

## Correlational Analysis of Dyad-Level Data in the Exchangeable Case

Dale Griffin  
University of Waterloo

Richard Gonzalez  
University of Washington

Many research problems in psychology require statistical methods that take into account the dependencies introduced by dyadic interaction. The authors provide correlational tools for dyadic data when the individuals within the dyads are both from the same class or category, such as 2 male adults. First, the authors provide significance tests for correlations between 2 variables when individuals are nested within dyads. Second, they provide a simplified method for decomposing the overall correlation into individual-level and dyad-level relations. Finally, the authors demonstrate these methods with dyadic data collected by L. Stinson and W. Ickes (1992) in a study of unstructured dyadic interactions.

There are many problems in psychology for which theory and data analysis at the level of the dyad, rather than the individual, are appropriate. Social psychologists talk about warm and cold interactions, as well as about warm and cold individuals. Clinical psychologists talk about happy and unhappy couples, as well as about happy and unhappy people. Developmental psychologists talk about cooperative and competitive sibling relationships, as well as about cooperative and competitive siblings. However, rather than collect data at the dyad level, researchers typically rely on data collected from the individuals within these dyads. In the case of correlational analyses, the nonindependent nature of dyadic data presents both statistical and conceptual challenges.

These challenges can be seen in a simple example. Stinson and Ickes (1992) observed 24 pairs of male students interacting in an unstructured waiting room situation. Each interaction was videotaped and coded on a number of dimensions, including the number of verbalizations and the number of gestures made by each individual. The researchers were interested in, among other things, the correlation between the frequency of students' verbalizations and gestures. Note that although the researchers had 48 individual scores on each variable to correlate, these scores were collected from 24 interdependent dyads. In

such a design, the degree of interdependence between dyadic partners must be taken into account in the data analysis. In the most extreme case, the two individuals in each dyad could be "carbon copies," and their scores completely redundant. In such a situation, there would be only 24 nonredundant scores on which to base a significance test. As this example suggests, a significance test for the correlation between the individual scores should be adjusted to take intradyadic similarity into account. One way to solve this problem is to adjust the effective sample size. We present a test based on this logic.

After the observed correlation is computed in a dyadic design, researchers face a second hurdle in interpreting its meaning. The interpretation of the observed correlation between two variables is unclear because it can reflect relations between the variables at an individual level, at a dyadic level, or both. In the prior example, a positive overall correlation between verbalizations and gestures indexes the tendency for talkative individuals to make many gestures (an individual-level relation) or for members of talkative dyads to jointly make many gestures (a dyad-level relation). The observed "overall" correlation for individuals interacting in dyads can be seen as a mixture of both the individual-level and the dyad-level relation. This approach involves a decomposition of the observed correlation into separate individual-level and dyad-level correlations (Kenny & La Voie, 1985).

The decomposition of the overall correlation into individual-level and dyad-level components is of interest because it permits research questions to be answered at two different levels of analysis. For example, in the Stinson and Ickes (1992) study, an individual-level question would be, "Did the individual who spoke more frequently in the dyad also gesture more frequently?" The corresponding dyad-level question would be, "In couples with members who both spoke frequently, did those members also both gesture frequently?" The techniques we present later give researchers the flexibility to ask questions at both levels.

This article focuses on dyadic correlational methods for *exchangeable* dyad members. When both members of each dyad are from the same class or category, the members of the dyads are said to be exchangeable or interchangeable. When members

---

Dale Griffin, Department of Psychology, University of Waterloo; Richard Gonzalez, Department of Psychology, University of Washington. Dale Griffin is now at the School of Cognitive and Computing Science, University of Sussex, Brighton, England.

This research was supported by grants from the Social Sciences and Humanities Research Council of Canada and from the National Science Foundation.

We thank John Davis, Tony Greenwald, Bill Ickes, David Kenny, Lisa Smith, and Jane Swanson for their helpful suggestions on a previous draft.

Correspondence concerning this article should be addressed to either Dale Griffin, School of Cognitive and Computing Sciences, University of Sussex, Falmer, Brighton, England BN1 9QH, or Richard Gonzalez, Department of Psychology, NI-25, University of Washington, Seattle, WA 98195. Electronic mail may be sent to daleg@cogs.susx.ac.uk (Griffin) or gonzo@u.washington.edu (Gonzalez).

are in the same class, there is no natural or nonarbitrary way to classify the two individuals in a dyad into two separate columns for correlational analysis. For example, in the Stinson and Ickes (1992) study, the two members of each dyad were male adults. Because of this, the intradyadic similarity in the exchangeable case is indexed by the intraclass correlation, which does not require the categorization of individuals within dyads (Fisher, 1925; Haggard, 1958). In contrast, in the "distinguishable" case, the two individuals in a dyad come from different classes, for example, when one member of each dyad is male and the other is female. In such a case, the similarity within dyads is typically assessed by the more familiar Pearson, or interclass, correlation.

### Overall Correlation for Exchangeable Dyads

The overall correlation  $r_{xy}$  is simply the Pearson product-moment correlation between each individual's score on  $X$  and that individual's score on  $Y$ . In other words, with  $N$  dyads, the correlation involves  $2N$  scores on each variable  $X$  and  $Y$ . However, the standard significance test for a correlation is not appropriate in a dyadic design. The significance test for designs with independent individual data depends on only the observed correlation and the total sample size (Hays, 1988). In a dyadic design, however, additional information about the intradyadic similarity within  $X$  and within  $Y$  is necessary to evaluate the significance of the overall correlation.

The challenge in computing an intradyadic similarity correlation in the exchangeable case is that there is no way to distinguish the two members of the dyad and, therefore, no way to assign one individual's score to one column and the other individual's score to a second column. The pairwise intraclass correlation conveniently overcomes this problem (Donner & Koval, 1980; Fisher, 1925). The pairwise intraclass correlation is so named because all possible within-group pairs of scores are used to compute the correlation. For example, with individuals Adam and Amos in the first dyad, there are two possible pairings: first Adam in column 1 and Amos in column 2, and second Amos in column 1 and Adam in column 2. With three exchangeable dyads (Adam and Amos, Bill and Bob, and Colin and Chris), the pairwise setup consists of the scores on  $X$  of Adam, Amos, Bill, Bob, Colin, and Chris in the first column (denoted  $X$ ) and the scores on  $X$  of Amos, Adam, Bob, Bill, Chris, and Colin in the second column (denoted  $X'$ ). Note that each pairing occurs twice but in opposite order (Adam in column 1 with Amos adjacent in column 2, then Amos in column 1 and Adam adjacent in column 2, etc.). Thus, with  $N = 3$  dyads, each column contains  $2N = 6$  scores because each member is represented in both columns. The two columns are then correlated using the usual product-moment method. That is, the pairwise intraclass correlation is computed as

$$\text{cov}(X, X') / \sqrt{\text{var}(X)} \sqrt{\text{var}(X')}.$$

Because  $X$  and  $X'$  are the same variables in different order,  $\text{var}(X) = \text{var}(X')$  and the pairwise intraclass correlation reduces to  $\text{cov}(X, X') / \text{var}(X)$ . Note that even though the pairwise intraclass correlation is computed as a product-moment

correlation, its test of significance is slightly different than the usual product-moment test.

The pairwise intraclass correlation indexes the similarity of individuals within dyads and is closely related to the more familiar analysis of variance estimate of the intraclass correlation (Fisher, 1925; Haggard, 1958). However, the pairwise method has several important advantages in the present situation. First, it is calculated in the same manner as the usual Pearson correlation: The two "doubled" columns are simply correlated in the usual manner, thus giving ease of computation, flexibility in terms of computer packages, and an intuitive link to general correlational methods. Second, in the case of equal group sizes, which is obviously the case in dyadic research, the pairwise intraclass correlation is the maximum likelihood estimator of the population intraclass correlation and has elegant distributional properties (Donner & Koval, 1980). Finally, the pairwise method generalizes from the intraclass correlation within a single variable to the pairwise correlation  $r_{xy}$  across two variables (the *cross-intraclass correlation*; Harris, 1913; Pearson, 1901).

Table 1 presents four columns of symbolic data representing the scores on  $X$  and  $Y$  for the members of four dyads. The first two columns are labeled  $X$  and  $X'$  to indicate that both columns contain the same data listed in different order. The third and fourth columns are labeled  $Y$  and  $Y'$  for the same reason. The calculation of the two intraclass correlations proceeds by using the usual product-moment method between  $X$  and  $X'$  (yielding  $r_{xx'}$ ) and  $Y$  and  $Y'$  (yielding  $r_{yy'}$ ), respectively. This setup also yields the correlation between  $X$  and  $Y'$ , the cross-intraclass correlation  $r_{xy'}$ , and the overall correlation  $r_{xy}$  itself. Figure 1 illustrates the correlations that can be computed from these four columns of pairwise data. We first present tests of significance on the correlations in Figure 1 and then turn to simulation results to assess the characteristics of these tests.

### Testing the Overall Correlation $r_{xy}$

A maximum likelihood procedure provided by Elston (1975) can be used to derive a significance test for the overall correlation  $r_{xy}$  (see Appendix A for the derivation of the test). Note that in the pairwise approach, the correlations  $r_{xy}$  and  $r_{x'y'}$  are

Table 1  
*Symbolic Representation of Pairwise Data Setup*

Dyad	Variable			
	$X$	$X'$	$Y$	$Y'$
1	$X_{11}$	$X_{12}$	$Y_{11}$	$Y_{12}$
	$X_{12}$	$X_{11}$	$Y_{12}$	$Y_{11}$
2	$X_{21}$	$X_{22}$	$Y_{21}$	$Y_{22}$
	$X_{22}$	$X_{21}$	$Y_{22}$	$Y_{21}$
3	$X_{31}$	$X_{32}$	$Y_{31}$	$Y_{32}$
	$X_{32}$	$X_{31}$	$Y_{32}$	$Y_{31}$
4	$X_{41}$	$X_{42}$	$Y_{41}$	$Y_{42}$
	$X_{42}$	$X_{41}$	$Y_{42}$	$Y_{41}$

*Note.* The first subscript represents the dyad, and the second subscript represents the individual. Categorization of individuals as 1 or 2 is arbitrary.

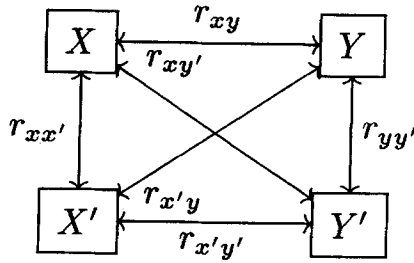


Figure 1. All possible pairwise correlations between variables  $X$  and  $Y$ .

equal to each other. Under the null hypothesis that  $\rho_{xy} = 0$ , the approximate large-sample variance of  $r_{xy}$  is  $1/N_1^*$ , where  $N_1^* = 2N/(1 + r_{xx'}r_{yy'} + r_{xy}^2)$ . Thus, the overall correlation  $r_{xy}$  can be tested using a  $Z$  test,<sup>1</sup> where

$$Z = \frac{r_{xy}}{\sqrt{\frac{1}{N_1^*}}}, \tag{1}$$

or, equivalently,  $Z = r_{xy} \sqrt{N_1^*}$ .

Intuitively,  $N_1^*$  can be thought of as the “effective sample size” for  $r_{xy}$  adjusted for dependent observations (see Rosner, 1982, and Eliasziw & Donner, 1991, for the development of this intuition). According to the null hypothesis,  $N_1^*$  ranges between  $N$  and  $2N$  when the product of the two intraclass correlations is nonnegative. Thus, under the null hypothesis, when there is complete dependence within dyads ( $r_{xx'} = r_{yy'} = 1$ ), the overall correlation and its significance test essentially reduce to the usual Pearson estimate using one individual from each dyad because the two individuals are identical. When there is complete independence within dyads on both variables ( $r_{xx'} = r_{yy'} = 0$ ), the overall correlation and its significance test essentially reduce to the Pearson estimate obtained when using both members and ignoring dyad membership. In the special case when one intraclass is negative and the other is positive,  $N_1^*$  can exceed  $2N$ .

*Testing the Cross-Intraclass Correlation  $r_{xy'}$*

The correlation between an individual’s score on variable  $X$  and his or her dyadic partner’s score on variable  $Y'$  is the cross-intraclass correlation. Note that in the pairwise approach, the correlation  $r_{xy'} = r_{x'y}$ . The cross-intraclass correlation assesses the strength of the relationship between two variables measured on different dyadic partners. Elston’s (1975) procedure can be used to derive a significance test for  $r_{xy'}$ . Under the null hypothesis that  $\rho_{xy'} = 0$ , the asymptotic variance of  $r_{xy'}$  is  $1/N_2^*$ , where  $N_2^* = 2N/(1 + r_{xx'}r_{yy'} + r_{xy}^2)$ . The cross-intraclass correlation  $r_{xy'}$  can be tested using a  $Z$  test, where

$$Z = \frac{r_{xy'}}{\sqrt{\frac{1}{N_2^*}}}, \tag{2}$$

or  $Z = r_{xy'} \sqrt{N_2^*}$ . Similar to  $N_1^*$ ,  $N_2^*$  can be thought of as the effective sample size for  $r_{xy'}$ , adjusted for dependent observations.

*Testing the Intraclass Correlations  $r_{xx'}$  and  $r_{yy'}$*

There are two intraclass correlations—one for variable  $X$  and the other for variable  $Y$ . In the special case of dyads, the intraclass correlation can range between  $-1$  and  $1$  and is interpretable throughout that range. For simplicity, we present significance tests in terms of  $r_{xx'}$ . The test for  $r_{yy'}$  can be performed by substituting  $r_{xx'}$  with  $r_{yy'}$ . Under the null hypothesis that the intraclass correlation is zero, the asymptotic standard error is  $1/\sqrt{N}$ . Thus the product  $r_{xx'}\sqrt{N}$  is asymptotically normally distributed and can be tested against the  $Z$  distribution.<sup>2</sup>

*Simulation*

We present simulation results for selected population values of the four pairwise correlations. The purpose of this simulation is to demonstrate that the asymptotic properties of the  $Z$  tests presented in this article behave relatively well for samples as small as  $N = 30$ . The simulation is based on the general multivariate pairwise algorithm described by Eliasziw and Donner (1991). The simulation was written using the S statistical language (Becker, Chambers, & Wilks, 1988) and used its built-in random number generator. For each set of population values, 1,000 samples of  $N = 30$  dyads were drawn from a multivariate normal distribution. Table 2 presents the average value of the population estimates over the 1,000 samples and the effective Type I error rates for a nominal  $\alpha = 0.05$  (two-tailed). The choices for  $N$  and  $\alpha$  were made because they represent typical values found in research. Table 2 is given in three parts: (a) The population intraclass correlations  $\rho_{xx'}$  and  $\rho_{yy'}$  are set to zero, and the correlations  $\rho_{xy}$  and  $\rho_{xy'}$  are varied across a few selected values; (b) the population overall correlation  $\rho_{xy}$  is set to zero, and the correlations  $\rho_{xx'}$ ,  $\rho_{yy'}$ , and  $\rho_{xy'}$  are varied across a few selected values; and (c) the population pairwise cross-intraclass correlation  $\rho_{xy'}$  is set to zero, and the correlations  $\rho_{xx'}$ ,  $\rho_{yy'}$ , and  $\rho_{xy}$  are varied across a few selected values. All statistical tests in this simulation were based on the  $Z$  ratios discussed in the text.

The results of the simulation suggest that for these parameter values the asymptotic tests perform well in the sense that effective Type I error rates were near the nominal  $\alpha$ , even when  $N = 30$ . Note that the pairwise intraclass correlation has a slight negative bias, which is expected in both the pairwise and the

<sup>1</sup> In this article, we focus on hypothesis testing, but an interval estimation approach could also be used. The hypothesis testing approach has the advantage of simplifying the standard error under the null hypothesis (see Appendix A). The calculation of confidence intervals requires more complicated standard errors.

<sup>2</sup> A slightly more complicated procedure can be used to test the pairwise intraclass correlations: The pairwise correlations are transformed to  $Z = 1/2 \log(1 + r_{xx'})/(1 - r_{xx'})$ , having a standard error of  $\sqrt{1/N - 3/2}$  (Fisher, 1921). The advantage of the  $r$ -to- $z$  transformation is that it allows the correlation to be tested against any value for the null hypothesis, whereas the method given in the text can only be used for testing correlations against the null hypothesis equaling zero.

Table 2  
Bias and Effective Type I Error Rates for Pairwise Correlations in the Dyadic Case

$\rho_{xy}$	$\rho_{xy'}$	Average estimate	Type I error rate
Null hypothesis that $\rho_{xx'} = \rho_{yy'} = 0$			
-.5	-.5	-.021	.050
-.5	0	-.014	.040
-.5	.5	-.019	.051
0	-.5	-.012	.049
0	0	-.012	.049
0	.5	-.011	.047
.5	-.5	-.015	.050
.5	0	-.027	.055
.5	.5	-.014	.046

$\rho_{xx'}$	$\rho_{yy'}$	$\rho_{xy}$	Average estimate	Type I error rate
Null hypothesis that $\rho_{xy} = 0$				
-.5	-.5	-.5	.000	.046
-.5	-.5	0	-.004	.036
-.5	-.5	.5	-.003	.051
0	0	-.5	.001	.048
0	0	0	.000	.041
0	0	.5	-.011	.049
.5	.5	-.5	.007	.064
.5	.5	0	.004	.041
.5	.5	.5	-.015	.059
-.5	.5	-.5	.003	.053
-.5	.5	0	-.007	.060
-.5	.5	.5	-.003	.044
0	-.5	-.5	.005	.050
0	-.5	0	-.001	.058
0	-.5	.5	.001	.044
0	.5	-.5	.019	.043
0	.5	0	.000	.048
0	.5	.5	-.021	.053

$\rho_{xx'}$	$\rho_{yy'}$	$\rho_{xy}$	Average estimate	Type I error rate
Null hypothesis that $\rho_{xy} = 0$				
-.5	-.5	-.5	.012	.055
-.5	-.5	0	.005	.038
-.5	-.5	.5	.000	.041
0	0	-.5	.006	.044
0	0	0	.005	.059
0	0	.5	-.016	.038
.5	.5	-.5	.026	.061
.5	.5	0	.004	.047
.5	.5	.5	-.016	.058
-.5	.5	-.5	.010	.043
-.5	.5	0	.008	.049
-.5	.5	.5	-.011	.048
0	-.5	-.5	.005	.043
0	-.5	0	.001	.047
0	-.5	.5	-.007	.060
0	.5	-.5	.016	.065
0	.5	0	.000	.054
0	.5	.5	-.010	.045

analyses of variance (ANOVA) formulations. This negative bias does not appear to occur in the estimation of either  $\rho_{xy}$  or  $\rho_{xy'}$ . More detailed simulation results are presented in Gonzalez and Griffin (1994).

### Separating Individual-Level and Dyad-Level Effects

Kenny (1994) presented a number of structural models that can be used to analyze dyadic correlational designs. In the next section, we focus on one of these models, the *group correlation model* (Kenny & La Voie, 1985), that serves to separate individual-level and dyad-level effects. We present a pairwise version of the Kenny and La Voie model that has a number of advantages in the dyadic case.

### A Latent Variable Model of Dyadic Data

In Figure 2, we present a simple latent variable model for the exchangeable dyadic design. In this model, each measured variable is coded in a pairwise fashion so that the variables  $X$  and  $X'$  (and, by the same logic,  $Y$  and  $Y'$ ) are identical except for order. The variance of a given, observed variable is assumed to result from two different latent (not measured) sources: (a) a dyadic component representing the portion of the variable that is shared between dyadic partners and (b) an individual component representing the portion of the variable that is unshared or unique between dyadic partners. The loading of each measured variable on the latent dyadic component is the square root of the pairwise intraclass correlation for that variable. The loading on the latent individual component is the complement of the dyadic loading, the square root of one minus the pairwise intraclass correlation for that variable.

As Figure 2 illustrates, there are two levels at which the variables can be related. The shared dyadic variance of  $X$  and  $Y$  can be related through the dyadic correlation  $r_d$ , and the unique individual variance of  $X$  and  $Y$  can be related through the individual-level correlation  $r_i$ . Our goal is to express the two unknowns,  $r_d$  and  $r_i$  in terms of the four observed pairwise correlations.

Using the "tracing rule" of path analysis (Kenny, 1979), or covariance algebra, it is possible to decompose the two pairwise  $XY$  correlations,  $r_{xy}$  and  $r_{xy'}$ . According to the model in Figure 2, the two correlations decompose as (a)  $r_{xy} = \sqrt{r_{xx'}} r_d \sqrt{r_{yy'}} + \sqrt{1 - r_{xx'}} r_i \sqrt{1 - r_{yy'}}$  and (b)  $r_{xy'} = \sqrt{r_{xx'}} r_d \sqrt{r_{yy'}}$ . In words, the correlation between an individual's  $X$  score and  $Y$  score ignoring dyadic membership is a weighted sum of two effects: (a) the dyad-level relation (the term involving  $r_d$ ) and (b) the individual within-dyad relation (the term involving  $r_i$ ). Therefore, according to this model, the overall correlation is a function of both individual-level and dyad-level components.

With these decompositions, simple algebra yields expressions for the individual-level correlation,  $r_i$ , and the latent dyad-level correlation,  $r_d$ :

$$r_i = \frac{r_{xy} - r_{xy'}}{\sqrt{1 - r_{xx'}} \sqrt{1 - r_{yy'}}} \quad (3)$$

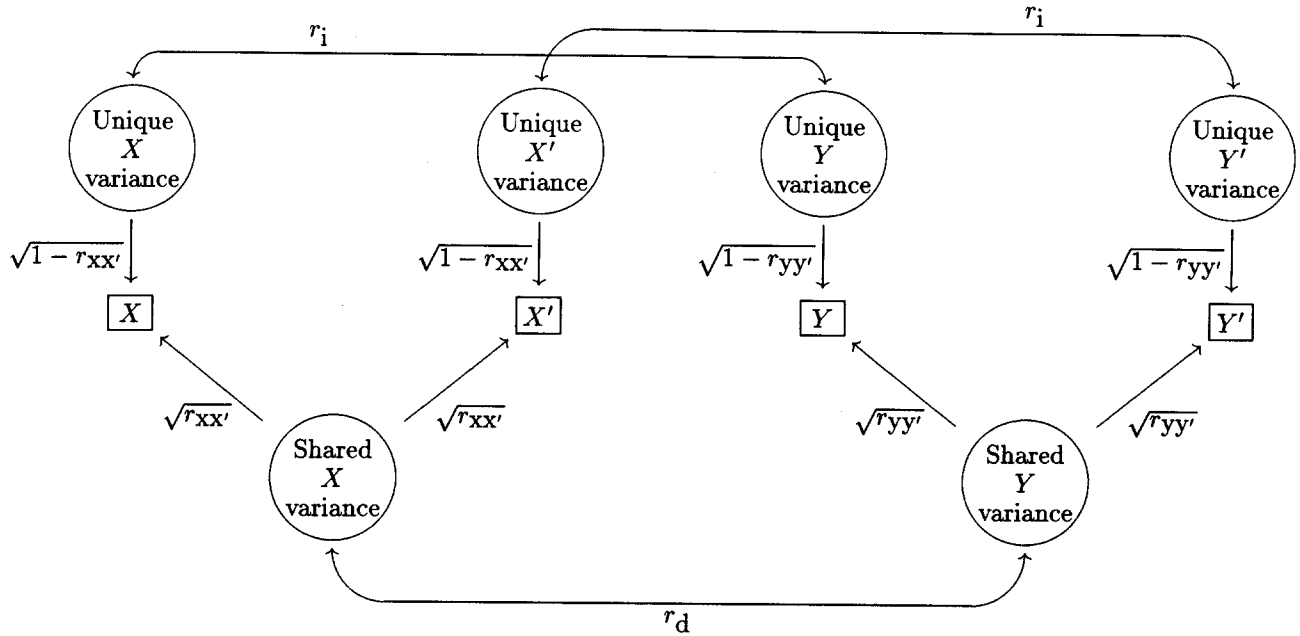


Figure 2. A latent variable model separating individual-level (unique) and dyad-level (shared) effects.

and

$$r_d = \frac{r_{xy'}}{\sqrt{r_{xx'}} \sqrt{r_{yy'}}}. \tag{4}$$

These expressions are intuitively meaningful. The numerator of the individual-level correlation  $r_i$  is the difference between the observed correlation  $r_{xy}$ , which combines dyad-level and individual-level effects, and the cross-intraclass correlation  $r_{xy'}$ , which contains only dyad-level effects. Thus,  $r_i$  is a measure of the individual-level relation uncontaminated by dyad-level effects. The numerator of the dyad-level correlation  $r_d$  is simply the pairwise correlation  $r_{xy'}$  corresponding to the direct measure of the dyad-level relations. The denominators, too, are conceptually straightforward; they correct the scale of the correlations for the fact that only “part” of each observed variable is correlated. When the individual components of variables  $X$  and  $Y$  are correlated, the denominator adjusts for the proportions of variance in the observed  $X$  and  $Y$  that correspond to the *non-shared* effects ( $\sqrt{1 - r_{xx'}}$  and  $\sqrt{1 - r_{yy'}}$ , respectively). Similarly, when the dyadic components of the variables  $X$  and  $Y$  are correlated, the denominator adjusts for the proportions of variance in the observed  $X$  and  $Y$  that correspond to the shared dyadic effects ( $\sqrt{r_{xx'}}$  and  $\sqrt{r_{yy'}}$ , respectively).

Both of the latent variable correlations are *disattenuated* correlations, although only the dyad-level correlation  $r_d$  is in the usual form of the disattenuated correlation (which consists of a correlation divided by the square root of the product of the reliabilities of each variable.) Because of this difference in form, the individual-level correlation  $r_i$  lends itself to a very straightforward significance test, whereas the dyad-level correlation  $r_d$  presents challenges because, in some situations, it can exceed the value of one.

### Testing the Underlying Correlations $r_i$ and $r_d$

In the dyadic case,  $r_i$  can be computed by correlating the deviation scores on  $X$  and on  $Y$ . That is, the dyad mean on  $X$  is subtracted from each  $X$  score, the dyad mean on  $Y$  is subtracted from each  $Y$  score, and then the  $2N$  deviations on  $X$  are correlated with the  $2N$  deviations on  $Y$ . Alternatively,  $r_i$  can be computed as the correlation between the signed difference of the two dyad members on  $X$  and the signed difference of the two dyad members on  $Y$ . For dyads, Equation 3, the deviation method, and the correlation of the signed differences yield identical values for  $r_i$ , which can be tested using the usual Pearson correlation table (or the associated  $t$ -test formula) with  $N - 1$  degrees of freedom (Kenny & La Voie, 1985).

The latent dyad-level correlation  $r_d$  is a disattenuated version of  $r_{xy'}$ , which complicates the significance test and also leads to cases where sample values of  $r_d$  are outside the range  $\pm 1.0$ . Although the computation of the latent dyad-level correlation may be descriptively useful, interpretation should probably be restricted to the raw score correlation  $r_{xy'}$ . It turns out that the significance test for  $r_d$  is a generalization of the above test for  $r_{xy'}$ . Under the null hypothesis that  $\rho_d = 0$ , the asymptotic variance of  $r_d$  is  $1/D^*$ , where  $D^* = [2N/(1 + r_{xx'}r_{yy'} + r_{xy'}^2)]r_{xx'}r_{yy'}$ . The ratio  $r_d/\sqrt{1/D^*}$ , or  $r_d\sqrt{D^*}$ , can be tested as a standard normal deviate (i.e., the  $Z$  test). Intuitively,  $D^*$  can be thought of as the effective sample size for  $r_d$  adjusted for disattenuation. When  $r_{xx'}$ ,  $r_{yy'}$ , or both are small,  $r_d$  tends to be large as a consequence and may even exceed 1.0. Because the model is based on the assumption of dyadic similarity, these parameters should only be estimated when both intraclass correlations are significantly positive. In general, the practice of restricting the application of this model to cases where both intraclass correlations

are significantly positive should reduce the occurrence of out-of-bounds values for  $r_d$ .

The cross-intraclass correlation  $r_{xy'}$  may be the most useful index of dyadic relations because it provides the same information as  $r_d$  without the interpretational complexities associated with a latent variable correlation. The  $p$  values for testing  $r_{xy'}$  and  $r_d$  against the null hypothesis are always identical. Note that  $r_{xy'}$  should be interpreted as a dyadic correlation only when both intraclass correlations  $r_{xx'}$  and  $r_{yy'}$  are significant, indicating significant dyadic variance.

The pairwise dyadic model is closely related to the ANOVA model proposed by Kenny and La Voie (1985) for the group correlation case. Our correlation  $r_i$  is identical to their "individual-level" correlation. Our  $r_d$  is virtually the same as their "group-level" correlation; in practice, this difference is negligible.<sup>3</sup>

For the dyadic case, the pairwise model is computationally more straightforward than the general group model and has the added advantage of providing a significance test for the dyadic correlation—something lacking in the general group model.<sup>4</sup> Furthermore, the pairwise model provides  $r_{xy'}$  (the pairwise cross-intraclass correlation) as an additional index of dyadic relations. Although  $r_{xy'}$  indexes the strength of the dyad-level relation, it has all the benefits of a raw score correlation: It is easy to compute, easy to test for significance, not subject to out-of-bounds values, and, therefore, just as straightforward to interpret as any observed correlation.

### The Mean-Level Correlation

At first blush, it may appear that the correlation between the means of each dyad on the two variables should yield an estimate of the dyad-level correlation. Contrary to this intuition, the *mean-level* correlation  $r_m$  reflects both individual and dyad-level effects and is best thought of as a "total" correlation. One way to see this is to express the mean-level correlation in terms of the path model in Figure 2. Using the tracing rule, the mean-level correlation

$$r_m = \frac{r_{xy} + r_{xy'}}{\sqrt{1 + r_{xx'}} \sqrt{1 + r_{yy'}}}. \quad (5)$$

Equation 5 is equivalent to the correlation between the  $N$  dyad means on  $X$  and the  $N$  dyad means on  $Y$ . The benefit of expressing the mean-level correlation in terms of its components is to demonstrate how  $r_m$  can be influenced by both individual-level and dyad-level components.

The mean-level correlation  $r_m$ , similar to the overall correlation  $r_{xy}$ , reflects a weighted average of individual-level and dyad-level components. The weights of each component vary depending on the degree of intradyadic similarity. When both intraclass correlations equal zero,  $r_m = r_{xy} = r_i$ ; when both intraclass correlations equal one,  $r_m = r_{xy} = r_d$ . That is, with complete independence within dyads on both variables, both the mean-level correlation and the overall correlation equal the pure individual-level correlation. With complete dependence within dyads, both the mean-level correlation and the overall correlation equal the pure dyad-level correlation. As the intraclass correlations increase from zero to one, both  $r_m$

and  $r_{xy}$  become more heavily weighted toward the dyad-level relation; although for any given value of the intraclass,  $r_m$  is more affected by dyad-level relations than is  $r_{xy}$ .

The mean-level correlation  $r_m$  should not be used as an index of dyad-level relations because it can be significantly positive or negative, even when  $r_d = 0$ . A positive dyad-level correlation exists only when the tendency of both dyad members to be high on  $X$  is matched by the tendency of both dyad members to be high on  $Y$ . However, this is only one of several circumstances that can lead to a positive value of  $r_m$ , indicating that a high average value on  $X$  is matched with a high average value on  $Y$ . For example, a positive mean-level correlation results when the tendency of one member to be extremely high on  $X$  is matched with the tendency of that member to be extremely high on  $Y$ , regardless of the score of his or her dyadic partner on either variable.

### Simulation

In the latent variable model of Figure 2, the population correlations  $\rho_i$  and  $\rho_d$  are underlying structural parameters that give rise to the four observed correlations depicted in Figure 1 (i.e.,  $r_{xx'}$ ,  $r_{yy'}$ ,  $r_{xy}$ , and  $r_{xy'}$ ). Because the population values of  $r_i$  and  $r_d$  must themselves be bound between  $-1$  and  $1$ , there are constraints placed on the population values of  $r_{xx'}$ ,  $r_{yy'}$ ,  $r_{xy}$ , and  $r_{xy'}$ . For instance, under this model, the intraclass correlations  $r_{xx'}$  and  $r_{yy'}$  cannot be negative.

We selected two values for  $\rho_{xx'}$  and  $\rho_{yy'}$  (.50 and .75) and conducted a simulation similar to the one presented in the section on pairwise correlations (i.e., 1,000 samples of 30 dyads from a multivariate normal distribution).<sup>5</sup> Table 3 presents the results of this simulation. Six runs were conducted under the null hypothesis that  $\rho_i = 0$ , and  $\rho_d$  was set to equal either  $-.5$ ,  $0$ , or  $.5$ ; the remaining six runs were conducted under the null hypothesis that  $\rho_d = 0$ , and  $\rho_i$  was set to equal either  $-.5$ ,  $0$ , or  $.5$ .

The simulation results suggest that for these parameter values, the estimation and testing of  $\rho_i$  and  $\rho_d$  performed well. Two decision rules were used in this simulation. First, whenever a sample intraclass correlation was negative, that sample was eliminated from the 1,000 samples. That is, we treated samples that included negative intraclass correlations as inconsistent with the model and, thus, dropped that sample from the simulation. Second, whenever the sample estimate of  $\rho_d$  was greater

<sup>3</sup> In our terms, Kenny and La Voie's (1985) estimator of  $r_d$  can be expressed as  $[r_{xy'} + r_{xy}/(2N - 1)]/[\sqrt{1/(2N - 1) + r_{xx'}} \cdot \sqrt{1/(2N - 1) + r_{yy'}}]$ . Note that as  $N$  gets large, the Kenny and La Voie group-level correlation applied to the special case of dyads approaches the pairwise  $r_d$ .

<sup>4</sup> For the special case of dyads, the  $r_d$  presented in this article is identical to the maximum likelihood estimator for the group-level correlation suggested by Gollub (1991). Gollub also suggested that the individual- and group-level correlations could be estimated and tested from the information provided by computer programs designed to handle multilevel data.

<sup>5</sup> For a discussion of the effects of violating the multivariate normal distribution, see Wilcox (1994).

Table 3  
Bias and Effective Type I Error Rates for the Latent Variable Model in Figure 2

$\rho_{xx'} = \rho_{yy'}$	$\rho_d$	Average estimate	Type I error rate	No. of samples consistent with the model
Null hypothesis that $\rho_i = 0$				
.50	-.5	.000	.044	996
.75	-.5	.002	.046	1,000
.50	0	.000	.062	991
.75	0	-.004	.054	1,000
.50	.5	.001	.049	940
.75	.5	.001	.055	1,000
$\rho_{xx'} = \rho_{yy'}$	$\rho_i$	Average estimate	Type I error rate	No. of samples consistent with the model
Null hypothesis that $\rho_d = 0$				
.50	-.5	.028	.037	974
.75	-.5	.017	.054	1,000
.50	0	-.005	.041	991
.75	0	.002	.049	1,000
.50	.5	-.049	.045	992
.75	.5	-.014	.044	1,000

than one (which can occur when either the intraclass correlations are close to zero or the sample estimate of  $\rho_{xy}$  is close to one), that sample was eliminated from the 1,000 samples. Note that estimates of  $\rho_i$  cannot be out-of-bounds because in the dyadic case,  $r_i$  can be computed as a Pearson correlation of deviation scores.

When the population intraclass correlations were set to .75, there was only 1 sample out of a possible 6,000 that was discarded (due to an out-of-bounds estimate of  $\rho_d$ ). However, when the population intraclass correlation was set to .50, the frequency of discarded samples increased. Most discarded samples were due to out-of-bounds estimates of  $\rho_d$ , but a few ( $M = 4, N = 1,000$ ) were due to estimates of the intraclass correlation that were negative. The total number of model consistent samples is in the last column of Table 3. Average estimates and effective Type I error rates were computed over the total number of model consistent samples.

On the basis of this simulation, we recommend that the latent variable model in Figure 2 be applied only when the intraclass correlations are significantly positive. Using the significance of the intraclass correlations as a hurdle for testing the latent variable model minimizes the occurrence of out-of-bounds values of  $\rho_d$  estimates. We recommend that the intraclass correlations be tested with an  $\alpha = .05$  (see Bowen & Huang, 1990; Gollob, 1991; Kenny & La Voie, 1985, for a discussion).

Illustrations and Examples

In the following section, we use data collected by Stinson and Ickes (1992) to illustrate the pairwise correlational analysis of dyads in the exchangeable case. From the data collected, we chose three variables (all expressed as frequencies): verbalizations, gazes, and gestures. Research questions might include the following:

1. Over all 48 individuals, were the three variables significantly related? This question can be answered by calculating the overall correlation between each pair of variables,  $r_{xy}$ .
2. During the period of interaction, did the newly created dyads show *emergent norms* (Kenny, 1994)? That is, did dyad members resemble each other on frequency of verbalization, frequency of directed gazes, or frequency of gestures? This question can be answered by calculating the intraclass correlations for each of the variables.
3. Are the three variables related at the level of the dyad? For example, did the members of those dyads who both spoke frequently also both gesture frequently? This question can be answered by calculating the latent dyad-level correlation,  $r_d$ , or its raw score counterpart,  $r_{xy'}$ , between each pair of variables. Dyad-level correlations are interpretable only when both variables have significantly positive intraclass correlations.
4. Are the three variables related at the level of the individual? For example, within dyads, did the individual who spoke more frequently also gesture more frequently? This question can be answered by calculating the individual-level correlation,  $r_i$ , between each pair of variables. The techniques presented in this article can be used to answer each of these four questions.

The first step in the analysis is to organize the data input for the pairwise setup. The  $2N = 48$  observations on each variable are listed in two columns but in opposite order in each column, so the scores for the two dyad members on a given variable are side by side in adjacent columns. These variables are correlated using the standard Pearson correlation. The resulting pairwise correlation matrix among the three variables is presented in Table 4. Note that even though the calculation of the correlations themselves can be accomplished with a standard statistical package, significance tests should be conducted with the formulas presented here.

Over all individuals, were the three variables significantly related? Examination of Table 4 reveals that all three overall correlations are positive and moderately large: The overall correlation between verbalization frequency and gaze frequency was .386, the overall correlation between verbalization frequency and gesture frequency was .449, and the overall correlation between gaze frequency and gesture frequency was .474. Recall that the significance test of the overall correlation  $r_{xy}$  depends on the effective sample size  $N_1^* = 2N / (1 + r_{xx'}r_{yy'} + r_{xy'}^2)$ . Between verbalizations and gazes,  $N_1^* = 48 / [1 + (.841)(.570) + .471^2] = 28.22$ ; between verbalizations and gestures,  $N_1^* = 48 / [1 + (.841)(.226) + .479^2] = 33.81$ ; and between gazes and

Table 4  
Pairwise Correlation Matrix for Randomly Sampled Strangers

Variable	Verb	Verb'	Gaze	Gaze'	Gest	Gest'
Verb'	<b>.841</b>	—				
Gaze	.386	.471	—			
Gaze'	.471	.386	<b>.570</b>	—		
Gest	.449	.479	.474	.325	—	
Gest'	.479	.449	.325	.474	<b>.226</b>	—

Note. Intraclass correlations are in boldface. Verb = number of verbalizations; Gaze = number of gazes; Gest = number of gestures.

gestures,  $N_1^* = 48/[1 + (.570)(.226) + .325^2] = 38.88$ . The effective sample size is smallest for the correlation between verbalizations and gazes because of the relatively large intraclass correlations, indicating that the individual scores within dyads are largely redundant. The effective sample size is largest for the correlation between gazes and gestures because those two variables have relatively small intraclass correlations so that each dyad member's score provides relatively independent information. The resulting significance tests are  $Z = .386/\sqrt{1/28.22} = 2.05, p < .05$ ;  $Z = .449/\sqrt{1/33.81} = 2.61, p < .05$ ; and  $Z = .474/\sqrt{1/38.88} = 2.96, p < .05$ , respectively. All three overall correlations are significantly positive; however, at this point, it is not clear which level of analysis gave rise to these correlations.

The computation and interpretation of the underlying correlations begin with an examination of the relevant pairwise intraclass correlations. During the period of interaction, did the individuals in the dyads resemble each other on the three measured variables? The intraclass correlations for verbalizations and gazes are .841 and .570, respectively. The resulting  $Z$  ratios under the null hypothesis that the population intraclass is zero (4.12 and 2.79, using  $r_{xx}\sqrt{N}$ ) are both significant ( $p < .01$ ). In contrast, the intraclass correlation for gestures is relatively small (.226) and translates into a  $Z$  ratio of 1.11, marginally significant only by the most optimistic standard ( $p < .15$ , one-tailed). Thus, although we can conclude that dyad members were similar on the number of verbalizations and gazes, we are uncertain whether the members were similar on the number of gestures they performed. In other words, there is significant, dyad-level variation in the first two variables, but there may not be in the third. As a result, dyad-level correlations that include frequency of gestures must be interpreted with caution.

Having determined that there is dyad-level variance in at least two of the three variables of interest, we now turn to the calculation and testing of the two dyadic correlations  $r_{xy'}$  and  $r_d$ . Were there dyad-level correlations among the three variables? The raw score dyad-level correlation  $r_{xy'}$  can simply be read from the pairwise correlation matrix. For example,  $r_{xy'}$  between verbalizations and gazes is .471. The effective sample size is  $N_2^* = 2N/(1 + r_{xx}r_{yy'} + r_{xy}^2) = 48/[1 + (.841)(.570) + .386^2] = 29.48$ , and the resulting  $Z$  ratio is  $.471/\sqrt{1/29.48} = 2.56, p < .05$ . Knowing that the raw score dyadic correlation  $r_{xy'}$  is significant implies that the latent dyadic correlation  $r_d$  is also significant. The calculation of  $r_d$  proceeds by disattenuating  $r_{xy'}$  for the reliability of the observed variables as indicators of the underlying dyadic factor (as indexed by the intraclass correlations). In the case of verbalizations and gazes,  $r_d = .471/\sqrt{(.841)(.570)} = .680$ . The effective sample size corrected for disattenuation is  $D^* = 48/[1 + (.841)(.570) + .386^2] = 14.13$ , and the resulting  $Z$  ratio is  $.680/\sqrt{1/14.13} = 2.56, p < .05$ , which is equivalent to the  $Z$  statistic for  $r_{xy'}$ .

The raw score dyadic correlation  $r_{xy'}$  between verbalization frequency and gesture frequency is .479, and the dyad-level correlation ( $r_d$ ) between these two variables is 1.10, which is slightly "out of bounds." Such out-of-bounds values are most likely to occur when the intraclass correlation for one or both of the variables is marginal or nonsignificant (as in the case of

gestures). Testing  $r_{xy'}$  against its standard error ( $N_2^* = 34.49$ ) or  $r_d$  against its standard error ( $D^* = 6.55$ ) yields the test statistic  $Z = 2.82, p < .01$ . Similarly, the cross-intraclass correlation  $r_{xy'}$  between gaze frequency and gesture frequency is .325, and the latent dyad-level correlation ( $r_d$ ) between these two variables is .906. Testing  $r_{xy'}$  against its standard error ( $N_2^* = 35.46$ ) or  $r_d$  against its standard error ( $D^* = 4.57$ ) yields the test statistic  $Z = 1.94, p < .10$ . In summary, all three dyad-level relations are positive, and two are large enough to be clearly significant. The significant, positive values for  $r_{xy'}$  (and  $r_d$ ) indicate that dyads in which both members speak frequently are also dyads in which both members look at each other frequently and gesture to each other frequently.

Were the three variables related at the level of individual within dyads? The computation of the individual-level correlation,  $r_i$ , between verbalizations and gazes is straightforward:  $(r_{xy} - r_{xy'})/(\sqrt{1 - r_{xx'}}\sqrt{1 - r_{yy'}}) = (.386 - .471)/[\sqrt{(1 - .941)}\sqrt{(1 - .570)}] = -.325$ . In contrast to the positive dyad-level correlation between verbalization and gaze, the individual-level correlation is negative. That is, the dyad member who speaks more often tends to be the dyad member who looks at the other less often (or, to turn the relation around, the dyad member who listens more often looks at the other more often). This despite the fact that dyads in which there is frequent speaking also tend to be dyads in which there is frequent gazing. However, the individual-level correlation is only marginally significant,  $t_{(N-1)} = (r_i\sqrt{N-1})/(\sqrt{1 - r_i^2}) = (-.325\sqrt{23})/\sqrt{1 - .325^2} = 1.65, p < .10$ . The individual-level correlations for the other pairs of variables are relatively small and nonsignificant. For verbalizations and gestures,  $r_i = -.086$ ; for gestures and gazes,  $r_i = .258$ . All three values of  $r_i$  are markedly discrepant from the corresponding values of  $r_d$  and  $r_{xy'}$ , underlining the importance of separating out the dyad-level and individual-level relations.

Recall that all three overall correlations were moderate and positive. However, each represents a different combination of underlying dyadic and individual-level correlations. Verbalizations and gazes were negatively related at the individual level but positively correlated at the dyad level; verbalizations and gestures were unrelated at the individual level but positively correlated at the dyad level; and finally, gazes and gestures were positively correlated at both the individual and dyadic levels.

### Research Design and Dyadic Correlations

In the previous study, participants were randomly assigned to dyads. This design feature gives the researcher the power of a randomized experiment in an unstructured interaction study. Unlike studies of naturally occurring dyads where dyadic effects may be due to selective affiliation of similar individuals, in this design, the researchers can be confident that both the significant intradyadic similarity and the significant dyadic correlations were due to processes that occurred during the dyadic interaction.

Note the distinct implications of random sampling of dyad members and random assignment to dyads. Random sampling means that each dyad is a randomly sampled subset of some larger possible population rather than a complete population in itself. Random assignment means that individuals are randomly



paired off into dyads. It is possible to have random sampling without random assignment. For example, when an educational researcher randomly chooses two individuals from each school, dyad-level similarity and dyad-level relations may be due to preexisting differences in the populations sampled. In contrast, when individuals are randomly assigned to dyads, any resulting intradyadic similarity must be the result of processes occurring during dyadic interaction, not because of preexisting differences in varying populations. The analyses presented here apply to either context; however, causal inferences must be restricted to designs featuring random assignment to dyads.

### Conclusion

In this article, we discussed and provided significance tests for the following correlations: the overall correlation  $r_{xy}$ , which combines the dyadic- and individual-level relations between the two variables; the intraclass correlations  $r_{xx'}$  and  $r_{yy'}$ , which index the amount of dyadic or shared variance in each variable; the individual-level correlation  $r_i$ , which indexes the extent to which one member's unique tendency on  $X$  is related to that individual's tendency on  $Y$ ; the raw score dyadic correlation  $r_{xy'}$ , which indexes the dyadic relation uncorrected for the degree of shared variance; and the latent dyadic correlation  $r_d$ , which indexes the extent to which the two individuals shared tendency on  $X$  is related to their shared tendency on  $Y$ . Appendix B summarizes the sequence of computations.

Psychological processes that are measured on individuals within dyads are likely to be determined by both levels of analysis. Researchers should realize that the traditional correlational methods do not separate dyadic and individual-level processes. At the moment, little theoretical attention has been paid to the question of how dyad-level or group-level processes differ from individual-level processes. We hope that analytic methods that separate the relative contribution of dyad or group-level processes from individual-level processes (and allow for significance testing separately at each level) will encourage more precise and complex theories of interpersonal behavior.

### References

- Becker, R. A., Chambers, J. M., & Wilks, A. R. (1988). *The new S language*. Pacific Grove, CA: Wadsworth.
- Bowen, J., & Huang, M.-H. (1990). A comparison of maximum likelihood with method of moment procedures for separating individual and group effects. *Journal of Personality and Social Psychology, 58*, 90-94.
- Donner, A., & Koval, J. J. (1980). The estimation of intraclass correlation in the analysis of family data. *Biometrics, 36*, 19-25.
- Eliaszewicz, M., & Donner, A. (1991). A generalized non-iterative approach to the analysis of family data. *Annals of Human Genetics, 55*, 77-90.
- Elston, R. C. (1975). On the correlation between correlations. *Biometrika, 62*, 133-140.
- Fisher, R. A. (1921). On the "probable error" of a coefficient of correlation deduced from a small sample. *Metron, 1*, 3-32.
- Fisher, R. A. (1925). *Statistical methods for research workers*. Edinburgh, Scotland: Oliver & Boyd.
- Gollob, H. F. (1991). Methods for estimating individual- and group-level correlations. *Journal of Personality and Social Psychology, 60*, 376-381.
- Gonzalez, R., & Griffin, D. W. (1994). *An approximate significance test for the dyad-level correlation*. Unpublished manuscript, University of Washington, Seattle, and University of Sussex, Brighton, England.
- Haggard, E. A. (1958). *Intraclass correlation and the analysis of variance*. New York: Dryden Press.
- Harris, J. A. (1913). On the calculation of intra-class and inter-class coefficients of correlation from class moments when the number of possible combinations is large. *Biometrika, 9*, 446-472.
- Hays, W. L. (1988). *Statistics* (4th ed.). New York: Holt, Rinehart & Winston.
- Kendall, M., & Stuart, A. (1966). *The advanced theory of statistics* (2nd ed.). London: Griffin.
- Kenny, D. A. (1979). *Correlation and causality*. New York: Wiley.
- Kenny, D. A. (1994). *The analysis of data from two-person relationships*. Unpublished manuscript, University of Connecticut, Storrs.
- Kenny, D. A., & La Voie, L. (1985). Separating individual and group effects. *Journal of Personality and Social Psychology, 48*, 339-348.
- Pearson, K. (1901). Mathematical contributions to the theory of evolution ix. On the principle of homotyposis and its relation to heredity, to the variability of the individual, and to that of the race. *Philosophical Transactions of the Royal Society of London, Series A, 197*, 285-379.
- Rosner, B. (1982). On the estimation and testing of interclass correlations: The general case of multiple replicates for each variable. *American Journal of Epidemiology, 116*, 722-730.
- Stinson, L., & Ickes, W. (1992). Empathic accuracy in the interactions of male friends versus male strangers. *Journal of Personality and Social Psychology, 62*, 787-797.
- Wilcox, R. R. (1994). Estimating Winsorized correlations in a univariate or bivariate random effects model. *British Journal of Mathematical and Statistical Psychology, 47*, 167-183.

## Appendix A

## Derivation of the Significance Test

Elston (1975) provides a multivariate generalization of the *delta method* (which is based on a Taylor series expansion) for deriving the asymptotic variance of a function of covariances and variances under the assumption of multivariate normality. We applied Elston's method to the pairwise setup presented in this article.

For the two variables  $X$  and  $Y$ , define the corresponding pairwise variables  $X'$  and  $Y'$ , respectively. Define the population intraclass correlations as  $\rho_{xx'}$  and  $\rho_{yy'}$ , the population overall correlation as  $\rho_{xy}$ , and the population cross-intraclass correlation as  $\rho_{xy'}$ . These correlations are each a function of a covariance and two variances, so Elston's (1975) general technique is applicable. Define the  $4 \times 4$  population variance-covariance matrix of the four variables as

$$\Sigma = \begin{pmatrix} \Sigma_x & \Sigma_{xy} \\ \Sigma_{xy} & \Sigma_y \end{pmatrix}$$

such that  $\Sigma$  can be partitioned into  $2 \times 2$  submatrices. Let

$$\Sigma_x = \begin{pmatrix} \text{var}_x & \text{cov}_{xx'} \\ \text{cov}_{x'x} & \text{var}_{x'} \end{pmatrix},$$

$$\Sigma_y = \begin{pmatrix} \text{var}_y & \text{cov}_{yy'} \\ \text{cov}_{y'y} & \text{var}_{y'} \end{pmatrix},$$

$$\Sigma_{xy} = \begin{pmatrix} \text{cov}_{xy} & \text{cov}_{xy'} \\ \text{cov}_{x'y} & \text{cov}_{x'y'} \end{pmatrix}.$$

Recall that in the pairwise case,  $\text{var}_x = \text{var}_{x'}$ ,  $\text{var}_y = \text{var}_{y'}$ , and  $\text{cov}_{xy} = \text{cov}_{x'y}$ , and all three submatrices are symmetric.

We sketch the derivation of the asymptotic variance of  $r_{xy}$ . All other asymptotic variances presented in the article follow a similar derivation. Define the sample estimate of  $\text{cov}_{xy}$  as  $v_{xy}$ , the sample estimate of  $\text{var}_x$  as  $v_x$ , and the sample estimate of  $\text{var}_y$  as  $v_y$ . Because  $\rho_{xy}$  is estimated by  $r_{xy} = v_{xy} / \sqrt{v_x v_y}$ , the asymptotic variances and covariances of these three parameters (creating a  $3 \times 3$  variance-covariance of parameters) must be estimated according to rules provided in Kendall and Stuart (1966). Compute the gradient of  $r_{xy}$  with respect to the three parameters in  $r_{xy}$ , and then pre- and postmultiply the variance-covariance of parameters by the gradient. The result is an approximate asymptotic variance of  $\rho_{xy}$ , which (after algebraic simplification) can be expressed as Equation A1.

According to the null hypothesis that  $\rho_{xy} = 0$ , the entire term within the left-hand curly brackets disappears, leaving the simplified variance  $(1 + \rho_{xx'}\rho_{yy'} + \rho_{xy'}^2)/2N$ . To test the significance of a sample estimate  $r_{xy}$ , the population parameters are replaced by their consistent estimators, and the approximate sampling variance of  $r_{xy}$  according to the null hypothesis that  $\rho_{xy} = 0$  is  $(1 + r_{xx'}r_{yy'} + r_{xy'}^2)/2N$ . This simplified variance is preferred over the full variance in situations with a small number of dyads. Under these conditions, the ratio

$$Z = \frac{r_{xy}}{\sqrt{\frac{1 + r_{xx'}r_{yy'} + r_{xy'}^2}{2N}}}$$

is approximately  $N(0, 1)$  and can be tested as a  $Z$  statistic against the standard normal distribution. Note that when constructing confidence intervals around the sample estimate  $r_{xy}$ , one should estimate the standard error using the full variance formula rather than the standard error that was simplified under the assumption of the null hypothesis.

$$\frac{\rho_{xy}^2 \{ \rho_{xy}^2 + \rho_{xy'} + 1/2[\rho_{xx'}^2 + \rho_{yy'}^2 - 4] - 2[\rho_{xx'} + \rho_{yy'}]\rho_{xy'}(\rho_{xy})^{-1} \} + \{1 + \rho_{xx'}\rho_{yy'} + \rho_{xy'}^2\}}{2N} \quad (\text{A1})$$

## Appendix B

## Steps in Analyzing Dyadic Data Using the Pairwise Correlation Model

1. Set up pairwise ("double-counted") columns of data ( $X, X', Y, Y'$ , etc.).
2. Compute the four basic pairwise correlations for each pair of variables: the two intraclass correlations,  $r_{xx'}$  and  $r_{yy'}$ ; the overall correlation,  $r_{xy}$ ; and the cross-intraclass correlation,  $r_{xy'}$ .
3. Apply the  $Z$  test to the overall correlation  $r_{xy}$  using  $N_1^*$  and to the cross-intraclass correlation  $r_{xy'}$  using  $N_1^*$ .
4. Test the intraclass correlations for significance. If one or both are nonsignificant (i.e.,  $p > .05$ ), then dyad-level processes are unimportant for those variables; the latent dyad-level correlation  $r_d$  would then be meaningless.
5. Compute and test individual-level correlation  $r_i$  using the standard  $t$  test with  $N - 1$  degrees of freedom.
6. If both intraclass correlations are significant (see Step 4), compute the latent dyad-level correlation  $r_d$  and interpret  $r_{xy'}$  as its raw-score counterpart.

Received September 20, 1993

Revision received May 22, 1995

Accepted May 23, 1995 ■