(p)]^2\int_{w_{UP}(\varepsilon)}^{p}[1+k_1\bar{\Gamma}(x)]^{-2}dx\right\}$$

$$
\begin{aligned}
w_{PC}(q,p,\varepsilon) = w_{PP}(q,\varepsilon) + \lambda_1^{\varepsilon}\big\{\bar{\Gamma}(\check{p}^{\varepsilon})V^{P}(w_{PP}(q,\varepsilon),\varepsilon) \\
- [\Gamma(p) - \Gamma(\check{p}^{\varepsilon})]\mathbb{E}[V^{C}(\varepsilon x, x, \varepsilon)|\check{p}^{\varepsilon} < x < p] \\
- \bar{\Gamma}(p)V^{C}(\varepsilon p, p, \varepsilon)\big\}.
\end{aligned}
$$

and

$$
w_{CC} = \varepsilon\left\{q - k_1 \int_q^p \bar{\Gamma}(x)dx\right\}
$$

The new result is that reservation wages for employment in a wage posting firm rise since the value of unemployment rises more than the value of employment in the minimum posting firm due to the difference in offer arrival rates across states. In addition, the effect "trickles up" the posted wage distribution (to see this inspect equation 2.9). The result overall is that differences in wage growth for different skill types are amplified compared to the case of equal job-offer hazards. Now low skilled workers earn disproportionately higher wages in all entry-level positions in WP firms and in entry positions at SA firms.