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**The Ricardian Model**

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## ABSTRACT

### **The Ricardian Model**

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This essay was written for the *Princeton Encyclopedia of the World Economy*. The Ricardian Model describes a world in which goods are competitively produced from a single factor of production, labor, using constant-returns-to-scale technologies that differ across countries and goods. With only two goods and two countries, the standard textbook model shows that countries will export the good in which they have comparative advantage. Equilibrium takes two forms, one with both countries completely specialized and gaining from trade, the other with one country producing both goods and neither gaining nor losing from trade. The model is easily extended to more than two goods or more than two countries, but not both. Important extensions have been provided by Dornbusch, Fischer, and Samuelson (1977) to a continuum of goods with two countries, and by Eaton and Kortum (2002) to a continuum of goods with many countries and random technologies.

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## **The Ricardian Model \***

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The Ricardian model is the simplest and most basic general equilibrium model of international trade that we have. It is usually featured in an early chapter of any textbook on international economics. Historically, it is the earliest model of trade to have appeared in the writings of classical economists, at least among models that are still considered useful today.

It is indeed still useful. In spite of being superseded over the years by models with much more complexity (more factors of production, increasing returns to scale, imperfect competition), the Ricardian model often provides the platform for the introduction of today's new ideas. Dornbusch, Fischer, and Samuelson (1977) examined a continuum of goods first in a Ricardian model. Eaton and Kortum (2002) incorporated an ingenious and elegant treatment of geography into a Ricardian model. Melitz (2003) started a small revolution in trade theory by modeling heterogeneous firms within what was essentially a Ricardian model.

The Ricardian model itself, as a new idea, came many years after Ricardo. David Ricardo, in 1816 according to Ruffin (2002), introduced only a portion of the model that now bears his name, focusing primarily on the amounts of labor used to produce traded goods and, from that, the concept of comparative advantage. The first appearance of the Ricardian model, according to Ruffin again, was in Mill (1844).

In this essay, I will first describe the simplest Ricardian Model as it is understood today, including its assumptions, its main implications, and the mode of analysis most commonly used to illustrate it. I will then, with much less detail, describe various extensions to the simple model.

### ***The Simple Ricardian Model***

The simple Ricardian model depicts a world of two countries, A and B, each using a single factor of production, labor L, to produce two goods, X and Y. Technologies display constant returns to scale, meaning that a fixed amount of labor,  $a_g^c$  is needed to produce a unit of output of each good,  $g=X,Y$ , in each country,  $c=A,B$ , regardless of how much is produced in total. All markets are perfectly competitive, so that goods are priced at cost in countries that produce them,  $p_g^c = w^c a_g^c$ , where  $w^c$  is the competitive wage in country  $c$ . Labor is available in fixed supply in each country,  $L^c$ ; it is immobile between countries but perfectly mobile within each. The Ricardian Model typically leaves demands for goods much less fully specified than supplies, though a modern formulation might specify for each country a utility function,  $U^c = U^c(C_X^c, C_Y^c)$ , which the representative consumer maximizes subject to a budget constraint. Utility functions might, or might not, be assumed in addition to be identical across countries, homothetic, or even Cobb-Douglas, although most properties of the model's solution do not require any of these assumptions.

The most basic use of the model compares the equilibria in autarky with those of free and frictionless trade. In autarky, since both goods must be produced in each

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\* I have benefited from helpful comments from Roy Ruffin, ...

country, prices are given immediately by the costs stated above, and further analysis is needed only if one wants to know quantities produced and consumed. If so, the linear technology implies a linear production possibility frontier (PPF) that also serves as the budget line for consumers in autarky. The autarky equilibrium is as shown in Figure 1, where “~” indicates autarky and  $Q$  represents production.

Comparison of the two countries in autarky depends primarily on their relative costs of producing the two goods, which in this model defines their comparative advantage. For concreteness, assume that country  $A$  has comparative advantage in good  $X$ :

$$a_X^A / a_Y^A < a_X^B / a_Y^B, \text{ so that}$$

$$\tilde{p}_X^A / \tilde{p}_Y^A < \tilde{p}_X^B / \tilde{p}_Y^B. \text{ Without further}$$

assumptions about preferences, little more can be said about autarky, but if preferences are identical and

homothetic, with positive elasticity of substitution, then one can infer that

$$\tilde{Q}_X^A / \tilde{Q}_Y^A > \tilde{Q}_X^B / \tilde{Q}_Y^B$$

With free and frictionless trade, prices must be the same in both countries. Two kinds of equilibrium are possible, depending on the supplies and demands for goods in the two countries. One kind of equilibrium has world relative prices, denoted here by “~”, strictly between the relative prices of the two countries in autarky:

$$\tilde{p}_X^A / \tilde{p}_Y^A < \tilde{p}_X^A / \tilde{p}_Y^A < \tilde{p}_X^B / \tilde{p}_Y^B. \text{ In that case, each country must specialize in producing}$$

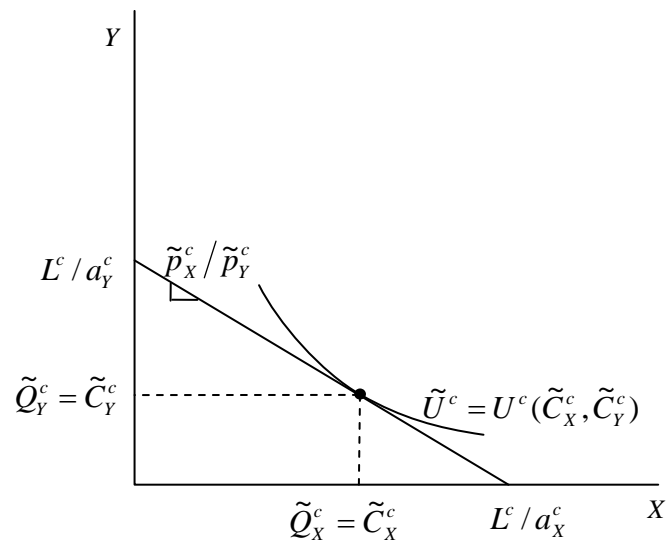


Figure 1:  
Ricardian Model Equilibrium in  
Autarky

only the good for which its relative cost is lower than the world relative price, thus the good in which it has comparative advantage. Each must necessarily export that good.

With such complete specialization, outputs of the goods are determined by labor endowments and productivities, so equality of world supply and demand must be achieved from the demand side. That is, world prices are determined such that the two countries' demands sum to the quantity produced in one of them. These demands derive from the expanded budget constraints of each country's consumers, reflecting the value at world prices of the single good that the country produces. Consumers can now, unless they wish to consume only that single good, consume more of both goods than they did in autarky. Whether they choose to do so or not depends on the extent to which they substitute toward the cheaper good now imported from abroad, but in any case they reach

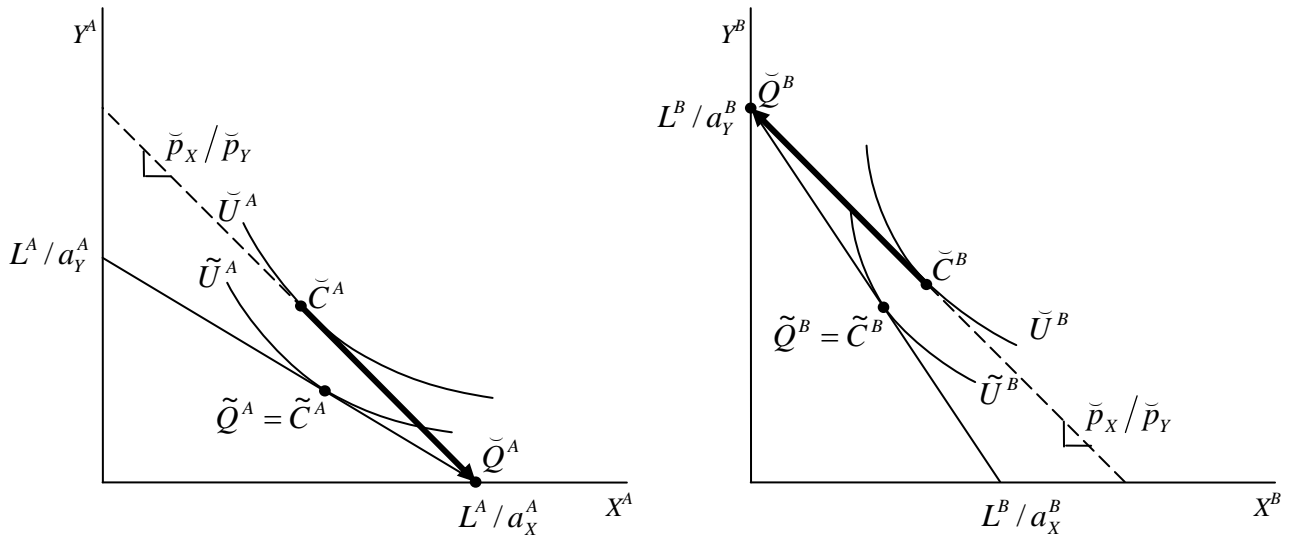


Figure 2:  
Free Trade Equilibrium with Complete  
Specialization

a higher indifference curve and are better off. All of this is shown in Figure 2. For this to be an equilibrium, the quantity of each good exported by one country must equal the

quantity imported by the other, so the heavy arrows showing net trade in each panel of the figure must be equal and opposite.

Such an equilibrium with specialization will arise only if the two countries' capacities to produce their respective comparative-advantage goods correspond sufficiently closely to world demands for the goods. If this is not the case – if one country's labor endowment is too low and/or its labor requirement for producing its comparative-advantage good is too high for it to satisfy world demand – then while that country will specialize, the other country (call it the larger one, although that is not strictly necessary) will not. Instead of world relative prices settling between the two autarky levels as above, prices will exactly equal the autarky prices of the larger country, and that country will produce both goods. At those prices, producers in the larger country will be indifferent among all output combinations on the PPF, and output in the large country will be determined instead by the need to fill whatever demand is not satisfied by the smaller country.



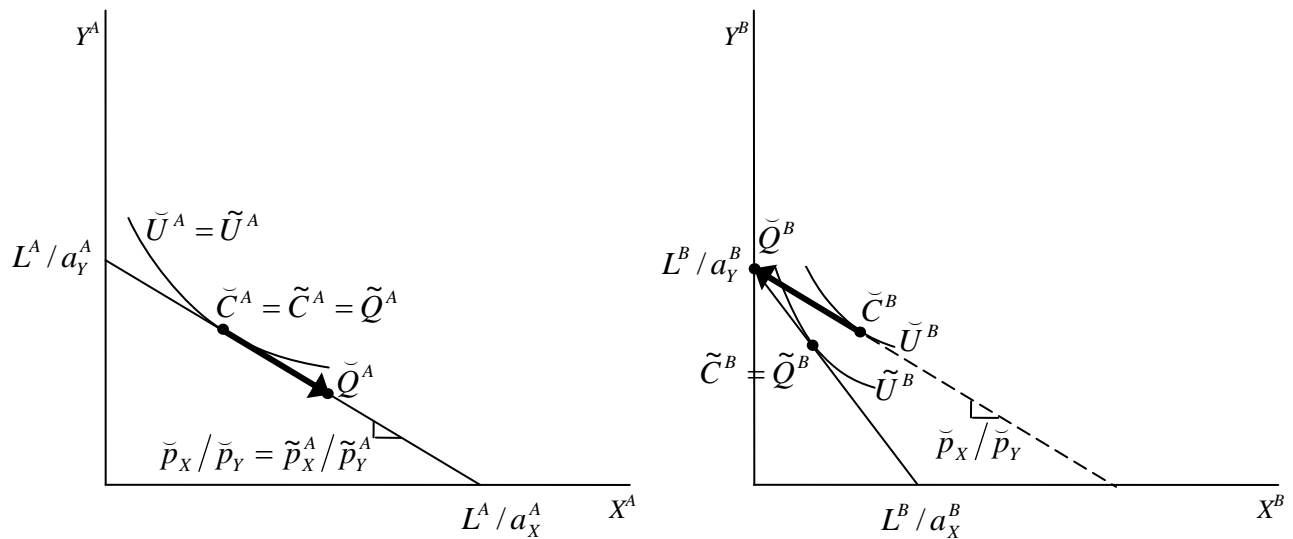


Figure 3:  
Free Trade Equilibrium with Country A  
Incompletely Specialized

Such an equilibrium is shown in Figure 3, where in comparison to Figure 2 country B's labor endowment has been made smaller and both countries' preference for good Y has been increased. As a result, country B is too small to meet world demand for good Y, even at country A's autarky prices. Therefore the free trade equilibrium has country A consuming where it did in autarky, while its production,  $\tilde{Q}^A$ , moves down along its PPF so that, again, its trade vector can be equal and opposite to that of country B. Note that, in this trading equilibrium, the larger country neither gains nor loses from trade.

The following are some of the implications of this simple model, some of which have been illustrated above, while others can be derived rather simply:

Effects of trade:

- Each country exports the good in which it has comparative advantage, as defined by having a lower relative autarky price than the other country.

- Trade causes each country to expand its production of the good it exports, with labor being reallocated to it from the import-competing industry.
- Trade causes the relative price of a country's export good to rise, except in the case of a "large" country, defined here as one whose trading partner is too small to meet its demand for imports.
- Consumption and welfare are unchanged by trade in a large country; in any country that is not large, consumers buy more of one or both goods and welfare increases.
- Because all income accrues to labor, which earns the same wage in both industries due to mobility, conclusions about welfare or utility apply equally well to the real wage.

Comparative Statics of Trading Equilibria (assuming that both goods are normal goods):

- An increase in the labor endowment of a country, holding other labor, technology, and tastes constant, hurts the growing country and benefits the other.
- A fall in the labor required by a country to produce its export good, holding other technology, endowments and tastes constant, benefits the other country but may either benefit or harm ("immizerize") the growing country.
- A rise in the labor required by a country to produce its import good has no effect if it does not produce that good; if it does produce it (like country A

in Figure 3), the world price of that good rises, that country is harmed, while the other country gains.

- A change in preferences, in either country, in favor of one of the goods has no effect on prices or production if one of the countries is incompletely specialized. If both are specialized, however, then the relative price of that good rises, improving the terms of trade of the country that exports it.

### ***Extensions of the Simple Ricardian Model***

Before considering several extensions of the simple model above, it is reasonable to ask what extensions would *not* be acceptable, in that they would lead to a model that would no longer be “Ricardian,” as trade economists understand the term. Ricardo himself might disagree, were he alive, but the essential features of a Ricardian Model seem to be two: that production uses only homogeneous labor as a primary input; and that comparative advantage arises from differences across goods and countries in the technology for producing goods from that labor. Both of these requirements distinguish a Ricardian Model from the other principal model of trade theory, the Heckscher-Ohlin or Factor-Proportions Model. With primary factors other than labor (including different kinds of labor based on skill and/or industry of location), a model takes on features that differ in essential ways from the Ricardian Model. On the other hand, with only homogeneous labor as a factor of production, if technologies do not differ across countries then there is no scope for comparative-advantage based trade.

### *More Goods and/or Countries*

Therefore, keeping the number of factors at one, the most obvious things to extend in the simple model are to add to the numbers of goods and/or countries. This is relatively easily done, as long as one does not try to do both.

With two countries and many goods, the goods can be ranked in a chain of comparative advantage based on the ratios of their unit labor requirements in the two countries. That is, if one numbers  $N$  goods such that  $a_1^A/a_1^B < a_2^A/a_2^B < \dots < a_N^A/a_N^B$ , then country A has comparative advantage in the low end of this ranking while country B has it in the high end. Further, one can show that under free trade, each country will specialize in and export goods in its respective end of the chain, with at most one good (and perhaps no good) being produced in common by both countries. The division between A's exports and B's exports depends on country sizes, technologies, and tastes, much as in the choice between Figures 2 and 3 above. For example, the larger is the labor endowment and/or efficiency of country A compared to B, the further up the chain will A produce and export.

Similarly, with two goods and many countries, the countries can be ranked in a chain of comparative advantage based on the ratios of their unit labor requirements for producing the two goods. That is, if one numbers  $M$  countries such that

$a_X^1/a_Y^1 < a_X^2/a_Y^2 < \dots < a_X^M/a_Y^M$ , those countries in the low end of this ranking will specialize in and export good X to those in the high end, which export good Y. Again there will be at most one country (and perhaps no country) that produces both goods. And the division between X-exporters and Y-exporters depends on country sizes, technologies, and tastes.

Unfortunately, extending to more than two of both goods and countries is not so simple or intuitive. Jones (1961) seems to have done about as well as one can, showing that an efficient assignment of countries to goods will minimize the product of their unit labor requirements. This certainly suggests the importance of comparative advantage, in the form of low relative unit labor requirements, which is perhaps all that one should hope for from a many-good, many-country Ricardian Model (though see below for Eaton and Kortum's solution to this problem).

### *A Continuum of Goods*

A less obvious, but much more useful, extension of the Ricardian model was provided by Dornbusch, Fischer and Samuelson (1977) – hereinafter DFS – who took the number of goods to infinity, in the form of a continuum. Indexing goods by the continuous variable  $j$  on the interval  $[0,1]$ , they specified technologies for each of two countries as  $a^c(j)$  representing the amount of labor required in country  $c$  to produce one unit of good  $j$ . The ratio of these in the two countries of the model, ordered monotonically in a function  $A(j)$ , then plays the same role as the chain of relative labor requirements mentioned above for the many goods case. But with a continuum of goods, the good at the dividing line between a country's exports and its imports is of negligible importance for labor markets, since it employs a negligible amount of labor, and this removes the need to consider whether a good is produced in both countries. Such a good now always exists, as the dividing line between one country's exports and its imports, but it is of negligible importance for employment.

This simplicity is helpful in itself, but the more important advantage of the continuum model is that it facilitates the analysis of the range of goods that a country will

export and import, something that the two-good model could not usefully address. One finds, for example, that an expansion of the labor endowment of one country relative to the other will cause it to expand its exports, not just by exporting more of what it already exported (though that happens too), but by exporting goods that it previously imported.

The model in its simplest form is depicted in Figure 4, which is taken directly from DFS's Figure 1. The downward sloping function  $A(j)$  is the ratio of the two countries' unit labor requirements, ordered so that country A's comparative advantage declines with rising  $j$ . Letting  $\omega = w^A/w^B$ , for any given value of this relative wage, free and frictionless trade will lead to country A producing and exporting all goods with  $A > \omega$  and importing goods with  $A < \omega$ . To determine the equilibrium value of  $\omega$  one needs assumptions about demand, which are reflected in the upward sloping curve  $B(j; L^B/L^A)$ . Assuming that preferences are identical and Cobb-Douglas, this curve measures the relative wage at which demands for each country's range of goods produced would equal their supplies (or, equivalently, the relative wage at which values of a country's exports and imports will be equal). This requires simply that the ratio of expenditures on the two sets of goods equal the ratio of the incomes of those who produce them. As the definition of this market-clearing relative wage shown in Figure 4 indicates, it depends positively on  $\mathcal{G}(j)$ , the fraction of income spent on the goods produced by country A, which in turn rises with the fraction of goods that A produces. It also depends positively on the relative size (labor force) of country B, since the larger that is, for a given division of goods between the two countries, the higher must be the relative wage in A to keep the expenditure ratio constant.

Figure 4 immediately yields the result mentioned above, that as a country's labor force rises relative to the other country (shifting the  $B$  curve up or down), its share of goods produced increases as well, while in addition its relative wage falls. Likewise, if a country becomes more productive in producing all goods (its  $a^i(j)$  shifts down, shifting the  $A$  curve up or down), it also produces more goods but its

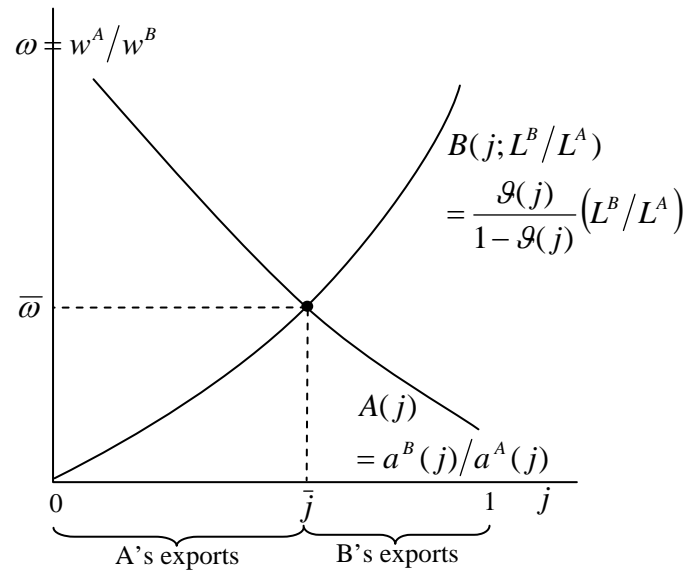


Figure 4:  
Ricardian Model with a  
Continuum of Goods

relative wage increases. Other exercises are possible with the simple model, and DFS extend the model in a variety of directions to illuminate many issues that could not be readily addressed in models with a finite number of goods.

Most notably, they incorporate transport costs, giving rise to a third endogenous range of goods, in addition to those exported by country A and by country B: nontraded goods, whose costs differ too little between countries to overcome the barrier of transport costs. This is particularly nice, since it implies that as a country's relative productive capacity rises, some of the goods it previously imported become nontraded, while it begins to export some of the goods that were previously nontraded.

#### *Multiple Countries with Random Technologies*

A limitation of the DFS model is that it applies to a world of only two countries, and because of its reliance on ratios of values in those countries it is not readily extended

to more, although some (e.g., Wilson (1980)) have had some success. A breakthrough was provided by Eaton and Kortum (2002), however, who extended the DFS model to an arbitrary number of countries by assuming that, in effect, the labor productivities of each good and country are determined randomly. Specifically, they let labor productivity,  $z^i(j) = 1/a^i(j)$ , be determined by a random draw from a probability distribution, such that each country has some probability, regardless of its overall technical ability and its wage, of having a lower cost than any other country. This probability, then, translates into the fraction of the continuum of goods that the country is able to produce and export under free and frictionless trade. More importantly, by including transport costs for each pair of countries, each country has a fraction of goods that it will be able to produce even without necessarily the lowest costs, since they only need to cost less than goods from other countries inclusive of transport cost. Furthermore, if transport costs are low enough that a country imports anything, then it will also export some fraction of goods as well, since if necessary the wage will fall until some fraction of goods can be exported to one or more countries for delivered prices below those countries' domestic prices. This formulation therefore extends the Ricardian model not only to multiple countries but to a context that can account for bilateral trade.

The complete Eaton-Kortum model, even in its simplest form excluding intermediate inputs and a separate non-manufacturing sector both of which Eaton and Kortum include, is beyond the scope of this essay. Suffice it to say that the model generates equations for prices and trade shares that provide the basis for empirical estimation as well as being susceptible to solution and comparative static analysis by numerical methods. The model provides an elegant and parsimonious theoretical



justification for the gravity model of bilateral trade flows while at the same time illustrating the interaction between the forces of comparative advantage that give rise to trade and the geographical resistance to those forces in the form of transport and other costs of trade that limit trade and direct it over particular geographical routes.

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