Modeling, Team Based Computer Lab Materials in Differential Equations: Implementation and Outcomes

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Preface: On Learning

- And Teaching
- What Matters:
  - Student engagement
    [Freeman, et al.; CBMS report; Laursen, et al.]
  - Student interaction [Laursen, et al.]

- In this talk, we look mostly at differential equations computer labs.
  - Technology may: aid exploration, allow computation, facilitate communication, provide assessment, and/or motivate students
    [CUPM curriculum guide]
  - *and, we assert, "works" by promoting engagement and interaction*.
    - *. . . which has implications for instructional effectiveness and assessment.*

Context

- Calculus II prereq. differential equations course
  - Format: Large lecture (100 student; 3$\times$50 min), Smaller lab (25 student; 1$\times$50 min).
  - 4 credit.
  - Students (fall 2016):
    75–80% Engineers ($\sim$10% math majors. . .)
    20% first-year, 60% second year
    70% male.
  - Lecture: highly instructor dependent; some technology, lots of blackboard.
    - *. . . limited opportunities for engagement and interaction*
      $\rightarrow$ *focus on labs.*

Differential Equations Labs

- Labs: 25 students, 1×50 min/wk, graduate student instructor.
- Original implementation from early 90s.
  - Using *MATLAB*—all engineering students take *MATLAB* programming course, Engin 101.
- By 2016: labs & recitations, perception that labs are disconnected from course, largely point-and-click...
- Challenges:
  - Connection with course.
  - (Limited) Available time.
  - Student affect and expectations.

Course Revision: Goals

- ~~Reduce class sizes to 18!~~
- Improve student learning:
  Increase student engagement.
- Extend material and students' understanding of connections to different course material, and other mathematics courses.
- Improve connection between labs and course.
- Update the course as a whole:
  Make it more conceptual, and more "modern."
- Center lab materials on "real-world" applications.

Course Revision: Changes

- In summer 2016: full course revision:
  - New text, greater dynamical systems emphasis, improved topic sequencing.
  - More demanding homework and exams.
- . . . and New labs
  - Rewritten largely by a post-doc over the summer, to be
  - More demanding, with significant mathematical content, and
  - Strongly application based.
  - *Made possible by a MathWorks Grant*.
- With labs held in renovated lab space.
  - *Made possible by the College*.

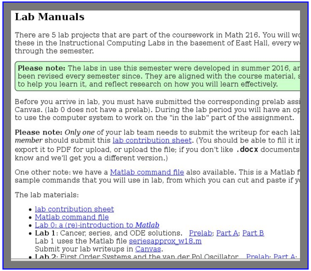
The New Labs

- Topics
  0. Introduction to *Matlab* (model: sprinter's velocity);
  1. Series approximations, of solutions, and to linearize equations (cancer tumor growth);
  2. Systems and phase planes, linearization and the difference between linear and nonlinear behavior (van der Pol circuit);
  3. Linearization, 2nd order equations, forced behavior (laser intensity);
  4. Numerical methods and error, stiffness (RLC circuit with impulse forcing);
  5. Nonlinear behavior, bifurcations, and chaos (Lorenz equations).
- Schedule

Monday	Tuesday	Wednesday	Thursday	Friday
1/1 No class	1/2 No class	1/3 First class: *Intro, phase line, direction fields* B&B 1.1–1.3	1/4 No lab	1/5 *Separable equations* B&B 2.1
1/8 *First-order linear equations* B&B 2.2	1/9 Lab 0	1/10 *Modeling & applications* B&B 2.3	1/11 Lab 0	1/12 *Existence & uniqueness* B&B 2.4 WebHW0 & 1 due
1/15 MLK Day No class	1/16 Lab 1 prelab due Lab 1 workday 1	1/17 *Autonomous equations, phase line* B&B 2.5	1/18 Lab 1 prelab due Lab 1 workday 1	1/19 *Systems of 2 linear algebraic equations* B&B 3.1 WebHW2 due
1/22 *Systems of 2 linear algebraic equations* B&B 3.1, cont.	1/23 Lab 1 workday 2 Written HW 1 due	1/24 *Systems of 2 linear DEs* B&B 3.2	1/25 Lab 1 workday 2 Written HW 1 due	1/26 *Theory of systems of 2 linear DEs* B&B 3.2, 3.3 WebHW3 due
1/29 *Theory of systems* B&B 3.3	1/30 Lab 1 writeup due Lab 2 prelab due Lab 2 workday 1	1/31 *Systems with complex eigenvalues* B&B 3.4	2/1 Lab 1 writeup due Lab 2 prelab due Lab 2 workday 1	2/2 *Systems with repeated eigenvalues* B&B 3.4, 3.5

Lab Structure

- Heavy investment in collaborative, engaged work
  - . . . while trying to retain student accountability
- Two-week cycle
  - Week 1:
    - Lab writeup from previous lab due; Pre-lab due (familiarize students with material, some math, checked for presence—2 pts);
    - Part A (1st workday) work (pairs within a team of four work on complementary parts of first half of material; pairs present to or collaborate with each other at the end of the period or start of next);
  - Week 2:
    - Part B (2nd workday) work (groups of four work together on remainder of lab; group is responsible for lab writeup—8 pts)



Lab Manuals

There are 5 lab projects that are part of the coursework in Math 216. You will wo
these in the Instructional Computing Labs in the basement of East Hall, every we
through the semester.

Please note: The labs in use this semester were developed in summer 2016, an
been revised every semester since. They are aligned with the course material, s
to help you learn it, and reflect research on how you will learn effectively.

Before you arrive in lab, you must have submitted the corresponding prelab assi
Canvas. (lab 0 does not have a prelab). During the lab period you will have an op
to use the computer system to work on the "in the lab" part of the assignment.

Please note: *Only one* of your lab team needs to submit the writeup for each lab
member should submit this lab contribution sheet. (You should be able to fill it in
export it to PDF for upload, or upload the file; if you don't like `.docx` documents
know and we'll get you a different version.)

One other note: we have a Matlab command file also available. This is a Matlab f
sample commands that you will use in lab, from which you can cut and paste if yo

The lab materials:

- lab contribution sheet
- Matlab command file
- Lab 0: a (re)-introduction to *Matlab*
- **Lab 1**: Cancer, series, and ODE solutions. Prelab; Part A; Part B
  Lab 1 uses the Matlab file seriesapprox_w18.m
  Submit your lab writeups in Canvas.
- Lab 2: First Order Systems and the van der Pol Oscillator. Prelab; Part A;

Example: Lab 5, Part A

LAB 5: THE LORENZ SYSTEM AND WEATHER PATTERNS, PART A

1. Matlab

MATLAB commands we use in this lab include the following.

1.1. **disp.** Displays text to the command window. For example,

```
>> disp('This is text sent to the command window')
```

1.2. **ode45.** Finds a numerical approximation to a differential equation or system of equations:

```
>> [tsol,xsol] = ode45(f_handle, [tmin tmax], init_cond);
```

1.3. **plot.** Plot one vector against another; e.g.,

```
>> plot( tsol, xsol(:,1) );
```

1.4. **plot3.** Plot a three-dimensional figure; input are a vector of x-values, a vector of y-values, and a vector of z-values. Successive (x, y, z) triples from these vectors are graphed in 3-space:

```
>> plot3( xvec, yvec, zvec );
```

For example, if `xsol` is a solution variable from `ode45` when solving a system of three equations, we can plot the trajectory in the three-dimensional phase space with

```
>> plot3( xsol(:,1), xsol(:,2), xsol(:,3) );
```

if `x0` is the initial condition used in the solution, we could add that by using `plot3` to plot the point:

```
>> hold on;
>> plot3( [x0(1)], [x0(2)], [x0(3)], '.', 'MarkerSize', 20 );
```

Three-dimensional graphics figures have a rotate-tool button ( ) on the tool bar; clicking that will allow clicking and dragging the graph to rotate the image. As you do this, in the bottom left corner of the graph that *azimuth* and *elevation* of the viewpoint are shown. If you get an orientation you like (e.g., `120,20`), you can set that in your *MATLAB* code with

```
>> view([120,20]);
```

(This is sort of like the `axis` command for 2D plots: it sets the azimuth (angle from the $-y$ axis) and elevation (angle above the xy-plane) of the viewer.)

2. Background

In this lab we consider the Lorenz equations,

$$
\begin{aligned}
x' &= \sigma(-x+y) \\
y' &= rx - y - xz \\
z' &= -bz + xy,
\end{aligned} \tag{1}
$$

which were proposed as a model three-dimensional system with applications to weather modeling. These equations model the motion of a layer of fluid when the temperatures at the top and bottom boundaries of the layer differ. The variables are x, a measure of the intensity of the motion of the particles in the fluid; y, measuring the temperature difference between ascending and descending particles; and z, a measure of the distortion from vertical in particles' motion. The coefficients σ, b, and r are all positive, and represent different characteristics of the system: in particular, r is proportional to the difference in temperature between the boundaries of the layer. (The other parameters, σ and b, depend on the gas and geometry of the layer.) In this lab, we study how the behaviors of solution trajectories change for different values of r, and will see chaotic behavior and a new (to us) type of bifurcation called "period doubling."

3. Part A

Exercises in this section are to be completed by pairs. At the end of Workday 1, pairs should present their solutions to each other as indicated. Note that material from Part A appears in one of your written homework problems and will be relevant for Part B.

For all exercises, we consider the Lorenz system, (1), with $\sigma = 10$ and $b = \frac{8}{3}$. Values of r, and consideration of the linearization of the system, are indicated in the exercises.

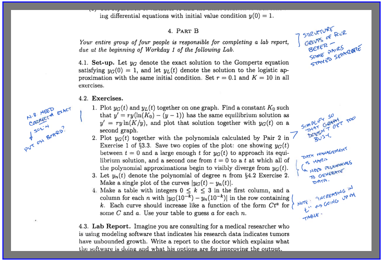
4. Pair 1 Exercises

Pair 1 Exercise 1. Review with your partner your work in Exercise 2 of the prelab, determining the linear stability of the critical point $(0, 0, 0)$. Suppose we solve (1) with $r = 0.5$ and the initial condition $(x(0), y(0), z(0)) = (0.5, 0.5, 0.5)$. Based on your linear analysis, what do you expect the component plots of solutions to the system to look like? As $t \to \infty$, what values do you expect x, y, and z to approach?

Solve (1) numerically using `ode45` and plot x, y and z as functions of t to confirm that they do what you expected. For systems of two equations we also considered plots in the phase plane: graphs of y vs x. Here, because there are three state variables, we have a phase space instead of a phase plane. Use `plot3` (see the *MATLAB* section, above) to graph the trajectory you obtained in the phase space. Note that you can rotate the 3D figure by clicking the rotate-tool button ( ) and then clicking and dragging the graph.

Implementation

- Summer 2016—revision
  - Syllabus and course revision by course coordinator.
  - Lab rewrites by post-doc, Sarah Kitchen.
- Fall 2016—implentation!
- Winter '17, Fall '17, Winter '18
  - Lab updates: update clarity and accessibility of materials.
  - Improve instructor support: graduate student lab manual, prompts graduate students, instructors.
- Assessment
  - Weekly meetings with lab graduate students;
  - End of term student survey about labs;
  - Teaching evaluation questions.


ing differential equations with initial value condition $y(0) = 1$.

4. Part B

Your entire group of four people is responsible for completing a lab report, due at the beginning of Workday 1 of the following Lab.

STRUCTURE GROUPS OF FOUR BETTER — SOME PAIRS STAYED SEPARATE

4.1. **Set-up.** Let y_G denote the exact solution to the Gompertz equation satisfying $y_G(0) = 1$, and let $y_L(t)$ denote the solution to the logistic approximation with the same initial condition. Set $r = 0.1$ and $K = 10$ in all exercises.

4.2. **Exercises.**

N.B. NEED CORRECT EXACT SOL'N PUT ON BOARD?

1. Plot $y_G(t)$ and $y_L(t)$ together on one graph. Find a constant K_0 such that $y' = ry(\ln(K_0) - (y-1))$ has the same equilibrium solution as $y' = ry\ln(K/y)$, and plot that solution together with $y_G(t)$ on a second graph.
2. Plot $y_G(t)$ together with the polynomials calculated by Pair 2 in Exercise 1 of §3.3. Save two copies of the plot: one showing $y_G(t)$ between $t = 0$ and a large enough t for $y_G(t)$ to approach its equilibrium solution, and a second one from $t = 0$ to a t at which all of the polynomial approximations begin to visibly diverge from $y_G(t)$.
3. Let $y_n(t)$ denote the polynomial of degree n from §4.2 Exercise 2. Make a single plot of the curves $|y_G(t) - y_n(t)|$.
4. Make a table with integers $0 \le k \le 3$ in the first column, and a column for each n with $|y_G(10^{-k}) - y_n(10^{-k})|$ in the row containing k. Each curve should increase like a function of the form Ct^a for some C and a. Use your table to guess a for each n.

SIMPLIFY SO THAT GRAPH DOESN'T GET TOO BUSY.

DATA MANAGEMENT IS HARD NEED POLYNOMIALS TO GENERATE DATA.

NOTE: "INCREASING IN t" ⟺ GOING UP ON TABLE.

4.3. **Lab Report.** Imagine you are consulting for a medical researcher who is using modeling software that indicates his research data indicates tumors have unbounded growth. Write a report to the doctor which explains what the software is doing and what his options are for improving the output.

Observations

- Students' background with *MATLAB* was weaker than we expected (Expected: ≈80% with knowledge; actual ≈50%).
- Students do not like courses becoming more challenging (unexpectedly).
  - Instructors' teaching evaluations reflected this.

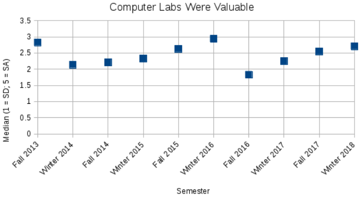
> *. . . [It] was like looking both ways before crossing the street and then getting hit from behind by an airplane.*
>
> *–student eval*

- Course expectations are significantly higher, and students largely rise to them.
  - (But) *Actual student learning is difficult to assess*.
- Multiple iterations were required to get materials to a desired level of clarity.

Assessment: Student, 1

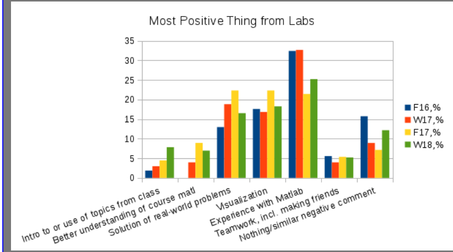
- Student surveys: looked at how productive and how connected to the course students felt labs were.
- With similar data from teaching evaluation questions.
- *All results are basically flat.*

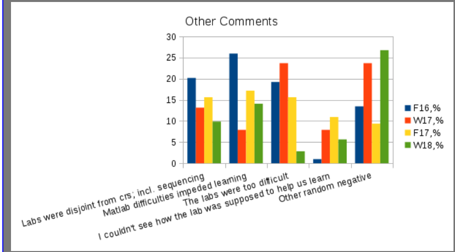
Goals: 1. Learning;
2. Lab/Course Connection;
3. Update Course;
4. Real-World Applications



Assessment: Student, 2

- Student surveys: *What was the most positive thing that you got from the labs (what did you learn from them)?*
  (90–115 comments/semester)
- Student surveys: *Other Comments*
  (65–105 comments/semester)
- Student surveys: *How well did the structure of the lab work for you?*
  (90–110 comments/semester)
  - *Worked fine:*
    F'16, 41%; W'17, 36%; F'17, 40%; W'18, 46%





Assessment: Instructor

> *Goals: 1. Learning;*
> *2. Lab/Course Connection;*
> *3. Update Course;*
> *4. Real-World Applications*

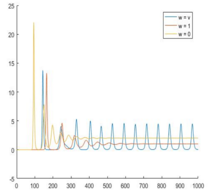
- Observations from GSI meetings
  - Student engagement is key, and hard to promote uniformly.
  - Issues continue with students reading: many sources of confusion are well-explained in lab materials.
    - Prelabs have significant value in framing the mathematics and lab structure, but are under utilized by students.
  - Students (largely) learn *Matlab*.
- And, at least anecdotally, this is (now) a solid, coherent course.

Assessment Conclusions

- Assertion: *Meaningful pedagogical change moves a learning environment in the right direction*.
  - With course revision, students are (actively) working on more substantial problems in lab, and doing so more collaboratively.
  - We have no direct evidence of improved student learning, but *the research on student engagement, collaboration, & learning is clear*.
- Assertion: *Student response data provide (non-definitive) insight on outcomes*.
  - With course revision, there is some positive impact on student learning, affect, and understanding of the course.
  - But *changing students' overall and in-class understanding of learning and a course is hard*.
- Articulation is hard
  - Our lectures/labs have: different instructors, different rooms, and different context (lecture vs. interactive computer lab).

Looking Back, Moving Forward

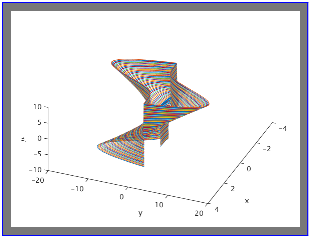
- There are positive take-aways from the lab implementation:
  - Students are working more substantial problems in lab, working in a structured, collaborative environment, and doing so in a stronger, more conceptual course.



Graph of P(t) with A(t) and multiple values for ω.

- Improved pedagogy is an ongoing effort.
  - Development, and implementation, takes time.
  - Course materials cannot be static.
  - Instructor buy-in and engagement is as important as student engagement.
- Moving forward: we need to address
  - Retaining energy in and focus on the labs and course, and
  - Maintaining the course and materials.

Questions and Information



Thank you!

- Questions, comments:
  `glarose@umich.edu`
- Materials:
  `http://www.math.lsa.umich.edu/courses/216/`
  (and, pending: *MathWorks*)