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Contract Enforcement and Firms' Financing

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Abstract

This paper studies how the degree of contract enforcement in a country influences firms' financing decisions. We first document empirical facts on debt financing for two new firm-level datasets in the United Kingdom and Ecuador. In the United Kingdom, small firms borrow more relative to their assets than large firms, whereas in Ecuador small firms borrow less. We build a dynamic model of firms' debt financing where debt is constrained by the likelihood of default, which varies across firms and economies with different degrees of enforcement. Because of their low firm values, small firms are mostly affected by abundance or scarcity of economy-wide loans generated by weak or strong contract enforcement. We calibrate our model to the datasets in the two countries and find that our mechanism can quantitatively account for the patterns observed in the data.

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1 Introduction

Do firms finance their assets differently across countries? In the United Kingdom small firms use more debt financing than large firms, whereas in Ecuador small firms use less debt financing than large firms. The stark difference in the financing patterns across countries is exemplified in the behavior of small firms: their ratio of debt to assets in Ecuador is half of that in the United Kingdom. We develop a model that can rationalize the empirical regularities where firms' debt financing is constrained by lack of enforcement in financial contracts. Firms in economies with weak contract enforcement face limited loans that are especially restricted for the smaller firms because they have larger default probabilities.

We document the financing patterns of firms in Ecuador and the United Kingdom for two new firm-level comprehensive datasets. We find that debt relative to assets is on average higher for firms in the UK than for firms in Ecuador and that the relation between debt and size varies across countries. In the United Kingdom debt to assets ratios of smaller firms are on average 1.0, whereas they are 0.6 for the large firms. In contrast, in Ecuador debt to assets ratios of smaller firms are 0.4, whereas they are 0.7 for the larger firms. We also document the relation between size, growth, and exit. Both countries feature the common size-growth relation: small firms grow faster than large firms conditional on survival.[1] We also find that in Ecuador small firms have higher exit rates than large firms, and that the debt levels of the exiting firms are larger than those of the continuing firms conditional on size. We choose these countries because of data availability and also because we want to contrast firms' financing in economies with different degrees of contract enforcement.

The paper develops a dynamic model of firms' debt financing to study the patterns found in the data. Debt is constrained by the likelihood of default, which varies across heterogeneous firms and economies with different degrees of contract enforcement. Firms use debt to finance capital and for precautionary motives because they want to avoid firm failure after a sequence of low shocks. In economies with good enforcement, default is unlikely and loan availability is large for all firms. The ease of loans allows small firms to keep a disproportionately smaller buffer of precautionary savings as the economy-wide borrowing opportunities are larger for them. Thus, the ratio of debt to assets of small firms is higher than that of large firms. In economies with weak enforcement, loans are limited because of high probabilities of default. Firms do not have the tools to smooth fluctuations and firm value is low as firms engage in costly accumulation of assets to partially insure against firm failure. Small firms use debt most conservatively given their highest likelihood of failure because of their low and volatile

[1] The relation between size, growth, and exit has been documented extensively for U.S. firms. See, for example, Rossi-Hansberg and Wright (2005) and Klette and Kortum (2004).

value. Thus, the ratio of debt to assets of small firms is lower than that of large firms. We quantify these mechanisms by calibrating our model to Ecuador and the UK and find that the model provides a unified rationale for the relation between debt financing and size that is dependent on the degree of contract enforcement.

The framework is a dynamic model of heterogeneous firms similar to Albuquerque and Hopenhayn (2004) but with incomplete markets as in Cooley and Quadrini (2001). Firms finance capital using their own income and borrowing from creditors. Firms' productivity consists of two components: a permanent component and an i.i.d. component. We assume loan contracts cannot be contingent on the shock realization and thus are simply debt contracts. After observing their shocks, firms choose to either repay the debt due and remain in operation or default and get a constant default value. If they remain in operation, firms choose investment, the new debt contract, and dividends. Low productivity shocks are costly because they might lead to firm failure or costly private equity issuances.

The set of equilibrium loans available incorporates the likelihood of default events. The incentives to repay or default are determined by the firm's value relative to the value of default, controlled by the degree of contract enforcement. Weak contract enforcement acts like a tax on borrowing, which limits loans for all firms. Small firms are affected more because their firm value relative to the tax is smaller, and thus the set of loans available relative to their assets is stricter. Strong contract enforcement acts like a subsidy on borrowing, which enables all firms to issue more debt. Likewise, it expands the set of available loans disproportionately for small firms given that the subsidy relative to their value is large. We show analytically for the case with no uncertainty that firms' borrowing capacities relative to their assets is increasing with size when enforcement is weak, and is decreasing with size when enforcement is strong. These patterns of borrowing capacities relative to assets also carry over to the case with uncertainties.

Given the set of loans available, firms' choices over borrowing depend on their current income and investment needs, and precautionary incentives to self-insure for the future. When firms have a large income, they finance their investment largely with inside cash while reducing their levels of debt. This is optimal because lower levels of debt reduce default probabilities, interest rate premia, and equity issuance costs. In fact, these firms use their income to invest, save in the risk-free bond, and distribute dividends. Firms with low levels of income borrow large loans to finance all the investment and to roll over part of the loan due while distributing zero dividends. With incomplete markets, the large set of loans available in strong enforcement economies is beneficial for firms because they can avoid the costly accumulation of assets for precautionary motives.

We calibrate the full model to match certain features of the firm size distribution in Ecuador. We then increase the degree of contract enforcement and compare the results to the British dataset. We find that the enforcement friction and incomplete markets can deliver the features of the data in terms of the debt to asset ratios observed in both countries. The model can quantitatively account for the relation between size and debt to asset ratios ranging from 0.4 for small firms to 0.7 for large firms in Ecuador. The model with a stronger degree of enforcement can also account for the decreasing relation between debt to assets and size with ratios matching those in the UK data. Both precautionary savings motives and endogenous firm-specific borrowing limits that depend on the degree of contract enforcement allow the model to deliver these results. The model reproduces qualitatively the growth-size relation similar to the data: small firms grow faster than large firms. The model also generates the fact that small firms tend to exit more than large firms and that the exiting firms have larger debt to asset ratios as in the data. We also find that the degree of contract enforcement has important real implications on the output and firm value, especially for small firms. A small firm in a good enforcement economy can produce twice as much and can have an 8% higher value than if it were located in a weak enforcement economy.

Finally, we use our datasets to test two implications of our model regarding the determinants of the firms' debt to assets ratios and default decisions. We find that consistent with the model's predictions, low-income firms in both countries have on average larger debt to assets ratios the following period after controlling for size. In terms of exit, we also find evidence that consistent with the model, firms with low profit to assets ratios and high leverage ratios are more likely to exit the following period after controlling for size. Thus, these further tests are consistent with the mechanisms of the model that firms use debt to smooth income and that firms with large debt and low income are more likely to fail.

The paper is related to the literature that studies the implications of financial frictions and agency problem for the dynamics and sizes of firms. Our model is closest to Cooley and Quadrini (2001), who study how financial frictions can rationalize the relation of exit and growth with size. However, we concentrate on how the degree of contract enforcement across countries can help explain the differential financing patterns of small firms. Moreover Cooley and Quadrini's model imposes an upper bound on the level of liquid assets, which limits the precautionary motive for debt use. Albuquerque and Hopenhayn (2004) focus on the effects of enforcement problems and solve for the optimal state contingent contract. Our environment is different from theirs in that we consider an incomplete set of assets. Incomplete markets allow for firms in the model with a history of bad shock to decrease their value while increasing debt holdings through time and allow precautionary savings to play a role.

Clementi and Hopenhayn (2006) and Quadrini (2004) study financial constraints that arise due to informational asymmetries between the lender and the entrepreneur and show that moral hazard considerations can rationalize borrowing constraints, which makes investment sensitive to cash flows. They show that information asymmetries can also provide a rationale for the relation between growth and size. DeMarzo and Fishman (2007) study the effects on investment within a large set of agency problems between equity and debt holders. They find that the positive relation between investment and current and past cash flows is robust to a large class of optimal contracting models that feature informational asymmetries. Moreover, they find that leverage ratios are larger for small firms. Our model generates a similar dependency of investment to cash flow due to imperfections in the degree of enforcement in contracts; however, our model provides a rationale for why small firms can have higher or lower leverage ratios relative to large firms depending on the degree of contract enforcement.

The paper is also related to a vast literature on the optimal capital structure of firms in corporate finance, which is motivated by the classic Modigliani-Miller irrelevance propositions.[2] Among various strands of theories on capital structures, our model falls into the dynamic trade-off theory. Two close works in this strand of literature are Hennessy and Whited (2005) and Miao (2005). Hennessy and Whited (2005) develop a dynamic trade-off model with the presence of taxes and financial distress costs. They model corporate taxes to be progressive, as in the United Kingdom. Their model predicts that larger firms use debt more because they face higher tax rates, which is at odds with the size-leverage relation we document for the United Kingdom. Miao (2005) develops a model where firms choose debt and exit decisions to optimally respond to stochastic productivities. However, the debt decision is static as all firms choose only the same perpetuity level of debt when they enter. In his model, small firms with low productivity shocks have higher leverage ratios because their equity value is small. In our paper, the firm's debt decision is dynamic, firms can choose to exit, and the interest on debt is time varying, reflecting endogenous default probabilities. In addition, our framework rationalizes that small firms can have smaller debt to asset ratios because debt is more constrained for them given their higher default probabilities.

2 Empirical Facts

We have compiled a new dataset of firms from two countries: Ecuador and the United Kingdom (UK). The countries differ in terms of GDP per capita and contract enforcement

[2] See Harris and Raviv (1991) for a comprehensive review.

indexes. GDP per capita in 2005 in US dollars adjusted for PPP was 4,272 in Ecuador and 32,005 in the UK. In Ecuador enforcement of contracts is much weaker than in the UK in terms of the efficiency of the judicial system, rules of law, and risks of contract repudiation as documented by La Porta et al. (1998).[3] Also, as compiled by the Business Environment Risk Intelligence, the broad contract enforcement index for Ecuador is 0.65, whereas for the UK it is 1.25, close to the highest index of 1.28 for the United States.

We document empirical regularities regarding the financing patterns of firms in these countries and the dynamics of firms in terms of growth and exit. For Ecuador, we have balance sheets for a panel dataset of over 25,000 firms for the years 1996 to 1999 from the tax records in the local regulatory agency: Superintendencia de Companias del Ecuador. The dataset includes the universe of firms registered as legal entities in Ecuador from all sectors except agriculture and real estate. For the UK, we have balance sheet data for a panel of over two million firms for the years 2000 to 2005 from the Amadeus Database. The dataset covers all sectors in the economy. The majority of firms in both countries are private; less than 4% of firms in the UK dataset and less than 1% of firms in the Ecuador dataset are public. To analyze the data, we clean two samples by restricting them to include firms that report positive and non-missing assets and liabilities each year. We also take out all firms in the financial sector following Rajan and Zingales (1995) and throw away outlier firms with leverage ratios above 10.[4] This procedure cuts the sample in Ecuador to about 20,000 firms per year, and in the UK to about 600,000 firms per year. For the sales growth statistics, we further restrict both samples to firms that report positive and non-missing revenues.

Our measure of firm size is the book value of total assets for the firm. Figure 1 plots the firm size distribution for the two countries with the clean sample. The histogram for Ecuadorian firms is for 1996, and the histogram for British firms is for 2000.[5] The histogram is plotted by constructing 40 equally spaced bins across all firms in each country with the exception of the largest 10% of firms.[6] The firm size distributions for both countries present similar patterns: most firms are small, and the distribution is highly skewed to the right. The ratio of the 90th to 10th percentile asset is 423 in Ecuador and 487 in the United Kingdom.

Although the shape of the distribution is very similar across countries, overall asset levels are different. The median asset in Ecuador is 63,762 US dollars, but in the UK it is 184,952

[3] In La Porta, Lopez-de-Silanes, Shleifer and Vishny (1998), indicators regarding enforcement for Ecuador range between 5.2 and 6.7, whereas for the UK they range from 9.1 to 10, on a scale from 1 to 10.

[4] The percentage of firms with leverage ratios greater than 10 is smaller than 1% for the two countries in all years.

[5] Firm size distributions are very stable across time in both countries.

[6] We exclude the largest 10% of firms for illustrating the asset histogram more clearly; the empirical analysis in-cludes the whole sample of firms.

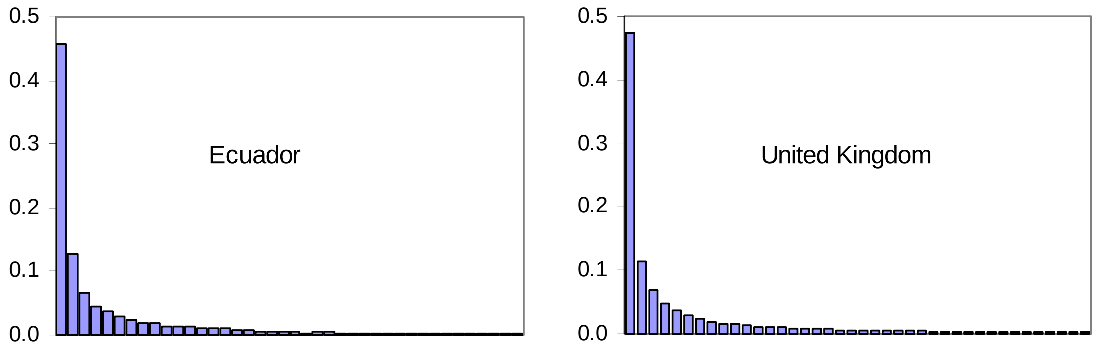

Figure 1: Asset Distributions

US dollars. Furthermore, asset levels in each of the histogram bins are uniformly larger in the UK; for example, the median asset of the first bin in the UK is 25,517 US dollars, whereas in Ecuador it is 9,625 US dollars. These observations are not surprising given the large ratio of GDP per capita between these two countries.

It is worth noting that although our samples have a large number of small firms, they might still be underrepresented. In Ecuador our dataset does not cover the significant informal sector of the economy, and in the UK the Amadeus dataset coverage does not include the universe of registered firms – both of which bias the samples against the smallest firms. However, both samples cover a very significant portion of the economy in each country; the ratios of total sales relative to aggregate GDP in our samples are 1.90 in Ecuador and 2.55 in the UK.[7]

To analyze the financing patterns of firms, we divide the sample of firms in each country into quintiles according to assets and compute their debt to asset ratios, i.e., leverage ratios. Leverage is defined as the broad measure of total liabilities over total assets of the firm, and this is our measure for the capital structure of firms. We use this broad definition because it is a more consistent measure across countries and because it provides the largest sample of firms. We are also interested in the relation between size with growth and exit. Growth is defined as real sales growth per firm between one year and the next, where real sales are constructed using nominal sales deflated by the consumer price index (CPI) in each country. We are able to construct the exit measure for firms in Ecuador. We classify a firm as an exiter in one year if it does not re-appear in the sample after that year, and if the "status code" for that particular firm in 2005 indicates that the process of liquidation has started.

[7] As a reference point, the ratio of gross output to value added in the United States equals 2.12 in 1997.

We have codes of firms' status only for Ecuador and only for year 2005.

Table 1 reports descriptive statistics for the firms in Ecuador in 1996 and the United Kingdom in 2000. The mean debt to assets ratio for Ecuadorian firms, 0.57, is smaller than that for British firms, 0.76.[8] The arithmetic average yearly growth in sales in Ecuador, 64%, is larger than that in the UK, 39%. In the UK the leverage size relation is downward sloping: small firms have relatively higher leverage ratios than large firms.[9] In particular, the mean leverage ratio of firms in the first quintile is 1.01 and that of firms in the fifth quintile is 0.6.[10] In Ecuador the leverage-size relation is monotonically increasing, ranging from 0.41 for the smallest bin to 0.68 for the largest bin.

Table 1. Data Statistics[11]

	Leverage		Sales Growth		Exit Rate
	Ecuador	UK	Ecuador	UK	Ecuador
Quintile 1	0.41	1.01	1.30	0.67	0.12
Quintile 2	0.55	0.82	0.58	0.31	0.08
Quintile 3	0.60	0.71	0.37	0.29	0.06
Quintile 4	0.64	0.64	0.50	0.39	0.05
Quintile 5	0.68	0.60	0.38	0.28	0.03
Overall	0.57	0.76	0.64	0.39	0.07
Obs.	20,331	681,907	11,345	146,067	1,430

Figure 2 illustrates graphically the relation of size with leverage when firms are divided into a finer grid of 20 quantiles. The figure illustrates the remarkably distinct pattern of size and leverage across countries. The large firms in each country have similar debt to assets ratios. However, small firms appear very different across the two countries; for example firms with assets equal to $41,000 in the UK have on average a leverage ratio of 0.85, whereas in Ecuador their leverage is 0.58 (see Table 10 in Appendix 1). The difference in the size-leverage relation across these two countries is robust across years, sectors, number of bins, and

[8] Desai, Foley and Hines (2004) study multinational firms and find that affiliates are financed with less debt in countries with weak creditor rights. Here we find more broadly that firms use debt less in countries with weak contract enforcement.

[9] Rajan and Zingales (1995) document a monotonic increasing relation between leverage and size for the UK in a sample of the largest 608 firms. We find a similar pattern when considering only the largest 1,000 firms from our dataset.

[10] When leverage is greater than one, firms have negative equity which implies owners use personal funds to cover business losses. Herranz et al. (2007) document that 21% of the small firms in the United States have negative equity in 1998.

[11] Data for Ecuador are from 1996, and for the United Kingdom are from 2000. Leverage is defined as Total Liabilities/Total Assets. Growth is net real growth in sales deflated by CPI.

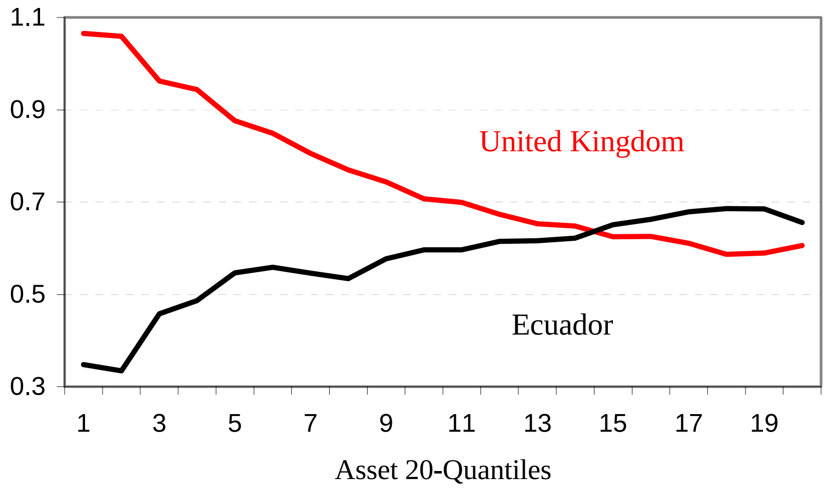

Figure 2: Firm Size and Debt to Asset Ratios

alternative size measures such as sales. Appendix 1 shows these robustness checks regarding the differential leverage-size relation in the data between Ecuador and the United Kingdom. Our model will address precisely how different degrees of contract enforcement across both countries can affect most significantly the debt limits of small firms.

In terms of growth and size, both countries present the patterns that have been documented extensively for other countries such as the United States: small firms grow faster than large firms conditional on surviving. Only the smallest firms in the United Kingdom have a significantly higher growth rate than larger firms. Rossi-Hansberg and Wright (2006) report similar patterns for the United States; employment growth rates for middle and large firms do not vary substantially, whereas employment growth for small firms is significantly higher. In Ecuador, growth rates for all firms are on average higher than those in the UK, and the difference in growth rates between firms in the fifth and second quintiles remains substantial. For Ecuador we can compute exit rates of firms. As reported in Table 1, in Ecuador small firms have higher exit rates than larger firms, as is the case in the United States (see Rossi-Hansberg and Wright 2006).

3 Model Economy

This section presents a dynamic model of heterogeneous firms to study firms' financing choices and dynamics. The model builds on Cooley and Quadrini (2001) and Albuquerque and Hopenhayn (2004). In the model, infinitely lived entrepreneurs own firms and decide on financing for their operations using the firms' income, debt, or private equity. Debt contracts are not enforceable, and entrepreneurs can default on the debt they owe. Debt contracts are also incomplete, and thus default occurs along the equilibrium.

3.1 Firms

Entrepreneurs in the economy have access to a mass of N project opportunities to produce consumption goods. We define each project opportunity as a firm. Entrepreneurs in the economy are infinitely lived and decide on entry, exit, and production and financing plans to maximize the lifetime value of dividends from the project. Entrepreneurs discount time at rate $\beta < 1$ and face an exogenous probability $\delta < 1$ of dying each period. Each entrepreneur owns at most one firm. Every period a fraction ξ of the projects are available to new entrepreneurs. New entrepreneurs always find it worthwhile to enter and operate the firm if the project drawn gives them positive expected present value. When an entrepreneur decides to exit, his firm ceases to exist.

Every period each firm that is in operation produces output y with a stochastic decreasing returns technology that transforms capital goods into consumption goods. For a given level of capital input K invested the previous period, the firm produces output y with a technology given by

$$y = z\varepsilon K^{\alpha} \tag{1}$$

where $0 < \alpha < 1$. The productivity level has two firm-specific components. One is the permanent productivity level z, which is drawn from the distribution $f(z)$. The other is the temporary productivity level ε, which is i.i.d. and drawn from the distribution $g(\varepsilon)$. Capital depreciates completely after production every period.

Entrepreneurs decide how to finance their needs of capital K every period from three options: internally with the firm's income, externally by entering into loan contracts with creditors, or by drawing from costly private equity. In the first period, the entrepreneur has to finance initial capital with external loans or private equity.

An operating firm of productivity z starts the period with a loan to be paid of size B_R and produces output $z\varepsilon K^{\alpha}$ after the productivity shock ε is realized. We define the firm's income x as the difference between output and debt repayment: $x = z\varepsilon K^{\alpha} - B_R$. The entrepreneur can raise supplementary funds by acquiring a new loan B' from the creditor and by costly private equity issuances. The entrepreneur can use the firm's income x plus the additional funds from loans or equity for investment K' or dividends D. When dividends are positive, investment is financed only with the firm's income and loans. The dividends are given by

$$D = z\varepsilon K^{\alpha} - B_R + B' - K'. \tag{2}$$

When dividends are negative, the firm finances its investment with private equity together with loans and income if any. New private equity is costly in that it takes resources γD to raise D, where the parameter $\gamma > 0$ refers to the costs of the equity issuance. We define the leverage of this firm as the ratio of total debt due this period to capital installed B_R/K if $B_R \geq 0$. If the firm starts with assets $B_R < 0$, the firm has no liabilities due, and thus its leverage ratio is equal to zero.

3.2 Incentives and Financing

Every period entrepreneurs can choose to default on their debt if they find it optimal. If the entrepreneur chooses to default, he loses the project and the firm exits. After default the entrepreneur gets a default value given by V^d. The default value summarizes all costs and benefits to the entrepreneur after he chooses to default and parameterizes the degree of contract enforcement in the economy. The benefits of default capture the idea that the entrepreneur can divert for his own benefit some of the firm's assets and output after failure. The benefits are higher when contract enforcement is weak because it is easier and less costly to cheat in economies that are characterized by a less adequate rule of law and a fragile judicial system. The costs of default exemplify reputation costs that transcend into other relations of the entrepreneur after default. For example, the entrepreneur might find it more difficult to restart credit relations with lenders after he has defaulted in the past. The interpretation is that the weaker the contract enforcement, the lower the costs of default because in these economies credit score systems are non-existent and it is easier for defaulters to restart credit relations.[12]

The timing of decisions within the period is as follows. At the beginning of the period, δ fraction of firms exit exogenously. All surviving firms receive the idiosyncratic shock ε.

[12] More generally, V^d can also stand for the entrepreneur's costs of owning the firm that are unrelated to the firm's main operation activities. Economies with weak enforcement are characterized by higher risk of expropriation and lack of compliance in supplier/customer contracts, which make it more costly for entrepreneurs to maintain firms in operation. Thus, weak enforcement in this broader sense also translates into a higher V^d.

An entrepreneur with debt B_R, capital K, and shocks z and ε decides whether to default or not. If the entrepreneur repays his debts, then the firm produces output $z\varepsilon K^\alpha$, and income is equal to $z\varepsilon K^\alpha - B_R$. The entrepreneur next decides on dividends D, capital investment K' for the following period, and a new loan B', given the set of loan contracts offered. If the entrepreneur defaults, he receives the default value V^d and the firm exits. Every firm that exits frees up a project opportunity.

In the recursive formulation of the entrepreneur's problem, the current income x and the permanent productivity level z of the project summarize all the information needed to make production, financing, and default decisions. The entrepreneur's default decision depends on the default value relative to the value of staying in the contract and keeping the firm and is given by

$$V(x,z) = \max\{V^c(x,z), V^d\}, \tag{3}$$

where $V(x,z)$ denotes the present value of the firm and $V^c(x,z)$ the present value conditional on repaying debt today. The entrepreneur's default decision can be represented by a binary variable $d(x,z)$ that equals 1 if default is chosen and 0 if repayment is chosen and is determined in the following way:

$$d(x,z) = \left\{ \begin{array}{cc} 0 & \text{if } V^c(x,z) \geq V^d \\ 1 & \text{otherwise} \end{array} \right\}. \tag{4}$$

Given that $V^c(x,z)$ is increasing in income x and V^d is independent of income, the default decision features a threshold property. There exists a threshold $x^*(z)$ such that $d(x,z) = 1$ for all $x \leq x^*(z)$ and $d(x,z) = 0$ for all $x > x^*(z)$, where $x^*(z)$ is defined by $V^c(x^*(z), z) = V^d$. All else equal, income is higher when productivity is higher, capital is larger, and outstanding debt is small. Thus, default is more likely for firms that have low productivity, small size, and high debt levels.

If the entrepreneur decides to repay his debt, he chooses dividends D, investment for next period K', and a loan contract with the creditor. The loan contract specifies the amount to be received from the creditor this period B' and the amount to be repaid the following period B'_R conditional on not defaulting. Given the set of loan contracts offered, the entrepreneur chooses the new loan, capital, and dividends $\{B', B'_R, K', D\}$ to maximize the value of staying in the contract:

$$V^c(x,z) = \max_{\{B', B'_R, K', D\}} \left(w + \beta(1-\delta) \int V(x'(\varepsilon'), z) g(\varepsilon') d\varepsilon' \right) \tag{5}$$

subject to

$$D = x + B' - K',$$

$$w(x,z) = \left\{ \begin{array}{cc} D & if\ D \geq 0 \\ (1+\gamma)D & if\ D < 0 \end{array} \right\},$$

$$x'(\varepsilon') = z\varepsilon' K'^{\alpha} - B'_R.$$

Firms have three sources of funds: they have the income x they start the period with, they can get loans B' from creditors, and they can raise new private equity D by paying the transaction cost γD. Given the costly equity issuances, all else equal, entrepreneurs prefer to avoid issuing private equity and using income and loans as sources of funds. The optimal debt decision is determined by trading off the costs and benefits of various loans within the set of contracts offered. Debt is beneficial for firms especially when income is low and insufficient to finance the desired investment. In general, the larger the investment requirements, the larger the debt needed. Debt can also be used for dividends, and this is especially attractive when loans are cheap and entrepreneurs discount the future heavily. Finally, debt can be used to avoid issuing private equity when the firm's income is negative. However, debt is costly because it can lead firms to financial distress and default. In particular, a large loan today implies a large repayment the next period that will be costly especially when the productivity shock is low. In this case, income might be so low that the entrepreneur defaults and loses the project. Thus, in anticipation of a possible default, the entrepreneur might find it optimal to reduce his loan the previous period such that default is avoided.

When deciding the optimal debt level, the entrepreneur takes into account not only default possibilities the next period, but also the likelihood of financial distress in future periods. The dynamic character of the debt decision is important because future expensive loans affect today's financing decisions. Our model of firm behavior mirrors the trade-offs present in the savings models of household behavior as in Huggett (1993) and Aiyagari (1994). Precautionary motives that give households incentives to reduce their debt are also present here; firms in our model have incentives to reduce debt to avoid financial distress.

Financing decisions also interact with the size of capital and output of each firm. In a world where contracts are perfectly enforceable but incomplete, the capital stock of each firm is such that the expected marginal product of capital equals the risk-free rate as follows:

$$zE(\varepsilon)\alpha K_{fb}(z)^{\alpha-1} = (1+r)/(1-\delta). \tag{6}$$

However, with enforcement frictions, investment also depends on the set of loan contracts available. In particular, investment can be distorted downward because the set of loans available is limited. For example, if a firm starts with a negative level of income x due to a large outstanding debt, the entrepreneur might want to borrow a big loan B' to avoid the equity costs and to keep the investment level at the unconstrained optimal. However, given that the set of loans is bounded due to a possible default, such a big loan might not be offered to the entrepreneur. Hence, the entrepreneur might have to reduce the level of investment, making the project inefficiently small. The positive relation between investment and income is a generic feature of our model.

Whenever existing firms exit either exogenously or endogenously, the projects are released to the new entrant entrepreneurs. Observing the permanent productivity z, the new entrepreneur decides to enter or not by comparing the expected value of the project with an outside option of zero. We define the value of the project with productivity z conditional on entering by $V^e(z)$. The new entrant starts with no income, and the first period dividend is given by $D = B' - K'$ given the optimal investment K' and loan B'. This implies that the value conditional on entering $V^e(z)$ is exactly equal to the value of the contract $V^c(x, z)$ with $x = 0$.

3.3 Loan Contracts

The contract between the entrepreneur and the creditor consists of a triplet (B', K', B'_R), where B' is the transfer of funds between the firm and the creditor the current period, and B'_R is the transfer the next period. If B' is positive, it represents the payment from the lender to the firm which is used for capital and dividends. B'_R is the associated payment that the firm promises the lender conditional on not defaulting. The contract is designed conditional on the capital input K' because default probabilities the following period are influenced by the level of capital. The entrepreneur can also save with the creditor. If B' is negative, it denotes a payment from the firm to the creditor as savings, and B'_R denotes the gross saving proceeding from the lender to the firm the next period.

Creditors in the model are assumed to be able to commit to loan contracts. They are risk-neutral, competitive, and discount time at the risk-free interest rate r. They behave passively and are willing to finance the firm's financing needs as long as they are compensated for the expected loss in case of default. Default probabilities vary across firms with different permanent productivity levels. Thus, for each firm with productivity z, creditors offer contracts (B', K', B'_R) such that

$$B' = \frac{B'_R(1-\delta)}{(1+r)} \left(1 - \int d(x'(\varepsilon'), z) g(\varepsilon') d\varepsilon' \right), \tag{7}$$

where $x'(\varepsilon') = z\varepsilon' K'^{\alpha} - B'_R$. The lender breaks even in the expected value for every contract, as the effective interest rate required incorporates the default premium consistent with default probabilities.

3.4 Equilibrium

Before defining the equilibrium of this economy, we make an assumption on the relation between interest rates and discount rates. The assumption imposes that the rate at which entrepreneurs discount the future is higher than the risk-free rate.

Assumption 1 The risk-free rate r is such that $1/\beta - 1 > r > 0$.

This condition can be interpreted as a general equilibrium property of economies with lack of enforcement and incomplete markets. If $\beta(1+r) = 1$, firms strictly prefer to accumulate assets rather than distribute dividends because of the additional benefits of assets in terms of avoiding firm failure. This would generate an excessive supply of loans that would in turn drive down the interest rate. We now define the recursive equilibrium:

Definition 1 *The recursive equilibrium for this economy is defined by*

1. *The policy functions of firms* $\{(B'(x,z), K'(x,z), B'_R(x,z)), D(x,z), d(x,z)\}$
2. *The menu of contracts* $(B', K', B'_R)_z$ *offered to firm* z *by the creditors*
3. *The distribution* $\Upsilon(x,z)$ *of firms over* (x,z)
4. *The mass of new entrants* $\xi(\Upsilon(x,z))$

such that:

1. *Taking as given the menu of contracts offered, the policy functions of firms satisfy their optimization problem.*
2. *Contracts available reflect the firm's default probabilities such that creditors break even in expected value.*
3. *The distribution of firms is consistent with individual decisions and shocks.*
4. *The mass of new entrants is equal to the measure of all the firms exiting either exogenously by death shocks or endogenously by defaulting.*

4 Optimal Debt Financing

The novelties of the model are the limited firm-specific borrowing opportunities arising endogenously from lack of enforcement and our emphasis on the firm's precautionary usage of debt. This section precisely studies the trade-offs entrepreneurs face when choosing the firm's optimal levels of debt and how the firm-specific restrictions on borrowing interact with contract enforcement. Risk-free debt is beneficial for entrepreneurs to finance investment and dividends given their impatience and costs of equity. However, entrepreneurs are also reluctant to borrow because large debt increases the future likelihood of financial distress: costly private equity issuance, default premium, and firm failure. Thus, entrepreneurs need to balance the benefits and costs associated with debt financing.

A key element in this trade-off is the endogenous limit on borrowing arising due to limited enforcement of contracts. When borrowing opportunities are abundant, entrepreneurs use debt financing more aggressively than when borrowing is constrained, because the likelihood of financial distress is lower with ample availability in loans. Yet, the set of loans available varies across firms and with the degree of contract enforcement. We show that borrowing capacity relative to assets increases with the size of firms in economies with strong contract enforcement, whereas it decreases with size when enforcement is weak. Thus, in weak enforcement economies, debt financing is especially costly for the small firms because they face the strictest borrowing opportunities due to a high likelihood of default. The endogenous borrowing capacities and the precautionary motive for debt management are the features that allow our model to rationalize the differential relation of debt to asset ratios and size across economies with different degrees of contract enforcement.

4.1 Limits on Borrowing

Lack of enforcement in debt contracts limits the borrowing capacities of firms because creditors do not provide loans that will not be repaid in all future states. The borrowing capacity of a firm in our model depends on the present value of the firm to the entrepreneur relative to the value of default. For simplicity, let's consider the case with no idiosyncratic uncertainty, no death shock ($\delta = 0$), and no equity issuance costs ($\gamma = 0$).

Without uncertainty, default probabilities are either zero or one, and there is no equilibrium default because contracts that will be defaulted upon with probability one, are not offered. Hence, the set of available contracts are offered at the risk-free rate with $B'_R = (1+r)B'$ for $B' \leq \overline{B}(z)$, where $\overline{B}(z)$ is the debt capacity of a firm with productivity z. Moreover, without uncertainty and $\beta(1+r) < 1$, entrepreneurs borrow to the firm's debt capacity because they enjoy having dividends up front. The assets of the firm are thus equal to the level of capital $K_{fb}(z)$, which is constant over time at the first best level as its return is equalized in equilibrium to the constant return on bonds. The value function of a firm with productivity z and current debt repayments B_R is thus equal to

$$V^c(zK_{fb}(z)^\alpha - B_R, z) = [zK_{fb}(z)^\alpha - B_R + \overline{B}(z) - K_{fb}(z)] + \beta V^c(zK_{fb}(z)^\alpha - (1+r)\overline{B}(z), z).$$

Given that more productive firms have larger capital, as long as debt capacity is weakly increasing in productivity, these firms also have larger values. The borrowing limit for a firm with productivity z is the level of debt that makes the contract value equal to the default value, and is given by

$$\overline{B}(z) = \frac{zK_{fb}(z)^\alpha - K_{fb}(z)}{r} - \frac{1-\beta}{r}V^d. \tag{8}$$

Therefore, large and productive firms can borrow more than small and unproductive firms, independent of the degree of contract enforcement in the economy. Also, independent of the productivity, the stronger the degree of contract enforcement (lower V^d), the larger the borrowing capacity for all firms.

In terms of the relation between size and the ratio of debt to assets, what matters is whether debt or capital increases faster with the firm's productivity, which depends crucially on the degree of enforcement. The maximum (also equilibrium) loan relative to capital for a firm with productivity z is

$$\frac{\overline{B}(z)}{K_{fb}(z)} = \frac{1+r-\alpha}{\alpha r} - \frac{1-\beta}{r}\frac{V^d}{K_{fb}(z)}. \tag{9}$$

The following proposition summarizes how the degree of contract enforcement affects the relation between size and leverage ratios.

Proposition 1 *In the case without uncertainty, $\gamma = 0$ and $\delta = 0$, leverage ratios are increasing with firm size when enforcement is weak, $V^d > 0$, and decreasing with firm size when enforcement is strong, $V^d < 0$.*

Proof. See Appendix 2.

The proposition shows that leverage increases with size only in economies with weak contract enforcement, positive V^d, whereas it decreases with size in economies with strong contract enforcement, negative V^d. Moreover, the leverage-size relation is monotonically decreasing with size as the degree of enforcement becomes stronger. Our result can be extended to more general specifications of the default value as long as its absolute value across firms increases less than proportionally to capital.[13]

The default value V^d encompasses both benefits and costs the entrepreneur receives after default. The idea is that in strong enforcement economies, the costs of default can outweigh any benefit from defaulting, which in turn translates into a negative default value. In weak enforcement economies, the benefits from defaulting are large and outweigh any costs from defaulting, which correspond to a positive default value. Good enforcement essentially acts like a lump-sum subsidy on borrowing for all firms. In this example, the subsidy improves the time path of dividends increasing the entrepreneur's welfare. Relative to assets, the subsidy benefits small firms the most, making the debt to asset ratios larger for them. Weak enforcement acts like a lump-sum tax on borrowing for all firms, which reduces welfare by deteriorating the time path of dividends. Relative to assets, the tax is most costly for small firms, which translates into lower ratios for these firms.

Deriving analytical expressions for borrowing capacities and maximum leverage ratios in the case with uncertain productivities is difficult due to lack of analytical solutions for the firm's decision rules of debt and investment. However, all these results regarding borrowing capacities, sizes, and enforcement carry through when we solve numerically the model for the more general case with uncertainty and equity issuance costs.

4.2 Precautionary Debt Financing

In the presence of uncertainty, lack of enforcement affects the set of loan contracts not only by limiting the risk-free loans offered, but also by affecting the interest rate charged on loans that have positive default probabilities. These features of debt contracts influence the firms' optimal debt choices, as they may not find it optimal to exhaust their borrowing opportunities because large debt demands default premium and increases the likelihood of financial distress. Thus, when firms face stochastic productivities, debt is used not only for investment and dividend purposes but also for precautionary purposes in terms of smoothing income and avoiding firm failure and equity costs. As in standard precautionary savings models (Aiyagari 1994; Huggett 1993), firms in our model have incentives to decrease their debt levels and even to build up some buffer stock of savings whenever possible to insure against the costs associated with a possible stream of bad shocks realizations.

This subsection presents the policy rules for debt, dividends, and investment decisions of a representative firm in our model. The detailed calibration is reported in Table 2 in the next section, and the results here are for the firms with the lowest permanent productivity

[13] Warner (1977) presents evidence of economies of scale in the direct costs of corporate bankruptcy for the railroad firms which were in the bankruptcy between 1933 and 1955.

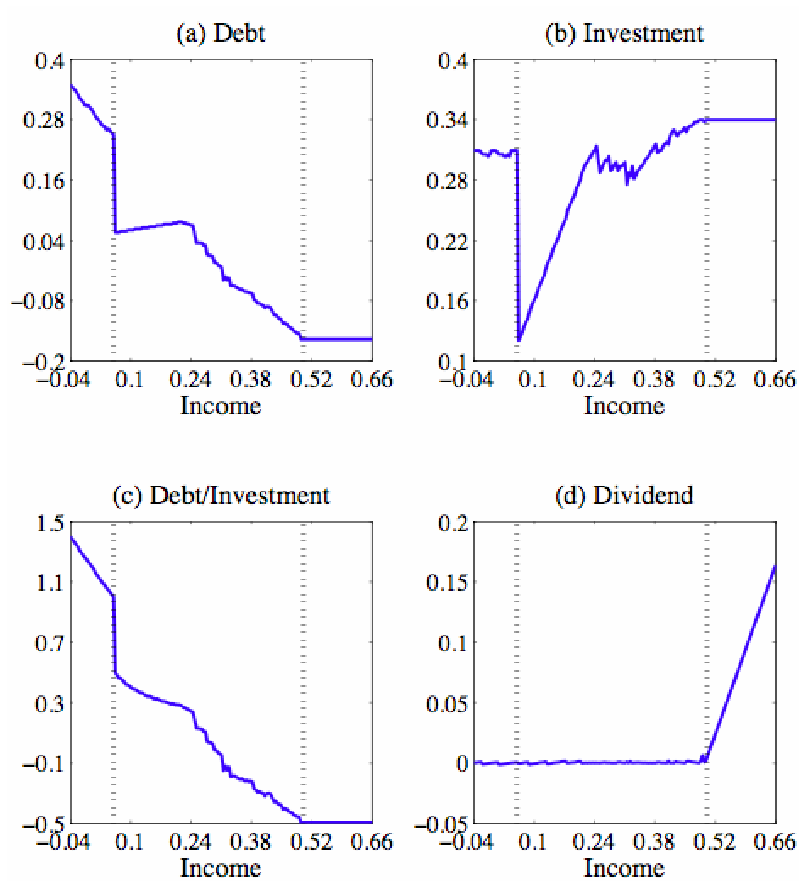

Figure 3: Decision Rules

in the Ecuador model economy.

Let's first look at the debt decision rule B', which is plotted in Panel (a) of Figure 3. Firms with high income do not hold any debt and actually save to insure against future low income shocks; their leverage ratios are equal to zero because they don't have any debt. On the other extreme, firms with low income use debt not only to invest but also to avoid the costs of private equity. Their leverage ratios are above one, as Panel (c) shows. Firms with intermediate levels of income use all their income plus new loans for investment purposes; their leverage ratios are between zero and one.[14] Thus, our model can rationalize the empirical fact that many firms have leverage ratios above one and equal to zero. As illustrated in Panel (c) of Figure 3, our model generates a monotonic negative relation between debt to assets and income as firms try to lower their debt whenever possible due to the precautionary motives.

[14]For this particular parameterization, firms do not use loans for dividends, but in alternative parameterizations, such as one with lower rate of time preference or less volatile shocks, firms use borrowing for dividends when the capital investment is already high enough.

The dividend decision rule is shown in Panel (d) of Figure 3. Only the firms with very high income pay positive dividends, whereas all others pay zero dividends. Paying out dividends is expensive for firms because it implies forgoing the opportunity to reduce debt and to improve the firms' position in the event of a stream of low shocks. Only when debt is low enough (i.e., high income) do firms find it optimal to pay dividends. Our model predicts that firms that distribute dividends are the ones with the lowest leverage ratios, which is consistent with the data in the United States.[15]

Lastly, we show that the precautionary motives also have an impact on the investment decision of firms, as shown in Panel (b) of Figure 3. When income is large enough, firms invest the first best level and also save the optimal buffer stock, while the rest is distributed as dividend. When income is below the sum of the optimal buffer and the first best investment, firms have three options: they can issue private equity, reduce the optimal buffer of savings, or invest a lower amount. Firms are reluctant to issue private equity because of the transaction costs. But firms adjust the other two margins simultaneously as they reduce the investment level and decrease their savings. Firms prefer to lower their investment to prevent an even further reduction in their savings such that they are better equipped to evade financial distress in the future. The reason why the firm is willing to be inefficiently small to avoid a future default is because the expected value of keeping the project $E(V(x'))$ is very large.

When income is low enough, it is no longer optimal for the firm to avoid default the next period because the debt to be repaid is too large. Thus, at very low income levels the entrepreneur anticipates the next period default in case of the low shock and adjusts the firm's investment to a more appropriate scale for only the high shock. This explains the jump in investment around the income level of 0.08. However, the optimal investment in the default region is still smaller than under the first best. The reason is that relative to the first best case, the expected marginal product of capital is lower because of the output loss in the defaulting state, whereas the expected marginal cost of capital is the same regardless of the default decisions because lenders break even.

The precautionary usage of debt in our model generates rich dynamics in the firms' income, assets, and investment. When a firm faces a sequence of good shocks, it decreases its debt, increases its income and investment, and becomes unconstrained over time. The model generates amplification of good shocks and growth for firms that have low levels of income. When a firm experiences a sequence of low shocks, it invests less and produces less and less output while increasing its debt over time. When its debt grows high enough, the

[15] Strebulaev and Yang (2006) document that about 9% of the public firms in the United States from 1962 to 2003 have leverage ratios of zero, and these firms tend to distribute higher dividends.

firm finds it optimal to default and exit. Thus, our model also amplifies bad shocks over time and generates high debt to asset ratios for exiting firms.

Although models of enforcement frictions with a complete set of assets generate underinvestment for firms with large debt levels (Quintin 2000; Albuquerque and Hopenhayn 2004), incomplete markets are essential for enforcement frictions to amplify bad productivity shocks over time. In those models the value of the firm does not go down with negative productivity shocks because state contingent assets allow the firm to pay off debt only when high shocks are realized. However, in our model a sequence of low shocks lowers the value of the firm because uncontingent debt has to grow over time to cover interest payments of an increasingly higher stock of debt. Our interpretation is that enforcement problems are exacerbated for firms that have a history of low productivities.

5 Quantitative Implications of the Model

We now calibrate the model to assess quantitatively our mechanism in reproducing the facts regarding firms' financing, growth, and exit observed in the dataset of firms in the United Kingdom and Ecuador. We first describe the calibration of the model, then present the quantitative results and perform empirical tests to provide further evidence for our mechanism.

5.1 Calibration

The calibration strategy we adopt here is as follows. We first parameterize the model to the Ecuadorian economy, and then recalibrate only the default value for the UK economy. These two economies are thus the same in terms of all the parameters except the default value V^d. This strategy isolates the impact of contract enforcement, which is the focus of this work.

The following parameters are chosen independent of our model. The interest rate r is set at 4% per annum. The decreasing returns parameter α is chosen to be 0.90 following the empirical estimates by Basu and Fernald (1997). The death rate δ is calibrated to the exit rate in the largest asset quintile of 3% in Ecuador. The equity issuance cost parameter γ is set to 0.3 following Cooley and Quadrini (2001).

All other parameters are calibrated such that our model produces relevant moments of the Ecuadorian firm dataset. We discretize the productivity space into two transitory shocks and five permanent shocks. The permanent z shocks have equal mass and are chosen to generate five first best capital levels, defined in Equation (5), equal to the mean asset levels of quintiles in the data. Without loss of generality, we assume that transitory shocks have a mean of one. Under this assumption, the remaining four parameters – the spread of the idiosyncratic transitory shocks ($\varepsilon_H - \varepsilon_L$), the probability of the low transitory shock p_L, the discount factor β, and the enforcement parameter V^d – are jointly estimated such that the model matches the following four moments in the data: the average sales growth rate of 63%, the average coefficient of variation for sales across firms of 0.35, the mean leverage ratio of 0.60, and the leverage ratio of the first asset quintile of 0.40.[16]

To study the United Kingdom, we only recalibrate the enforcement parameter V^d such that the model generates the average leverage ratio of the first asset quintile in the distribution of British firms. The value for V^d in Ecuador is calibrated to be 0.03, which corresponds to about 1% of average first best output of firms in the first quintile ($0.01 z_1 K_{fb}^{\alpha}$). The default value in the UK is -0.01, which corresponds to a loss of 0.3% of average first best output of firms in the first quintile ($-0.003 z_1 K_{fb}^{\alpha}$). Table 2 summarizes all the parameters.

Table 2. Parameter Values

		Value	Target
Interest rate	r	0.04	Annual US real interest rate
Technology	α	0.90	Basu and Fernald (1997)
Death rate	δ	0.03	Exit rate of the 5th quintile
Equity issue cost	γ	0.3	Cooley and Quadrini (2001)
Permanent prod.	$z_1, z_2, ...z_5$	1.0, 1.17, 1.33,1.49, 1.83	Mean asset level of each quintile
Temporary prod.	$\varepsilon_L, \varepsilon_H$	0.11, 1.13,	Mean sales coefficient of variance
	p_L	0.14	Mean sales growth rate
Discount factor	β	0.956	Mean leverage
Enforcement	V^d	0.03 for Ecuador	Ecuador leverage of 1st quintile
		-0.01 for the UK	UK leverage of 1st quintile

5.2 Quantitative Results

We simulate the model over 500 periods for 15,000 firms for both calibrations to Ecuador and the UK. The model delivers in the long run a stationary cross section distribution of firms, which we use to compute the model's statistics presented in Table 3. At every point in time, we divide the cross section of firms into five quintiles based on assets as in the data. In the model, firms' assets equal the firm capital K plus savings B_R for firms with $B_R < 0$. The table shows that the model reproduces the highly skewed asset distribution of firms,

[16]The coefficient of variation for sales is computed from the detrended time series of real sales of each firm for 1996-1999.

which is similar in the two economies. We compute for every asset quintile and for the entire distribution of firms average leverage ratios, sales growth rates, and exit rates. Table 3 also reports averages of these statistics across the last 100 periods for both calibrations.

Table 3. Model Results

Ecuador	Assets	Leverage	Growth	Exit Rate
Overall	99.11	0.59	0.71	0.03
Quintile 1	1.00	0.31	0.77	0.05
Quintile 2	4.88	0.51	0.71	0.03
Quintile 3	17.17	0.67	0.70	0.03
Quintile 4	52.82	0.72	0.69	0.03
Quintile 5	419.69	0.74	0.68	0.03
United Kingdom	**Assets**	**Leverage**	**Growth**	**Exit Rate**
Overall	101.03	0.82	0.72	0.03
Quintile 1	1.00	1.05	0.75	0.04
Quintile 2	4.89	0.79	0.74	0.03
Quintile 3	17.27	0.77	0.73	0.03
Quintile 4	52.87	0.76	0.72	0.03
Quintile 5	429.64	0.71	0.68	0.03

Let's first look at the model implications regarding firms' financing patterns. The calibration to the average leverage ratio in Ecuador is successful, as the model's statistic of 0.59 is very close to 0.57 in the data. Our UK model economy delivers a higher average leverage ratio of 0.82, which matches closely the ratio of 0.76 observed in the data. With stronger contract enforcement, the set of available loans in the British economy is much larger than that in Ecuador. For example, the maximum loan relative to the first best capital for a firm in the lowest productivity bin equals 2.4 in the UK, whereas it is only 1.3 in Ecuador. Furthermore, in equilibrium, firms in the UK borrow more not only because of the greater borrowing opportunities, but also because this gives them less incentive to engage in precautionary savings.

The model produces an upward size-leverage relation for Ecuador and a downward size-leverage relation for the United Kingdom (see Figure 4 for the contrasting patterns). Moreover, the model matches quantitatively the patterns in the data. For example, for the smallest quintiles the leverage ratio in Ecuador is much smaller than that in the UK, 0.31 versus 1.05, whereas leverage ratios of large firms are similar in both countries. The key mechanism of the

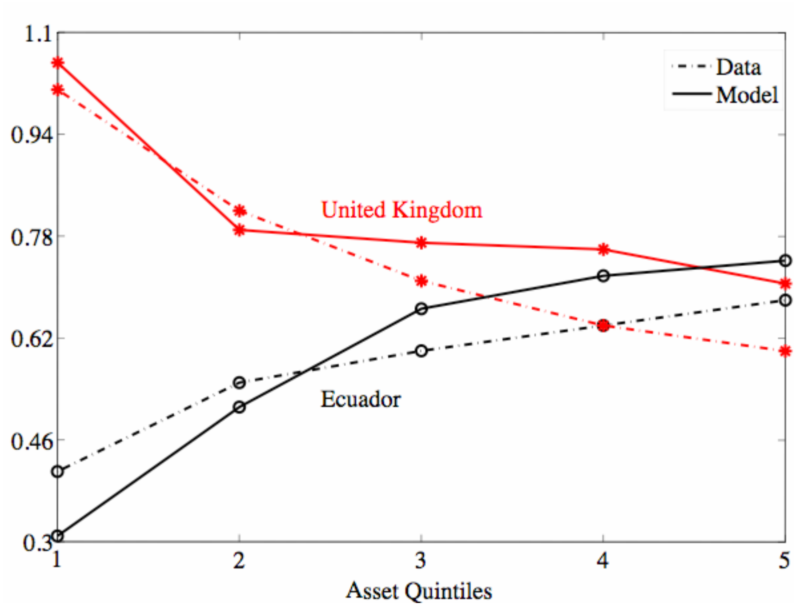

Figure 4: Size and Leverage in the Model

model that generates the contrasting size-leverage relations is how the borrowing capacity of firms varies with size under different degrees of contract enforcement. In weak enforcement economies, the borrowing capacity relative to assets is more limited for small firms than for large firms, whereas the opposite is true in strong enforcement economies, as shown in Proposition 1. Although in equilibrium firms do not borrow the largest available loan, the observed borrowing levels mirror the patterns of borrowing constraints because firms endogenize these limits when making financing decisions.

We next look at the model implications on firm dynamics. Both model economies generate that small firms grow faster than large firms qualitatively as in the data. However, quantitatively, the model underestimates the differences across firms in both countries. To generate the negative size-growth relation, the model requires that firms in lower asset bins are on average of less efficient scales in the limiting distribution. Sales growth rates of more inefficient firms are higher because they generally hold larger debt and under good shocks can reduce their debt and increase their capital toward the efficient level. However, exactly to avoid the above inefficiency firms borrow less and engage in precautionary savings. Thus, the model's limiting distribution has only a small fraction of firms with inefficient scales even in the lowest asset bins, which translates into small differences in sales growth across quintiles. Yet the model suggests that lack of contract enforcement can be a more predominant factor for the negative size-growth relation in the transition periods toward the stationary distribution. For example, in the first period of the UK economy when all firms are new entrants, the average growth rate of small firms is 0.80, whereas the average growth rate of large firms is 0.61.

In terms of exit rates, in the Ecuadorian model economy, small firms have higher exit rates than large firms as in the data. However, the model's exit rate for the first asset quintile of 5% is lower than that of 12% in the data. Firms in larger asset bins in Ecuador and all firms in the UK do not default in equilibrium. The reason for the relatively low default rates is that firms' debt management allows them to avoid failure. Firms accumulate assets and reduce their debt whenever possible to prevent default.[17] However, our model shows that the degree of contract enforcement does affect default rates. In an economy with weak enforcement, firms are constrained in their financial needs such that the leverage ratios are low and they default after a sequence of bad shocks. These same firms would remain in operation in an economy with strong enforcement because of the larger loans firms can use to respond to shocks.[18]

5.3 Welfare and Output Implications

Differential enforcement in contracts also impacts the level of output of firms and more importantly the welfare of entrepreneurs. In general, when asset markets are incomplete, the greater the borrowing opportunities, the better are the equilibrium allocations because firms have better tools to insure against fluctuations and failures. Now we illustrate quantitatively the welfare and output differences in our model economies. Given that welfare comparisons are very sensitive to the level of assets, we control for the distribution of income in our model and focus on the case where all firms in the two economies have the same level of income to isolate the role of contract enforcement.

Table 4. Welfare and Output Implications

Ratio UK/Ecuador	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
Welfare	1.087	1.019	1.006	1.001	1.00
Output	2.38	1.09	1.08	1.02	1.00

[17] Unfortunately, in our data we do not have information on the reasons of exit. As a reference point, in the United States the average exit rates for firms is about 4.5%, whereas the average bankruptcy rate is about 1%, as reported in Cooley and Quadrini (2001).

[18] This is consistent with evidence presented in Desai, Gompers, and Lerner (2003), who show that for European countries, better legal institutions and protection of property rights lead to lower firm exit rates.

Table 4 first reports the ratio of welfare in the UK model economy to that in the Ecuador economy. We find that lack of enforcement in contracts has significant welfare implications. First, welfare is higher for all entrepreneurs in the UK economy than for entrepreneurs in the Ecuador economy. Second, the difference in welfare is larger for entrepreneurs holding small firms. An entrepreneur in Ecuador with a firm whose assets lie in the first quintile of the distribution would get an equivalent of 8.7% higher dividends each period if the enforcement technology were similar to the one in the UK. The welfare difference diminishes for large firms because their borrowing opportunities become similar across the two countries. Furthermore, the welfare differences across economies are robust to different levels of firms' income.

Table 4 also shows the output differences across countries. We find that different degrees of contract enforcement can lead to substantial differences in the output of the small firms. Small firms in the UK economy can produce 2.4 times as much as small firms in Ecuador when the income of firms equals 15% of the first best average output of the lowest productivity firms ($x = 0.15 z_1 K_{fb}^{\alpha}$). The reason is that when income is low, firms in the UK can borrow loans to finance the efficient investment, whereas firms in Ecuador have to underinvest due to limited borrowing capacities. In contrast to the robust differences in welfare, the output differences across economies is sensitive to the income of firms: when income is large, firms in both countries produce similar levels of output, and when income is low, the weaker enforcement economy produces lower output. For our parameterization, when income is above 113% of the first best average output of the lowest productivity firms, the difference in output between the two economies disappears.

Another way that lack of contract enforcement can affect the economy's output and welfare is by restricting the creation of new firms. For example, in our economies, when the initial income of firms is negative and between $-0.1 z_1 K_{fb}^{\alpha}$ and $-1.3 z_1 K_{fb}^{\alpha}$ the small z_1 firms start their operation in the UK but never get started in Ecuador.[19] Although in our model we have assumed zero entry costs for firms, negative initial income can be interpreted as entry costs. Thus, our model implies that when entry costs are large, weak enforcement can severely limit the creation of new firms, which lowers the economy's output and welfare.[20]

5.4 Further Empirical Tests

Our model has several additional testable implications regarding the determinants of the firms' debt to asset ratios and default decisions. In the model, leverage and default are solely

[19] The evidence presented in Desai, Gompers and Lerner (2003) also shows that countries with better legal institutions and protection of property rights are associated with higher entry rates in Europe.

[20] Djankov et al. (2002) document that in 1999 the cost of establishing a new firm equals 3% of GDP per capita (or $680) in the United Kingdom, whereas in Ecuador the cost is 91% of GDP per capita (or $1,200).

determined by the firm's income and productivity. Firms with productivity z choose high leverage ratios when their income is low, as shown in Panel (c) of Figure 3. Firms choose to default if the income is below a threshold that varies according to z, that is $d(x,z) = 1$, if $x < x^*(z)$. Although income in the default state is a latent variable that is not observed in equilibrium, it is highly correlated across time, with an autocorrelation of 0.86 in both model economies. Thus, exit of a firm with productivity z at time t can be predicted using the last period's income x_{t-1}.

To test these implications using our dataset, we need to find empirical counterparts for the productivity and income of firms. Income x in our model resembles firms' profits in the data. Thus, we proxy x with profits, and to eliminate scale effects we use the ratio of profits to assets. The productivity of the firm z is tightly linked in the model to the size of the firm; thus, we proxy z with sales, assets, and liabilities.

We first compute the simple correlation between profit to asset ratios and leverage ratios in the data. The correlations in both countries are negative: -0.2 in Ecuador and -0.01 in the UK. These correlations are consistent with the model's prediction that high income firms choose lower leverage ratios. We next estimate a linear OLS regression of leverage on income and other control variables. The results in Table 5 show that in both countries, firms with lower profit to asset ratios have larger debt to asset ratios the consecutive year even when all the size measures are introduced as controls. The coefficient on the profit variable is significant and negative at the 1% level in both countries.[21] Moreover, size and profits can explain a large portion of the variability in leverage ratios of firms in the data, as shown by the R^2 of 0.39 in Ecuador and 0.33 in the UK.

Table 5. Linear Regressions of Leverage[22]

	p_t/A_t	$\log(A_t)$	$\log(B_t)$	$\log(S_t)$	Obs.	R^2
Ecuador	-0.268***	-0.199***	0.193***	0.002	11889	0.39
	(0.011)	(0.003)	(0.003)	(0.002)		
United Kingdom	-0.001***	-0.554***	0.574***	-0.062***	139605	0.33
	(0.0002)	(0.002)	(0.002)	(0.002)		

[21] This is consistent with Fama and French (2002), who find that within publicly traded firms in the United States, firms with lower earnings to assets ratios have higher leverage.

[22] The data are for all firms continuing in two consecutive years: for Ecuador the years are 1996 and 1997, and for the United Kingdom the years are 2000 and 2001. The variable p_t/A_t corresponds to profit to asset. $\log(A_t)$, $\log(B_t)$, and $\log(S_t)$ denote logged assets, liabilities, and sales, respectively. Standard errors are in parentheses, and *** denotes significance of the coefficient at 1% level. The regressions include a constant.

The final policy rule we want to test with our data is exit. Our model predicts that conditional on z, firms with lower income x and higher debt to asset ratios are more likely to default and exit, and that this effect is more severe for firms with small z. The predictions are broadly consistent with the characteristics of exit firms in Ecuador in that they have larger leverage ratios, lower profits, and are smaller than the surviving firms. Firms who exit in 1997 have an average leverage ratio of 0.70 and an average profit to asset ratio of -0.13 in 1996. In contrast, surviving firms have an average leverage ratio of 0.59 and an average profit to asset ratio of zero. Moreover, firms that exit in 1997 are on average smaller, as about 60% of them lie in the lowest two asset quintiles of the distribution of firms in 1996.

Table 6. Logit Exit Regression on Predictor Variables[23]

B_t/A_t	p_t/A_t	$\log(A_t)$	$\log(S_t)$	$\log(B_t)$	Obs.	Pseudo R^2
0.371***	-0.160***	-0.185***	-0.096***	0.025	13607	0.038
(0.085)	(0.058)	(0.048)	(0.025)	(0.039)		

Following Campbell, Hilscher, and Szilagyi (2006), we also estimate the probability of exit over the next period using a logit model to provide further evidence for the mechanism in our model that income predicts exit after controlling for additional variables. We assume that the marginal probability of exit follows a logistic distribution and is given by $P_{t-1}(d_{i,t} = 1) = 1/(1 + \exp(-\alpha - \beta X_{i,t-1}))$, where $d_{i,t}$ is an indicator that equals one if the firm exits in year t and $X_{i,t-1}$ is a vector of explanatory variables known at the end of the previous year. A higher $X_{i,t-1}$ implies a higher probability of exit. Table 6 presents the results. As predicted by the model, firms with higher debt to asset ratios, lower profits, and that are small in terms of assets and sales have a higher probability of exit.[24] In addition, the higher likelihood of exit for firms with larger leverage ratios and lower profits remains significant after including additional control variables that are significant in predicting exit such as the firm's age and the level of cash on hand.

[23]The data are for all firms in Ecuador in 1996. The dependent variable takes a value of 1 if the firm exits in 1997 or zero otherwise. The independent variables are debt to asset ratio B_t/A_t, profit to asset ratio p_t/A_t, log of assets A_t, log of sales S_t, and log of liabilities B_t. Standard errors are in parentheses and *** denotes significance of the coefficient at 1% level.

[24]This is consistent with results presented in Campbell, Hilscher, and Szilagyi (2006) for US firms that fail and go bankrupt. Public US firms with low net income relative to assets, high leverage ratios, and small share of all assets (small firms) are more likely to fail.

6 Conclusion

This paper studies how the degree of contract enforcement affects firms' financing choices. We start by documenting the empirical disparities of firms' financing choices in the United Kingdom and Ecuador, where the United Kingdom has much better enforcement in financial contracts than Ecuador. We find the stark contrasting patterns of firms' debt financing; small firms use debt more in their total financing than large firms in the United Kingdom, whereas the opposite is true in Ecuador. Besides this new finding, our work also documents empirical facts of firm dynamics regarding relations between growth, exit, and size for these two countries.

We next construct a theoretical model with heterogeneous firms to establish the link between the degree of enforcement in financial contracts and firms' debt financing. This model features the firm-specific borrowing capacities arising endogenously from lack of contract enforcement and the precautionary usage of debt. This model offers a novel dynamic trade-off theory on capital structure of firms with endogenous entry, exit, and borrowing constraints. Debt is useful for investment and dividend purposes, but high debt levels also increase the likelihood of firm failure when asset markets are incomplete and borrowing is limited. Thus, firms use debt with precaution and the optimal debt level is determined by trading off benefits and costs under their specific borrowing constraints.

More importantly, this work rationalizes the differential relation of debt to assets ratios and size across economies with different degrees of contract enforcement. In economies with good enforcement, default is unlikely and loan availability is large for all firms. The ease of loans is larger for small firms relative to their size, which further reduces their precautionary use of debt. Thus, the debt to assets ratio of small firms is higher than that of large firms. In economies with weak enforcement, loans are limited because of high probabilities of default, and firms have strong incentives to decrease debt and accumulate savings to insure against firm failure. Small firms use debt more conservatively given their higher likelihood of failure because their values are lower and more volatile. Thus, the debt to asset ratio of small firms is lower than that of large firms.

This work makes an effort to quantify the link between contract enforcement and firm financing choices. We calibrate the model and show that our mechanism can account for the patterns of capital structure observed in the data. At the same time, this work produces firm dynamics consistent with the empirical findings. We further test two key model implications using our dataset and find supports for our mechanism.

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Appendix 1

This appendix presents various robustness checks on the empirical finding of the size and leverage relations in Ecuador and the UK. First, we investigate the size-leverage relation for other years in the data samples of the two countries, which is reported in Table 7. There are small variations in the overall leverage ratios over years in both countries. However, the positive relation and the negative relation between size and leverage are robust for all the years.

Table 7. Robustness Check 1: Various Years

	Ecuador			United Kingdom		
Quintile	1997	1998	1999	2001	2002	2003
1	0.43	0.44	0.50	1.05	1.07	1.07
2	0.53	0.55	0.56	0.84	0.86	0.88
3	0.59	0.60	0.61	0.72	0.74	0.77
4	0.63	0.64	0.66	0.64	0.65	0.67
5	0.67	0.67	0.72	0.60	0.61	0.61
Overall	0.57	0.58	0.61	0.77	0.78	0.80
Obs.	21,567	21,920	23,509	765,611	866,615	1,020,462

Second, we explore the size-leverage relation for firms in three major sectors in both economies: manufacturing, service, and wholesale/retail. The result is presented in Table 8. Overall, leverage varies across sectors in both countries: the service sector has the lowest leverage in Ecuador, whereas the manufacturing sector has the lowest leverage in the UK. The positive size-leverage relation holds for all three sectors in Ecuador, whereas the manufacturing sector has less variation in leverage across quintiles than the service and wholesale/retail sector. The negative size-leverage relation also holds for all three sectors in the UK. The results for other years have similar patterns as reported here and are available upon request.

Table 8. Robustness Check 2: Various Sectors

	Ecuador (1996)			United Kingdom (2000)		
Quintile	Manuf.	Service	Wholesale/Retail	Manuf.	Service	Wholesale/Retail
1	0.55	0.31	0.40	1.12	1.21	1.25
2	0.59	0.44	0.55	0.74	0.86	0.84
3	0.58	0.46	0.59	0.64	0.73	0.70
4	0.60	0.51	0.62	0.59	0.65	0.64
5	0.61	0.65	0.68	0.59	0.61	0.63
Overall	0.59	0.47	0.57	0.74	0.81	0.81
Obs.	2,853	6,662	1,076	69,739	296,272	78,572

Third, we experiment with a different size measurement: sales. We rank firms by their sales to construct five quintiles and report the leverage ratios for each quintile in both countries for various years in Table 9. The size-leverage relation identified in both countries is independent of size measurement of sales or asset and the patterns are similar across years. Lastly, we also show the size-leverage relations are robust to 20-quantiles in both countries. Interestingly, the differential financing behavior of the small firms across the two countries is maintained when comparing them in terms of their dollar values level of assets. The leverage ratios and median assets statistics are reported in Table 10.

Table 9. Robustness Check 3: Sales as Size

	Ecuador				United Kingdom			
Quintile	1996	1997	1998	1999	2000	2001	2002	2003
1	0.45	0.46	0.47	0.49	0.86	0.88	0.88	0.90
2	0.59	0.60	0.62	0.63	0.89	0.91	0.89	0.88
3	0.64	0.64	0.64	0.67	0.85	0.85	0.86	0.87
4	0.67	0.66	0.67	0.71	0.81	0.84	0.86	0.86
5	0.66	0.65	0.67	0.73	0.74	0.75	0.75	0.76
Overall	0.60	0.60	0.61	0.64	0.83	0.84	0.85	0.85
Obs.	13,708	14,363	15,017	16,717	181,841	192,340	203,190	218,039

Table 10. Robustness Check 4: 20-Quantiles

	Ecuador (1996)		United Kingdom (2000)	
Quantile	Asset (thousand $)	Leverage	Asset (thousand $)	Leverage
1	1.31	0.35	2.12	1.07
2	2.65	0.33	6.68	1.06
3	4.62	0.46	12.87	0.96
4	7.52	0.49	19.57	0.94
5	11.32	0.55	28.59	0.88
6	16.23	0.56	40.60	0.85
7	22.43	0.55	57.08	0.81
8	30.41	0.53	80.30	0.77
9	40.56	0.58	112.62	0.74
10	55.08	0.60	156.99	0.71
11	74.29	0.60	218.07	0.70
12	102.70	0.61	302.34	0.67
13	145.24	0.62	418.97	0.65
14	203.86	0.62	586.76	0.65
15	292.23	0.65	842.91	0.62
16	436.68	0.66	1,255.26	0.63
17	675.24	0.68	1,968.42	0.61
18	1,161.76	0.69	3,334.43	0.59
19	2,222.57	0.69	7,046.46	0.59
20	15,705.33	0.66	347,651.45	0.61

Appendix 2

In this appendix, we provide the formal proof for Proposition 1. We first show the features of firms' optimal debt and investment decisions in Lemma 1. Investment is always at the first best level, and debt is always at the maximum loan allowed under the risk-free rate at the steady state. Then the proof of Proposition 1 follows directly from Lemma 1.

Lemma 1. In the case without uncertainty, $\delta = 0$, and $\gamma = 0$, for each firm with permanent productivity z, the optimal decisions in the steady state are given by

$$K'(\overline{x}, z) = K_{fb}(z) \text{ and}$$

$$B'(\overline{x}, z) = \frac{zK_{fb}(z)^{\alpha} - K_{fb}(z) - (1-\beta)V^{d}}{r}$$

under the assumption that $z > (1+r)\left((1-\beta)V^{d}\right)^{1-\alpha} \alpha^{-\alpha}(1+r-\alpha)^{\alpha-1}$ for all z.

Proof. Consider the case of no uncertainty with firms differing in size because of permanent differences in productivity z. Furthermore, we assume zero equity issuance cost $\gamma = 0$ and no death shock $\delta = 0$ for simplicity. In this case, the bond price that firms face will be either the inverse of the risk-free rate or zero because the firms either repay with probability 1 or default with probability 1 the next period with no uncertainty. So firms will pick only the contracts that they will not default on next period, that is, $V^{c}(x', z) \geq V^{d}$. We could rewrite the firm's problem as follows:

$$V^{c}(x, z) = \max_{\{B', K'\}} \left(x + B' - K' + \beta V^{c}(x', z)\right)$$

subject to

$$x' = zK'^{\alpha} - B'(1+r),$$

$$V^{c}(x', z) \geq V^{d}.$$

The first order conditions with respect to K' and B' are given by

$$1 = (\lambda + \beta)V^{c}_{x'}(x', z)\alpha z K'^{\alpha - 1}$$

$$1 = (\lambda + \beta)V^{c}_{x'}(x', z)(1+r)$$

where λ is the Lagrangian multiplier on the inequality constraint. From the above two first order conditions, clearly the optimal investment choice is $K_{fb}(z)$. Under $\beta(1+r) < 1$, firms always borrow to the limit. Thus, in the steady state we have $x(z) = x'(z) = \overline{x}(z)$ and $B(\overline{x}(z), z) = B'(\overline{x}(z), z) = \overline{B}(z)$.

Plugging in the optimal investment and the optimal debt policies, we have

$$V^c(\overline{x}(z), z) = [zK_{fb}(z)^\alpha - K_{fb}(z) - r\overline{B}(z)] + \beta V^c(\overline{x}(z), z).$$

Therefore,

$$V^c(\overline{x}(z), z) = \frac{zK_{fb}(z)^\alpha - K_{fb}(z) - r\overline{B}(z)}{1-\beta}.$$

At the borrowing limit, we have $V^c(\overline{x}(z), z) = V^d$. This means

$$\overline{B}(z) = \frac{zK_{fb}(z)^\alpha - K_{fb}(z)}{r} - \frac{1-\beta}{r}V^d.$$

Under the assumption that $z > (1+r)\left((1-\beta)V^d\right)^{1-\alpha}\alpha^{-\alpha}(1+r-\alpha)^{\alpha-1}$, $\overline{B}(z)$ is positive for any z and thus it is a constraint on borrowing. Note that if the above assumption is not satisfied for some z, $\overline{B}(z)$ will be zero, i.e., no borrowing is allowed, which is not an interesting case for our purpose. Finally,

$$\frac{\overline{B}(z)}{K_{fb}(z)} = \frac{1+r-\alpha}{\alpha r} - \frac{1-\beta}{r}\frac{V^d}{K_{fb}(z)}.$$

The proof of Proposition 1 follows directly from Lemma 1, since $K_{fb}(z)$ is strictly increasing in z.