R11.5 Flow and Dissolution in Porous Media

The concepts in the shrinking core model can also be applied to situations in which there is growth rather than shrinking. One example of this growth is the dissolution of pores in oil-bearing reservoirs to increase the oil flow out of the reservoir (recall Example 5-3). To model and understand this process, laboratory experiments are carried out by injecting HCl through calcium carbonate (i.e., limestone) cores. The carbonate core, the pore network, and the individual pores are shown in Figure R11.5-1. As acid flows through and dissolves the carbonate pore space, the pore radius increases so that the resistance to flow decreases, resulting in more acid flowing into the pore. Because there is a distribution of pore sizes, some pores will receive more acid than others. This uneven distribution results in even a greater dissolution rate of the pores receiving the most acid. This "autocatalytic-like" growth rate results in the emergence of a dominant channel, called a *wormhole*, that will be formed in the porous media.

Dissolution of limestone pore space by acid

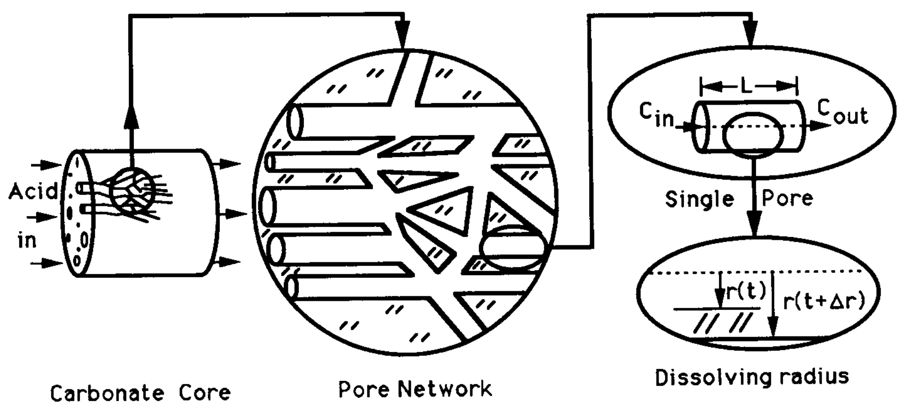

Figure R11.5-1 Network model of dissolving carbonate core.

The stoichiometric coefficient, $\vartheta_{c/a}$, times the moles of acid consumed in the pore in a time Δt is equal to the moles of material dissolved:

$$\vartheta_{c/a}[U\pi r^2(C_{\text{Ain}} - C_{\text{Aout}})\Delta t] = \rho_m 2\pi r L \Delta r \qquad \text{(R11.5-1)}$$

where $\vartheta_{c/a}$ is the moles of carbonate dissolved per moles of acid consumed, ρ_m is the carbonate molar density, and U is the fluid velocity.

The relationship between the inlet and outlet concentrations to the pore is analogous to Equation (11-61):

$$\frac{C_{\text{Aout}}}{C_{\text{Ain}}} = \exp\left(-\frac{2k_c}{rU}L\right) \qquad \text{(R11.5-2)}$$

We now combine Equations (R11.5-1) and (R11.5-2) with a mole balance on the acid to obtain the pore radius as a function of time.

Application of the mole balance to a specific pore in the network gives the radius of the pore as a function of time in increments of Δt:

Increase in pore radius

$$r_i(t+\Delta t) = r_i(t) + \frac{\vartheta_{c/a} Q_i C_{\text{Ain}}}{2\pi r_i L_i \rho_m} \left\{ 1 - \exp\left[-a \left(\frac{D_e L}{Q_i} \right)^{2/3} \right] \right\} \Delta t \qquad \text{(R11.5-3)}$$

where the subscript i refers to the ith pore, D_e is the effective diffusivity, $a = 1.75\pi$, and Q_i is the volumetric flow rate through the ith pore.

These equations can be coupled with the flow distribution through the porous media to yield the rate and pathway of the channel formation through the porous media. This channel can be visualized by filling the acidized carbonate pore space with molten woods metal, letting it solidify, and then imaging the etch channel by neutron radiography,[16] which is called a wormhole. A typical wormhole is shown in Figure R11.5-2a and the corresponding network simulation is shown in Figure R11.5-2b.

Wormholes etched by acid

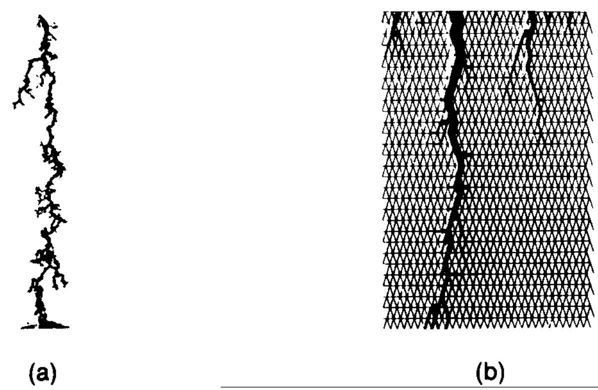

Figure R11.5-2 (a) Wood's alloy castings of a 0.127-m-length core showing the pathway that acid etched through the core. (b) Simulation results showing channeling in the network for the transport-limited case.

[16]H. S. Fogler and J. Jasti, *AIChE J.*, *36*(6), 827 (1990). See also M. L. Hoefner and H. S. Fogler, *AIChE J.*, *34*, 45 (1988), and S. D. Rege and H. S. Fogler, *AIChE J.*, *35*, 1177 (1989). C. Fredd and H. S. Fogler, *Soc. Petr. Engrg. J.*, **13**, p. 33 (1998). C. Fredd and H. S. Fogler, *AIChE J., 44,* p1933 (1998).