

Homework #1

1. For the hydrogen nucleus, the gyromagnetic ratio is $\gamma = 2.675 \times 10^8 \text{ T}^{-1} \text{ s}^{-1}$ (radians is an implicit unit here) or $\gamma/2\pi = 42.58 \text{ MHz/T}$.
 - a. Suppose the main magnetic field is 3T, determine the resonant frequency of the spins (in MHz or radian/s).
 - b. Suppose we wish to tip the magnetization by 90 degrees or $\pi/2$ by applying a rotating B1 field of strength $5 \times 10^{-6} \text{ T}$, determine the rotational frequency of the B1 field and its duration to achieve the desired tip angle.
 - c. Now, describe two different ways we can achieve a 45 degree tip angle by modifying the method of part b.
 - d. Suppose we now want to image the phosphorus nucleus. The gyromagnetic ratio for ^{31}P is about 1/3 that of ^1H , e.g., $\gamma_{\text{P}} = \gamma_{\text{H}}/3$. Answer parts a. and b. for the phosphorus nucleus.

Solution

- a. $f_0 = 127.7 \text{ MHz}$ or $\omega_0 = 8.026 \times 10^8 \text{ rad/s}$ or just $8.026 \times 10^8 \text{ s}^{-1}$.
 - b. $f_0 = 212.9 \text{ Hz}$ or $\omega_1 = 1337 \text{ rad/s}$. Duration for a $\pi/2$ pulse is $\pi/(2 \times 1337) = 1.17 \text{ ms}$.
 - c. We can get a 45 degree pulse by reducing the amplitude by $1/2$ or by reducing the duration by $1/2$.
 - d. $\omega_0 = 2.673 \times 10^8 \text{ rad/s}$, $\omega_1 = 446 \text{ rad/s}$. Duration for a $\pi/2$ pulse is 3.52 ms.
2. Consider two materials A and B with the same M_0 , but with T_1 and T_2 relaxation times (T_{1A} , T_{2A}) and (T_{1B} , T_{2B}). Let $\Delta S_{xy}(t) = M_{xyA}(t) - M_{xyB}(t)$ be the difference in transverse magnetization and $\Delta S_z(t) = M_{zA}(t) - M_{zB}(t)$ be the difference in longitudinal magnetization. Assume a $\pi/2$ excitation pulse. Find an expression for the time that maximizes each of the above differences.

Solution

To find the maximum of $\Delta S_{xy}(t) = M_{xyA}(t) - M_{xyB}(t) = M_0(e^{-t/T_{2A}} - e^{-t/T_{2B}})$, we must differentiate $\Delta S_{xy}(t)$ and find the value of t for which it is zero, that is, determine t such that $d\{\Delta S_{xy}(t)\}/dt = 0$. The solution is $t_{\text{opt}} = \ln(T_{2B}/T_{2A})(T_{2A}T_{2B})/(T_{2B}-T_{2A})$. This t_{opt} is the “echo time” that will yield the maximum difference between two tissues A and B, that is, will maximize the contrast to noise ratio. A similar solutions works for the T_1 's. To find the maximum of $\Delta S_z(t) = M_{zA}(t) - M_{zB}(t) = M_0(1 - e^{-t/T_{1A}} - 1 + e^{-t/T_{1B}})$, we must find t , such that $d\{\Delta S_z(t)\}/dt = 0$. The solution is $t_{\text{opt}} = \ln(T_{1B}/T_{1A})(T_{1A}T_{1B})/(T_{1B}-T_{1A})$. This t_{opt} is the “repetition time” that will yield the maximum difference between tissues A and B.

3. Using the table of tissue parameters below, we will investigate the pulse sequence parameters for T1- and T2-weighted pulse sequences. We will make use the relationship: $m = \rho(1 - e^{-TR/T1})e^{-TE/T2}$.

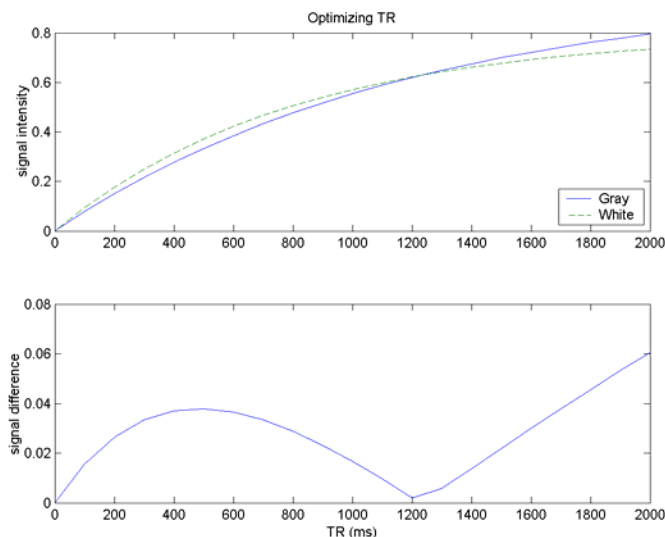
	T1	T2	ρ
Gray matter	1.2 s	70 ms	.98
White matter	800 ms	45 ms	.80

- For a T1-weighted pulse sequence, we usually make the TE as short as possible to remove all T2 weighting. For this part of the problem, please let $TE = 0$ (e.g. $e^{-TE/T2} = 1$). Determine the signal strength, m , vs. TR for TR = 0, 500, 1200 and 2000 ms.
- Which TR is “best?” Explain. Besides contrast, are there advantages/disadvantages from using a long TR?
- For a T2-weighted pulse sequence, we usually make the TR as long as possible to remove all T1 weighting. For this part of the problem, please let $TR = \infty$ (e.g. $e^{-TR/T1} = 0$). Determine the signal strength, m , vs. TE for TE = 0, 30, 80 and 150 ms. Put gray and white matter on the same plot (in different colors).
- Which TE is “best?” Explain. Besides contrast, are there advantages/disadvantages from using a long TE?

Solution

First, this problem is slightly different than problem 2 in that we now are considering ρ in addition to the T1's and T2's.

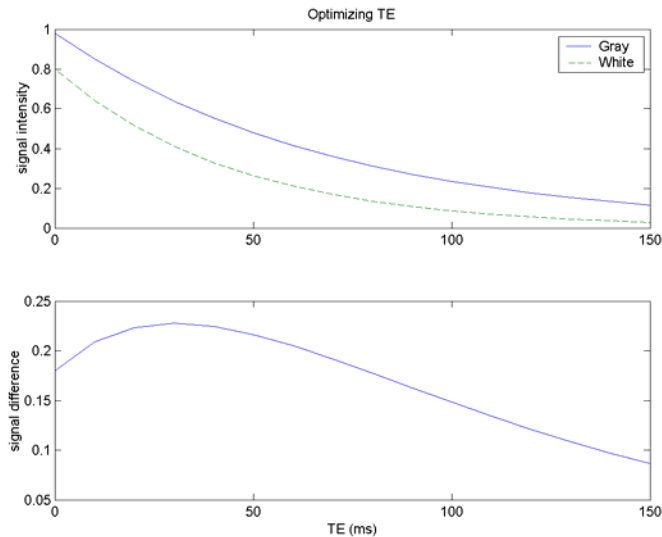
a.



- The best contrast comes for very long TR's, like 2000 ms. This is effectively a hydrogen density weighted scan – eliminate both T1 and T2 contrast. Unfortunately, this is a very time-consuming scan due to the long TR. A better

practical solution might be to choose the optimal T1-weighted image with a TR = 500 ms.

c.



d. Here the best TE is about 30 ms. Longer TE's can mean that fewer slice can be squeezed into a TR interval. Longer TE's also reduce the amount of signal and the images will have an overall noisier appearance.

4. Assume we have two objects located at $x = 0$ and $x = 10$ cm and we apply a frequency encoding gradient of 3×10^{-5} T/cm. Assume we are operating at 3T and that we are imaging ^1H .
- Determine the absolute frequency of these two objects.
 - Determine the frequency difference between them.

Solution

Here we will use the relationship $\omega(x) = \gamma(B + Gx)$. So, $\omega(0) = 802,500,000$ rad/s and $\omega(10) = 802,580,250$ rad/s (both approximately 128 MHz). The frequency difference is $\Delta\omega = 80,250$ rad/s or 12.8 kHz.

5. When doing slice selection, if the slice select (z) gradient is 10^{-5} T/cm and we have two slices positioned 5 cm apart. By how much frequency must we shift the transmitter to move the excited slice from one to the other? Assume ^1H .

Solution

Here we will use the relationship $\Delta\omega(\Delta z) = \gamma G \Delta z = 13,375$ rad/s or 2.1 kHz.

6. Suppose we wish to use a spin-warp acquisition to acquire an axial image of the human head. Suppose the size of the head is an oval of dimension 15 x 20 cm (x and y , respectively). We set our imaging field of view (FOV_x x FOV_y) to these dimensions.
- Describe the relationship between Δk_x and Δk_y .
 - If we want 1 mm spatial resolution, describe how many samples are required in k_x and k_y .
 - If we can arbitrarily assign x to phase or frequency encoding directions (and the same for y), which would be your preferred assignment and why?
 - If you could reduce the resolution in either the phase or frequency direction, which would you choose and why?

Solution

- Since $FOV_x/FOV_y = 3/4$, $\Delta k_x/\Delta k_y = 4/3$.
- For 1 mm sampling, we want $N_x = 150$ and $N_y = 200$.
- It is preferred to assign x to the phase direction since the total acquisition time for a spin-warp acquisition is $N_{phase} * TR$. This minimizes acquisition time.
- It is generally preferred to reduce resolution phase the phase direction. This reduces N_{phase} and thus total acquisition time.