Web 6.5.1   Startup of a CSTR

The startup of a fixed-volume CSTR under isothermal conditions is rare, but it does occur occasionally. We can, however, carry out an analysis to estimate the time necessary to reach steady-state operation. For the case when the reactor is well mixed and, as a result, there are no spatial variations in r_A, we begin with the general mole balance equation applied to Figure 4-14(a):

$$F_{A0} - F_A + r_A V = \frac{dN_A}{dt} \tag{6-8}$$

Conversion does not have any meaning in startup because one cannot separate the moles reacted from the moles accumulated in the CSTR. Consequently, we *must* use concentration rather than conversion as our variable in the balance equation. For liquid-phase ($v = v_0$) reactions with constant overflow ($V = V_0$), using $\tau = V_0/v_0$, we can transform Equation (6-8) to

$$C_{A0} - C_A + r_A \tau = \tau \frac{dC_A}{dt} \tag{6-9}$$

First-order reaction For a first-order reaction ($-r_A = kC_A$), Equation (6-9) then becomes

$$\frac{dC_A}{dt} + \frac{1 + \tau k}{\tau} C_A = \frac{C_{A0}}{\tau}$$

which, for the initial conditions $C_A = 0$ at $t = 0$ solves to

$$C_A = \frac{C_{A0}}{1 + \tau k} \left[1 - \exp\left[-[1 + \tau k]\frac{t}{\tau} \right] \right] \tag{6-10}$$

letting t_s be the time necessary to reach 99% of the steady-state concentration, C_{AS}

$$C_{AS} = \frac{C_{A0}}{1 + \tau k} \tag{6-11}$$

Rearranging Equation (6-10) for $C_A = 0.99 C_{AS}$ yields

$$t_s = 4.6 \frac{\tau}{1 + \tau k} \tag{6-12}$$

For slow reactions with small k ($1 \gg \tau k$)

$$\boxed{t_s = 4.6 \ \tau} \tag{6-13}$$

For rapid reactions with large k ($\tau k \gg 1$), time to reach steady state in an isothermal CSTR is

$$\boxed{t_s = \frac{4.6}{k}} \tag{6-14}$$

For most first-order systems, steady state is achieved in three to four space times.