For all problems, *SHOW ALL OF YOUR WORK*. Partial solutions and problems with missing steps will be marked wrong. Continue your work on the back of the page or extra sheet at the end of the exam if you need additional space. *You do not need but may use the normal graphing calculator functions of any graphing calculator, but not any differential equations functionality it may have.*

1. For each of the following differential equations, two solutions to the complementary homogeneous problem are given. Find the general solution for each.
   a. $y'' + 3y' + 2y = 3x$, $y_1(x) = e^{-2x}, y_2(x) = e^{-x}$. (10 points)

   b. $y'' + 4y' + 4y = 2e^{-2x}$, $y_1(x) = e^{-2x}, y_2(x) = xe^{-2x}$. (10 points)

2. Find all equilibrium solutions to the system

$$x' = x(3-x) - 2xy$$
$$y' = y(1-y) + xy.$$

   In what direction are solutions moving when $(x, y) = (1, 2)$? How is this related to a direction field for the system? (6 points)

3. The charge on a capacitor, $Q(t)$, in a simple LRC circuit is given by $LQ''(t) + RQ'(t) + \frac{1}{C}Q(t) = E(t)$, where L, R, and C are the inductance, resistance and capacitance of the inductor, resistor and capacitor in the system. Suppose that $R = 1\text{k}\Omega$, $C = 20\mu F$, $L = 0.35\text{h}$, and that the system is forced with an applied charge $E(t) = A\sin(\omega t)$.
   a. For what ω, if any, will this system exhibit resonance? Explain. (8 points)

   b. Now suppose $R = 0$ (the resistor is removed). If $\omega = 120\pi$ and $A = 117$, find the charge $Q(t)$ if $Q(0) = Q'(0) = 0$. Be sure that it is clear why you proceed as you do and how you arrive at your answer. Will your solution exhibit resonance, beats, or neither? Why? (10 points)

4. Suppose that the populations of two interacting species are given by the system

$$x' = x(1 - 2x) + xy$$
$$y' = y(2 - y) + xy$$

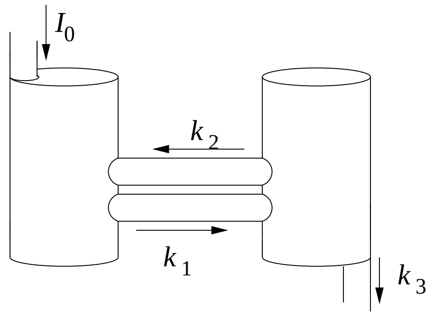
Use Euler's method with $h = 0.1$ to approximate $x(0.2)$ and $y(0.2)$ if $x(0) = 1$ and $y(0) = 1.5$. (12 points)

5. If $\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$, and $c = 4$, find $\mathbf{A}\mathbf{v} - c\mathbf{v}$. (5 points)

   a. Is this an eigenvalue problem? Explain. (5 points)

6. Solve the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ if $\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$. (12 points)

7. Consider the crude model of the lungs and blood shown in the figure to the right. If x_1 is the amount of some toxin in lungs (which comes from inhaling the toxin at a rate I_0), x_2 is the amount of that toxin in the bloodstream, and the constants k_j are the constants of proportionality for the indicated transfers of the toxins, the model predicts that



$$\begin{aligned} x_1' &= -k_1 x_1 + k_2 x_2 + I_0 \\ x_2' &= \quad k_1 x_1 - (k_2 + k_3) x_2. \end{aligned}$$

a. Explain why this makes sense. (5 points)

b. Rewrite the system in matrix notation. (5 points)

c. If $I_0 = 60$, $k_1 = k_2 = 2$ and $k_3 = 3$, solve it. (12 points)