R12.2 Trickle Bed Reactors

In a trickle bed reactor the gas and liquid flow (trickle) cocurrently downward over a packed bed of catalyst particles. Industrial trickle beds are typically 3 to 6 m deep and up to 3 m in diameter and are filled with catalyst particles ranging from $\frac{1}{2}$ to $\frac{1}{32}$ in. in diameter. The pores of the catalyst are filled with liquid. In petroleum refining, pressures of 34 to 100 atm and temperatures of 350 to 425°C are not
uncommon. A pilot-plant trickle bed reactor might be about 1 m deep and 4 cm in
diameter. Trickle beds are used in such processes as the hydrodesulfurization of
heavy oil stocks, the hydrotreating of lubricating oils, and reactions such as the
production of butynediol from acetylene and aqueous formaldehyde over a copper
acetylde catalyst. It is on this latter type of reaction, i.e.,

$$A(g, l) + B(l) \rightarrow C(l)$$ (12-24)

that we focus in this section. In a few cases, such as Fischer–Tropsch synthesis, the
liquid is inert and acts as a heat-transfer medium.

### R12.2A Fundamentals

The basic reaction and transport steps in trickle bed reactors are similar to
slurry reactors. The main differences are the correlations used to determine the
mass-transfer coefficients. In addition, if there is more than one component in the
gas phase (e.g., liquid has a high vapor pressure or one of the entering gases is
inert), there is one additional transport step in the gas phase. Figure 12-7 shows the
various transport steps in trickle bed reactors. Following our analysis for slurry
reactors we develop the equations for the rate of transport of each step.

1. **Transport from the bulk gas phase to the gas-liquid interface:** The
   rate of transport per mass of catalyst is
   \[ -r'_A = k_g a_i \frac{1}{(1 - \epsilon_b) \rho_p} [C_{A}(g) - C_{A_i}(g)] \] (12-25)
   where
   - \( a_i \) = interfacial area per volume of bed, \( m^2/m^3 \)
   - \( k_g \) = gas-phase mass-transfer coefficient, \( m/s \)
   - \( \rho_p \) = density of catalyst pellet, \( kg/m^3 \)
   - \( 1 - \epsilon_b \) = volume of solids/volume of bed (voids + solids)
   - \( C_{A}(g) \) = bulk gas-phase concentration of A, \( kmol/m^3 \)
   - \( C_{A_i}(g) \) = concentration of A at interface, \( kmol/m^3 \)

2. **Equilibrium at gas-liquid interface:**
   \[ C_{A_i} = \frac{C_{A_i}(g)}{H} \] (12-26)
   where \( C_{A_i} \) = concentration of A in liquid at the interface
   \( H = \) Henry's constant

3. **Transport from interface to bulk liquid:**
   \[ -r'_A = k_l a_i \frac{1}{(1 - \epsilon_b) \rho_p} (C_{A_i} - C_{A_{ib}}), \] (12-27)
   where
   - \( k_l \) = liquid-phase mass-transfer coefficient, \( m/s \)
   - \( C_{A_i} \) = concentration of A in liquid at interface
   - \( C_{A_{ib}} \) = bulk liquid concentration of A
Transport from bulk gas to gas-liquid interface to bulk liquid to solid-liquid interface

Diffusion and reaction in catalyst pellet

Figure 12-7 (a) Trickle bed reactor; (b) reactant concentration profile.

4. Transport from bulk liquid to external catalyst surface:

\[-r'_A = k_ea_c(C_{A_b} - C_{A_s}), \quad \text{mol/g cat}\cdot\text{s} \]  \hspace{1cm} (12-28)

where \( a_c \) [equation (12-18)] and \( C_{A_b} \) are defined as before

\( k_e \) = liquid-solid mass-transfer coefficient, m/s

\( C_{A_s} \) = concentration of A at solid-liquid interface

5. Diffusion and reaction in the pellet: If we assume first-order reaction in dissolved gas A and in liquid B, we have

\[-r'_A = \eta k C_{A_s} C_{B_s} \quad \text{mol/g cat}\cdot\text{s} \]  \hspace{1cm} (12-29)
where $\eta$ and $C_{A_i}$ are defined as before.

$k =$ specific reaction rate, m$^6$ of liquid/mol·g cat·s

Combining equations (12-25) through (12-29) and rearranging in an identical manner to that leading to the development of equation (12-8) for slurry reactors, we have

\[
-r'_A = \frac{1}{H} \left( \frac{1 - \epsilon_b}{k_e a_i} + \frac{1 - \epsilon_b}{k_r a_c} + \frac{1}{k_c} \frac{1}{k_{ug}} \frac{1}{\eta k C_{B_s}} \right) C_{A(g)} \text{ mol g cat} \cdot \text{s}^{-1}
\]

i.e.,

\[
-r'_A = k_{ug} C_{A(g)}
\]

where $k_{ug}$ is the overall transfer coefficient for the gas into the pellet (m$^3$ of gas/g cat·s). A mole balance on species A gives

\[
\frac{dF_A}{dW} = r'_A = -k_{ug} C_{A(g)}
\]

We next consider the transport and reaction of species B, which does not leave the liquid phase.

6. **Transport of B from bulk liquid to solid catalyst interface:**

\[
-r'_B = k_c a_c (C_B - C_{B_s}) \text{ mol g cat} \cdot \text{s}^{-1}
\]

where $C_B$ and $C_{B_s}$ are the concentrations of B in the bulk fluid and at the solid interface, respectively.

7. **Diffusion and reaction of B inside the catalyst pellet:**

\[
-r'_B = \eta k C_{A_i} C_{B_s}
\]

Combining equations (12-33) and (12-34) and rearranging, we have

\[
-r'_B = \frac{1}{k_c a_c + 1/\eta k C_{A_i}} \frac{1}{k_{us}} C_B \text{ mol g cat} \cdot \text{s}^{-1}
\]

\[
-r'_B = k_{us} C_B
\]
A mole balance on species B gives

\[
\frac{dF_B}{dW} = \nu_l \frac{dC_B}{dW} = \frac{d}{dW} = -k_{vl} C_B
\]  

(12-37)

One notes that the surface concentrations of A and B, \(C_{A_s}\) and \(C_{B_s}\), appear in the denominator of the overall transport coefficients \(k_{vg}\) and \(k_{vl}\). Consequently, equations (12-30), (12-32), (12-35), and (12-37) must be solved simultaneously. In some cases analytical solutions are available, but for complex rate laws, one resorts to numerical solutions.\(^1\) However, we shall consider some limiting situations.

### R12.2.2 Limiting situations

**Mass Transfer of the Gaseous Reactant Limiting** For this situation we assume that either the first three terms in the denominator of equation (12-30) are dominant, or that the liquid-phase concentration of species B does not vary significantly through the trickle bed. For these conditions \(k_{vg}\) is constant, and we can integrate the mole balance. For negligible volume change \(\varepsilon = 0\), then

\[
W = \frac{\nu_g \ln C_{A_{in}}}{k_{vg}} = \frac{\nu_g \ln \frac{1}{1 - X_A}}{k_{lg}}
\]

(12-38)

**Mass Transfer and Reaction of Liquid Species Limiting** Here we assume that the liquid phase is entirely saturated with gas throughout the column. As a result, \(C_{A_s}\) is a constant and therefore so is \(k_{vl}\). Consequently, we can integrate the combined mole balance and rate law to give

\[
W = \frac{\nu_l \ln C_{B_{in}}}{k_{vl}} = \frac{\nu_l \ln \frac{1}{1 - X_B}}{k_{vl}}
\]

(12-39)

### 12.2.3 Evaluating the transport coefficients

The mass-transfer coefficients, \(k_g\), \(k_l\), and \(k_c\) depend on a number of variables, such as type of packing, flow rates, wetting of particle, and geometry of the column, and as a result the correlations vary significantly from system to system. Consequently, we will not give all the correlations here but instead will give correlations for particular systems and refer the reader to four specific references

\(^1\)A number of worked example problems for three-phase reactors can be found in the article "Predicting performance of three phase catalytic reactors, by P. A. Ramachandran and R. V. Chaudhari, Chem. Eng., 87(24), 74 (1980).
where other correlations for trickle bed reactors may be found. Typical correlations are given in Table 12-3. Note that the correlation for organic particles tends to underpredict the transport coefficient.

The representative correlations given in Table 12-3 assume complete wetting of the catalyst particles. Corrections for incomplete wetting as well as flow regimes, pressure-drop equations, and other mass-transfer correlations can be found in the reviews by Shah, by Smith, and by Satterfield.\(^1\)

The plug-flow design equation may be applied successfully provided the ratio of reactor length \(L\) to particle diameter \(d_p\) satisfies the criteria (Satterfield, 1975)

\[
\frac{L}{d_p} > \frac{20}{\text{Pe}} n \ln \frac{1}{1 - X}
\] (12-40)

where \(\text{Pe} = \text{Péclet number} = d_p U / D_{Ax}\)

\(D_{Ax}\) = axial dispersion coefficient
\(n\) = reaction order

The CSTR design equations apply to the trickle bed when\(^3\)

\[
\frac{L}{d_p} < \frac{4}{\text{Pe}}
\] (12-41)

Techniques for determining the single-phase axial dispersion coefficient are given in Chapter 14.

**Example 12-3  Trickle Bed Reactor**

The hydrogenation of an unsaturated organic is to be carried out in a trickle bed reactor packed with 0.20-cm-diameter spherical catalyst particles.

\[
\text{H}_2 + \text{unsaturated} \rightarrow \text{saturated}
\]

The reaction in the pellet is first order in both hydrogen and the organic. Hydrogen and nitrogen are fed in equimolar portions at a total pressure of 20 atm and total molar rate of 10 mol/s. The reactor diameter is to be 1.0 m. The superficial liquid mass velocity is 5.0 kg/m\(^2\)·s. The corresponding pressure gradient through the bed is 25 kPa/m. As a first approximation, assume that the concentration of organic is constant and the pseudo-first-order specific reaction rate is \(3 \times 10^{-5} \text{ m}^3/\text{kg cat} \cdot \text{s}\) at 400 K.

(a) For each transport step, determine its fraction of the total resistance to mass transport and reaction.


\(^2\)Ibid.

\(^3\)P. A. Ramachandran and R. V. Chaudhari *Chem. Eng.*, 87(24), 74 (1980).
### Table 12.3. Sample Mass-Transfer Correlations for Trickle Beds

<table>
<thead>
<tr>
<th>Transport Step</th>
<th>Correlation</th>
<th>Typical Values</th>
<th>Reference</th>
<th>Eq. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas-to-gas interface</td>
<td>$k_g a_i = 2 + 0.91 E_g^{2/3}$</td>
<td>$k_g a_i = 7.4 \text{ s}^{-1}$</td>
<td>IEC Proc. Des. Dev., 6, 486 (1967)</td>
<td>(12-40)</td>
</tr>
<tr>
<td></td>
<td>$E_g = \frac{(\Delta P)}{U_g}$</td>
<td>$\Delta P = 10^{-2} \text{ psi/ft}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liquid interface to bulk liquid</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aqueous</td>
<td>$k_l a_i = 8.08 \left( \frac{G_l}{\mu} \right)^{0.41} \text{ Sc}^{1/2}$</td>
<td>$k_l a_i = 0.01 \text{ s}^{-1}$</td>
<td>Chem. Eng. Sci., 34, 1425 (1979)</td>
<td>(12-42)</td>
</tr>
<tr>
<td>Organic&lt;sup&gt;b&lt;/sup&gt;</td>
<td>$k_l a_i = 16.8 \frac{\text{Re}^{1/8}}{G_l^{0.22} \text{Sc}^{1/2} \text{ cm}^{-2}}$</td>
<td></td>
<td>Chem. Eng. Sci., 36, 569 (1981)</td>
<td>(12-43)</td>
</tr>
<tr>
<td>Bulk liquid-to-solid interface</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For Re &lt; 60</td>
<td>$\text{Sh}' = 0.815 \text{Re}^{0.82} \text{Sc}^{1/3}$</td>
<td>$k_l a_i \sim 0.2 \text{ s}^{-1}$</td>
<td>AIChE J., 24, 709 (1978)</td>
<td>(12-44)</td>
</tr>
<tr>
<td>For Re &lt; 20</td>
<td>$\text{Sh}' = 0.266 \text{Re}^{1/3} \text{Sc}^{1/3}$</td>
<td>for Re = 50 and $d_p = 0.5 \text{ cm}$</td>
<td></td>
<td>(12-45)</td>
</tr>
</tbody>
</table>

\[ Ga = \text{Galileo number} = \frac{d_p^2 \rho_l g}{\mu_l^2} \quad \text{Re}_l = \frac{G_l d_p}{\mu_l} \]

\[
g = 9.8 \text{ m/s}^2 \\
G_l = \text{superficial mass velocity of liquid, g/cm}^2 \cdot \text{s} \\
U_l = \text{superficial velocity of liquid } \langle v_l \rangle, \text{ cm/s} \\
\text{Sh}' = \frac{k_c d_p \alpha}{D_l}, \quad \alpha = \text{fraction of external surface that is wetted} \\
\]

\[
a_r = \frac{6}{d_p}, \quad a_c = \frac{6}{\rho_p d_p}, \quad \alpha = \frac{6(1 - \epsilon_b)}{d_p}, \quad \epsilon_b = \text{bed porosity} \\
\]

<sup>a</sup>Also see N. Midoix, B. I. Morsi, M. Purwasasmita, A. Laurent, and J. C. Charpentier, Chem. Eng. Sci. 39, 781 (1984), for a comprehensive list of correlations.<br>
<sup>b</sup>In some cases this gives a low estimate of $k_l a_i$; see M. Herskovitz and J. M. Smith, AIChE J., 29, 1 (1983); F. Turek and R. Lange, Chem. Eng. Sci., 36, 569 (1981).

(b) Calculate the catalyst weight necessary to achieve 55% conversion of the hydrogen.

**Additional data:**

- Liquid viscosity: $1.8 \text{ C}_{p} = 0.0018 \text{ kg/m} \cdot \text{s}$
- Liquid density: $840 \text{ kg/m}^3$
- Hydrogen liquid diffusivity in oil: $2.4 \times 10^{-9} \text{ m}^2/\text{s}$
- Organic diffusivity in organic product: $1.2 \times 10^{-9} \text{ m}^2/\text{s}$
Molecular weight of organic: 168 daltons
Hydrogen solubility: 0.008 kmol/m³·atm
Pellet porosity: 0.3
Pellet density: 1500 kg/m³
Bed porosity: 0.4

Assume that tortuosity = 1.5 and constriction = 0.8.

Solution
(a) Let A = H₂, B = unsaturated organic, and C = saturated organic:

\[ \text{A(g)} + \text{B(l)} \rightarrow \text{C(l)} \]

1. **Mole balance on H₂ (A):**

\[ \frac{dF_A}{dW} = r'_A \quad \text{(E12-3.1)} \]

2. **Rate law:** Assuming a constant liquid reactant concentration for low conversion of B,

\[ -r'_A = k_{og} C_A(g) \quad \text{(E12-3.2)} \]

with

\[ k_{og} = \frac{1}{H} \left( \frac{(1 - \epsilon_b) \rho_p}{H k_g a_i} + \frac{(1 - \epsilon_b) \rho_p}{k_i a_i} + \frac{1}{k_c a_c} + \frac{1}{\eta k} \right) \quad \text{(E12-3.3)} \]

\[ \epsilon_b = \text{bed porosity} = 0.4 \]

3. **Stoichiometry:** The isothermal gas-phase concentration is

\[ C_A = C_{A0} \left( \frac{1 - X}{1 + \epsilon X} \right) \frac{P}{P_0} \quad \text{(E12-3.4)} \]

\[ \epsilon = y_{A0} \delta = (0.5)(0 - 0 - 1) = -0.5 \]

4. **Pressure drop:**

\[ \frac{dP}{dL} = -\alpha = -25 \text{ kPa/m} = -0.246 \text{ atm/m} \]

\[ \frac{d(P/P_0)}{dW} = -\frac{\alpha}{P_0 A_c (1 - \epsilon_b) \rho_p} = -\beta \quad \text{(E12-3.5)} \]

\[ \frac{P}{P_0} = 1 - \beta W \quad \text{(E12-3.6)} \]

5. **Combining yields**

\[ F_{A0} \frac{dX}{dW} = k_{og} C_{A0} \left( \frac{1 - X}{1 + \epsilon X} \right) (1 - \beta W) \]
Integrating gives us

\[(1 + \varepsilon) \ln \frac{1}{1 - X} - \varepsilon X = \frac{k_{2e}}{u_0} \left( W - \frac{\beta}{2} W^2 \right) \]  \hspace{1cm} (E12-3.7)

6. Evaluating the parameters:

A. Solubility

\[ H' = 0.008 \text{ kmol/m}^3\text{·atm} \]

\[ P_{Ai}H' = C_{Ai} \quad \frac{C_{Ai}(g)}{H} = C_{Ai} \]

\[ H = \frac{1}{RTH'} = \frac{1}{\left( \frac{0.082 \text{ m}^3\text{·atm}}{\text{kmol·K}} \right) \left( 400 \text{ K} \right) \left( \frac{0.008 \text{ kmol}}{\text{m}^3\text{·atm}} \right)} = 3.81 \]  \hspace{1cm} (E12-3.8)

B. Internal diffusion and reaction

\[ \text{resistance} = R_R = \frac{1}{\eta k} \]  \hspace{1cm} (E12-3.9)

\[ \phi = R \sqrt{\frac{k_\rho_p}{D_e}} \]  \hspace{1cm} (E12-3.10)

\[ D_e = D_L \frac{\varepsilon \rho_e}{\tau} = 2.4 \times 10^{-9} \text{ m}^2/\text{s} \times \frac{(0.8)(0.3)}{1.5} = 3.84 \times 10^{-10} \text{ m}^2/\text{s} \]  \hspace{1cm} (11-1)

\[ \phi = 10^{-3} m \sqrt{\frac{(3 \times 10^{-5} \text{ m}^3/\text{kg} \cdot \text{s})(1500 \text{ kg/m}^3)}{3.84 \times 10^{-10} \text{ m}^2/\text{s}}} = 10.8 \]

For large values of the Thiele modulus,

\[ \eta = \frac{3}{\phi} = \frac{3}{10.8} = 0.277 \]

\[ R_R = \frac{1}{(0.277)(3 \times 10^{-5} \text{ m}^3/\text{kg} \cdot \text{s})} = 1.2 \times 10^5 \frac{\text{kg} \cdot \text{s}}{\text{m}^3} \]  \hspace{1cm} (E12-3.11)

C. Gas absorption

\[ \text{resistance} = R_g = \frac{(1 - \epsilon_b)\rho_g}{H_k \mu a_i} \]  \hspace{1cm} (E12-3.12)

\[ k_m a_i = 2 + 0.91 E_g^{2/3} \]

\[ E_g = \frac{\Delta P}{\Delta L} U_g \]
\[ v_0 = 10 \text{ mol/s} \times 0.0224 \text{ m}^3/\text{mol} \times \frac{1 \text{ atm}}{20 \text{ atm}} \times \frac{400 \text{ K}}{273 \text{ K}} = 0.0164 \text{ m}^3/\text{s} \]

\[ A_c = \frac{\pi D^2}{4} = 0.785 \text{ m}^2 \]

\[ U_e = \frac{v_0}{A_c} = \frac{0.0164 \text{ m}^3/\text{s}}{0.785 \text{ m}^2} = 0.021 \text{ m/s} \]

\[ E_s = 25 \text{ kPa/m} \left( \frac{0.021 \text{ m/s}}{0.52 \text{ kPa/s}} \right) = 10.9 \text{ ft} \cdot \text{lb}_t/\text{ft}^2 \cdot \text{s} \]

\[ k_s a_i = 2 + 0.91(10.9)^{2/3} = 6.47 \text{ s}^{-1} \]

\[ R_g = \frac{(1 - 0.4)(1500 \text{ kg/m}^3)}{(3.8)(6.47 \text{ s}^{-1})} = 36.5 \frac{\text{kg} \cdot \text{s}}{\text{m}^3} \]

D. Transport from gas-liquid interface to bulk liquid

\[ R_i = \frac{(1 - \epsilon_b) \rho_f}{k_s a_i} \quad \text{(E12-3.13)} \]

\[ \text{Re}_i = \frac{G d_p}{\mu_i} = \frac{(5 \text{ kg/m}^2 \cdot \text{s})(2 \times 10^{-3} \text{ m})}{0.0018 \text{ kg/m} \cdot \text{s}} = 5.55 \]

\[ \text{Ga} = \frac{d_p^3 \rho_f^2 g}{\mu_i^2} = \frac{(2 \times 10^{-3} \text{ m})^3(840 \text{ kg/m}^3)^2(9.8 \text{ m/s}^2)}{(0.0018 \text{ kg/m} \cdot \text{s})^2} = 17,073 \]

\[ \text{Sc} = \frac{\mu_i}{\rho_f D_L} = \frac{0.0018 \text{ kg/m} \cdot \text{s}}{(840 \text{ kg/m}^3)(2.4 \times 10^{-3} \text{ m}^2/\text{s})} = 893 \]

From the correlation for organic liquids,

\[ k_s a_i = D_L \left( 16.8 \frac{\text{Re}_i^{14} \text{Sc}^{1/2}}{\text{Ga}^{0.22}} \right) \text{ cm}^{-2} \quad \text{(E12-3.14)} \]

It has been noted\(^{\dagger}\) that this correlation gives a mass-transfer coefficient that is too low.

\[ k_s a_i = 2.4 \times 10^{-5} \text{ cm}^2/\text{s} \left[ \frac{16.8(5.55)^{14}(893)^{1/2}}{(17,073)^{0.22}} \right] \text{ cm}^{-2} \]

\[ = 2.17 \times 10^{-3} \text{ s}^{-1} \]

\[ R_i = \frac{(1 - 0.4)(1500 \text{ kg/m}^3)}{2.17 \times 10^{-3} \text{ s}^{-1}} = 4.15 \times 10^5 \frac{\text{kg} \cdot \text{s}}{\text{m}^3} \]

E. Resistance from bulk liquid to catalyst

\[
\text{resistance} = R_c = \frac{1}{k_c a_c} \quad \text{(E12-3.15)}
\]

\[
a_c = \frac{6}{\rho \rho d_p} = \frac{6}{(1500 \text{ kg/m}^3)(2 \times 10^{-3} \text{ m})} = 2.0 \text{ m}^2/\text{kg}
\]

\[
\text{Sh} = 0.266 \text{Re}^{1.15} \text{Sc}^{0.13} = (0.266)(5.55)^{1.15}(893)^{0.13} = 18.4
\]

\[
k_c = \frac{\text{Sh} D_L}{d_p} = \frac{(18.4)(2.4 \times 10^{-9} \text{ m}^2/\text{s})}{2 \times 10^{-3} \text{ m}} = 2.2 \times 10^{-5} \text{ m/s}
\]

\[
R_c = \frac{1}{k_c a_c} = 0.23 \times 10^5 \text{ kg} \cdot \text{s/m}^3
\]

F. Total and percentage resistances

\[
R_T = R_c + R_i + R_g + R_R
\]

\[
= (0.23 \times 10^5 + 4.15 \times 10^2 + 36.5 + 1.2 \times 10^5) \text{ kg} \cdot \text{s/m}^3
\]

\[
R_T = 5.58 \times 10^5 \text{ kg} \cdot \text{s/m}^3
\]

Individual resistances:

Gas absorption = 100 \( \frac{36.5}{5.58 \times 10^5} \) = 0.01%

Bulk liquid to catalyst = 100 \( \times \frac{0.23}{5.58} \) = 4.12%

Internal diffusion and reaction = 100 \( \times \frac{1.2}{5.58} \) = 21.50%

Gas interface to bulk liquid = 100 \( \times \frac{4.15}{5.58} \) = 74.37%

Total = 100%

(b) Calculate catalyst weight

\[
(1 + \epsilon) \ln \frac{1}{1 - \epsilon} = \epsilon X = \frac{k_{eg}}{v_0} \left( W - \frac{\beta}{2} W^2 \right) \quad \text{(E12-3.7)}
\]

\[
k_{eg} = \frac{1/\rho}{R_T} = \frac{1/(3.8)}{5.58 \times 10^5} = 4.6 \times 10^{-7} \text{ m}^2/\text{kg} \cdot \text{s}
\]

\[
k_{eg} = \frac{4.6 \times 10^{-7} \text{ m}^2/\text{kg} \cdot \text{s}}{0.0164 \text{ m}^3/\text{s}} = 2.8 \times 10^{-5} \text{ kg}^{-1}
\]
\[
\beta = \frac{\alpha}{A_c \rho (1 - \epsilon_b)} = \frac{0.246 \text{ atm/m}}{(0.785 \text{ m}^2)(20 \text{ atm})(1500 \text{ kg/m}^3)(0.6)} = 1.75 \times 10^{-5} \text{ kg}
\]

Substitution yields

\[
(1 - 0.5) \ln \frac{1}{1 - 0.55} - (-0.5)(0.55)
\]

\[
= 2.8 \times 10^{-5} \left( \frac{W - 1.75 \times 10^{-5} W^2}{2} \right)
\]

Solving for \(W\), we obtain

\[W = 34,500 \text{ kg}\]

The reactor volume corresponding to this catalyst weight is

\[V = \frac{W}{\rho (1 - \epsilon_b)} = 38.33 \text{ m}^3\]

The total height of the reactor

\[H = \frac{V}{A_c} = 48.83 \text{ m}\]

Four 1-m-diameter towers each 12.2 m in height connected in series will be sufficient.

Checking assumption of constant \(C_B\)

moles of \(\text{H}_2\) converted = \(F_{\text{AD}}X = (0.5)(10 \text{ mol/s})(0.55)\)

\[= 2.75 \text{ mol/s}\]

moles of organic leaving = \(5 \frac{\text{kg}}{\text{m}^2 \cdot \text{s}} \times (0.785 \text{ m}^2) \left(\frac{1000 \text{ mol}}{168 \text{ kg}}\right)\)

\[-2.75 \text{ mol/s}\]

\[= 23.36 - 2.75 = 20.6 \text{ mol/s}\]

Consequently, our assumption that the concentration of organic is essentially constant was valid.

The material presented in this example is meant to serve as an introduction to trickle bed reactors. Other worked-out trickle bed example problems can be found in the article by Ramachandran and Chaudhari.\(^1\) In addition, the hydrodesulfurization of a hydrocarbon in a trickle bed reactor is given in detail by Tarhan.\(^2\)

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\(^1\) Ramachandran and Chaudhari, op. cit.