Homework #1

Due: 8/16/04

1. For the hydrogen nucleus, the gyromagnetic ratio is $\gamma = 2.675\text{x}10^8\ \text{T}^{-1}\ \text{s}^{-1}$ (radians is an implicit unit here) or $\gamma/2\pi = 42.58$ MHz/T.
   a. Suppose the main magnetic field is 3T, determine the resonant frequency of the spins (in MHz or radian/s).
   b. Suppose we wish to tip the magnetization by 90 degrees or $\pi/2$ by applying a rotating B1 field of strength $5\text{x}10^{-6}$ T, determine the rotational frequency of the B1 field and its duration to achieve the desired tip angle.
   c. Now, describe two different ways we can achieve a 45 degree tip angle by modifying the method of part b.
   d. Suppose we now want to image the phosphorus nucleus. The gyromagnetic ratio for ^{31}P is about 1/3 that of ^{1}H, e.g., $\gamma_P = \gamma_H/3$. Answer parts a. and b. for the phosphorus nucleus.

2. Consider two materials A and B with the same M_0, but with T_1 and T_2 relaxation times (T_{1A}, T_{2A}) and (T_{1B}, T_{2B}). Let $\Delta s_{xy}(t) = M_{xyA}(t) - M_{xyB}(t)$ be the difference in transverse magnetization and $\Delta s_z(t) = M_{zA}(t) - M_{zB}(t)$ be the difference in longitudinal magnetization. Assume a $\pi/2$ excitation pulse. Find an expression for the time that maximizes each of the above differences.

3. Using the table of tissue parameters below, we will investigate the pulse sequence parameters for T1- and T2-weighted pulse sequences. We will make use the relationship: $m = \rho(1 - e^{-TR/T1})e^{-TE/T2}$.

	T1	T2	ρ
Gray matter	1.2 s	70 ms	.98
White matter	800 ms	45 ms	.80

   a. For a T1-weighted pulse sequence, we usually make the TE as short as possible to remove all T2 weighting. For this part of the problem, please let TE = 0 (e.g. $e^{-TE/T2} = 1$). Determine the signal strength, m, vs. TR for TR = 0, 500, 1200 and 2000 ms.
   b. Which TR is "best?" Explain. Besides contrast, are there advantages/disadvantages from using a long TR?
   c. For a T2-weighted pulse sequence, we usually make the TR as long as possible to remove all T1 weighting. For this part of the problem, please let TR = ∞ (e.g. $e^{-TR/T1} = 0$). Determine the signal strength, m, vs. TE for TE = 0, 30, 80 and 150 ms. Put gray and white matter on the same plot (in different colors).
   d. Which TE is "best?" Explain. Besides contrast, are there advantages/disadvantages from using a long TE?

4. Assume we have two objects located at $x = 0$ and $x = 10$ cm and we apply a frequency encoding gradient of $3x10^{-5}$ T/cm. Assume we are operating at 3T and that we are imaging ^1H.
   a. Determine the absolute frequency of these two objects.
   b. Determine the frequency difference between them.

5. When doing slice selection, if the slice select (z) gradient is 10^{-5} T/cm and we have two slices positioned 5 cm apart. By how much frequency must we shift the transmitter to move the excited slice from one to the other? Assume ^1H.

6. Suppose we wish to use a spin-warp acquisition to acquire and axial image of the human head. Suppose the size of the head is an oval of dimension 15 x 20 cm (*x* and *y*, respectively). We set our imaging field of view (FOV_x x FOV_y) to these dimensions.
   a. Describe the relationship between Δk_x and Δk_y.
   b. If we want 1 mm spatial resolution, describe how many samples are required in k_x and k_y.
   c. If we can arbitrarily assign *x* to phase or frequency encoding directions (and the same for *y*), which would be your preferred assignment and why?
   d. If you could reduce the resolution in either the phase or frequency direction, which would you choose and why?