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TRANSACTIONS AND STATISTICAL MODELING: DEVELOPMENTAL THEORY WAGGING THE STATISTICAL TAIL

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What new statistical techniques are available for developmental psychology? What hot new technique is available for me to use in my research? Such questions may appear reasonable, but I believe they place emphasis on the wrong attributes. I prefer to reframe such questions. What new theoretical and empirical properties are developmental psychologists testing and measuring? This changes the focus from asking about new statistical techniques to asking about theory, modeling and testing. It highlights the need to challenge current methodological assumptions and methods rather than to find ways to tweak existing methods to fit new empirical and theoretical questions.

In this chapter I focus on the key features of the transactional model. Some of these features require the development of new methodological techniques that provide appropriate tests. Once the key features of the transactional model are reviewed, I outline several statistical developments that facilitate the implementation of these features in statistical models. My hope is that this chapter will inspire developmental psychologists to think more broadly about the kinds of questions they ask and will inspire methodologists to think more broadly about the statistical techniques they develop.

I thank Arnold Sameroff for his illuminating discussions over several lunches about the transactional model.

BACKGROUND

When thinking about the use of a mathematical or statistical model, one might find it useful to make an analogy to a road map. A road map is helpful to the motorist driving between two cities. The map informs the motorist which streets and highways to take, which is typically the key information the motorist wants to know. A road map does not include every detail, nor is it a perfect replica of the original. The map is a simplified representation that contains only the features necessary for its intended use. The typical road map by itself would not be useful, for instance, in the study of social networks in a city. The road map was not designed to offer information about social networks, though one can imagine different kinds of maps that could be constructed by a researcher studying social networks. A modified road map that overlays key individuals and their social ties could be useful when the goal is to understand the location of the critical nodes of a social network in a geographical layout.

A statistical or mathematical model is analogous to a road map because it is a simplified, abstracted representation of a more complex phenomenon. The mathematical model is a useful representation in so far as the abstraction contains the features that are necessary to understand the phenomenon. Take the simple general linear model $Y = X\beta + \varepsilon$. This model is a simplification in the sense that a dependent variable Y is modeled as a linear combination of predictor variables X (where X is a matrix, including the unit vector that models the intercept) plus a stochastic term ε. One learns in introductory statistics courses (a) to check the assumptions of this model, (b) how residuals can diagnose problems with the equality of variance and normality assumptions, (c) that measures of influence can be used to check for outliers, and (d) to deal with curved data by transforming the dependent variable or the predictor(s) or by including polynomial terms as additional predictors. The simple general linear model can be extended further into other types of statistical distributions such as the generalized linear model that encompasses logistic, probit, Poisson, gamma, and other forms of regression. One learns (a) to extend the simple linear model into one with random-effects terms and multilevel equations that permit modeling different trajectories for each individual and (b) that the simple linear model can be extended to nonlinear regression and that it is even possible to mix and match elements from these various generalizations, such as a multilevel nonlinear regression equation with a Poisson distribution.

There are trade-offs in any modeling exercise. A model can be made more complex, and hence more representative, but it comes at the price of being harder to understand and less parsimonious. As a model is endowed with more complexity, the better it can fit a particular data set. Although complexity makes a model richer, it also makes it more likely that idiosyncratic features of the data dominate the fit, thus reducing the generalizability of the statistical conclusions that emerge from the estimation exercise. Navigating such trade-offs requires trial and error and a solid appreciation of the intended goals of the modeling exercise (i.e., the intended use of the road map).

Some people claim that a mathematical model forces one to be precise. I agree that precision is an important feature of a mathematical model. A lesser known property of mathematical models is that they allow one to be more general in the following sense. If a mathematical property is shown not to represent a phenomenon adequately, one also rejects every theory (not just current theory but theories yet to be developed) that makes use of the mathematical property that has been rejected by data. The usual empirical approach is to test theories one at a time. As new theories emerge, new empirical tests are needed. Today one tests a competitor theory against the old standard theory. If tomorrow a new theory emerges, one needs a new empirical test for the new contender. However, by focusing on properties of theories and testing those properties, one may make more general empirical and theoretical statements that cut across an individual study or test.

A good example of such generality from mathematical modeling occurs in research on similarity judgment. For a long time, the literature focused on geometric psychological representations that could account for similarity judgments. If a child judges two items as similar, those items should be represented as close in "psychological space." Two items judged as dissimilar could be represented as relatively far in psychological space. Through a series of judgments, researchers hoped to characterize the nature of psychological space. Such representations could be used, for example, to study the developmental trajectory of knowledge and meaning. Researchers struggled to find a mathematical representation to account for similarity judgments. Tversky (1977) showed that similarity judgments do not always obey key properties that characterize all distance metrics—symmetry and triangle inequality. *Symmetric judgments* occur when a child judges the similarity of Object A to B to be the same as the similarity of Object B to A (order of judgment is irrelevant). *Triangle inequality* can be viewed as a type of geometrical constraint: If a third Object C is "between" Objects A and B, then the direct distance between A and B cannot exceed the sum of the intermediate distances AC and CB (e.g., the distance between Seattle and Boston cannot be greater than the sum of the distances of Seattle to Ann Arbor and Ann Arbor to Boston). If those two properties are violated, the debate about which distance metric to use is silly because a distance metric for such data cannot exist. It would not be an issue of how well the model fits the data, but the model could be rejected outright for not satisfying the necessary properties of symmetry and triangle inequality. Not just existing theories but any theory yet to be developed that makes use of symmetry or the triangle inequality would be in jeopardy if either of those properties were found not to hold. It is not necessary to conduct new empirical studies to test new theories that make use of symmetry and triangle inequality. Once the empirical boundary conditions of those conditions are known, then one also knows something about the boundary conditions of any theory that makes use of those properties. Tversky (1977; also Tversky & Gati, 1982) provided convincing evidence of violations of both symmetry and the triangle inequality. This evidence provides one example of the benefits of a clear mathematical or statistical model. A good mathematical model can be simultaneously more precise and more general than the standard empirical approach.

There is much benefit from a dual approach that focuses on clear theoretical statements of psychological phenomena and the implementation of those statements in a mathematical or statistical model. In this chapter I highlight some key properties of the transactional model and discuss possible implementations of those properties in formal models. My discussion remains at the general level of process rather than at specific hypotheses about particular variables.

TRANSACTIONAL MODEL

Given my emphasis on having a clear theoretical statement before pursing statistical details, I need to outline the transactional model. There are several published pieces outlining, expanding, and applying the transactional model, so I refer the reader to various sources (Sameroff, 1995, 2000; Sameroff & Chandler, 1975; Sameroff & MacKenzie, 2003). The other chapters in this volume also provide excellent descriptions, illustrations, applications, and extensions of the transactional model. I outline the key pieces of the model that I believe are critical to specify a data analytic framework.

One definition of the transactional model comes from Sameroff (1995): "In the transactional model the development of the child is seen as a product of a continuous dynamic interaction of the child and the experience provided by his or her family and the social context" (p. 663). Sameroff and MacKenzie (2003) stated that

> transactions are documented where the activity of one element changes the usual activity of another, either quantitatively, by increasing or decreasing the level of the usual response, or qualitatively, by eliciting or initiating a new response, for example, when a smile is reciprocated by a frown. (p. 617)

The transactional model has several key properties, including multiple observations (time) over multiple variables of several interacting individuals. The nature of the social interaction creates a context that can also influence the variables. In this sense some variables are *endogenous* because they are response variables, which could also serve as predictor variables. The transactional model allows for heterogeneity of response (people differ from each other); it takes a dynamic perspective because it focuses on change over time; it is path dependent and nonlinear. It posits bidirectional reciprocal relationships between variables. The multivariate structure of the transactional model is rich. It includes person variables, genetic variables, biological variables, societal variables, environmental variables, parental and caregiver/socializer/teacher variables, cultural variables, neighborhood variables, and economic variables, as many of the chapters in this volume illustrate.

In this chapter I outline a few statistical properties that are relevant to implementing the transactional approach in an analytic framework, although I cannot review every statistical detail related to the transactional model in this short chapter. I extend here the article by Sameroff and MacKenzie (2003) that couched the transactional model within a few standard statistical models. Throughout the chapter I assume normally distributed interval data though at some points mention areas in which both models for other distributions (such as Poisson for counts) and extensions to nonparametric models exist. The chapter is organized around the key properties of the transactional model—a multivariate, dynamic, endogenous, heterogeneous, path-dependent system involving multiple individuals.

DESCRIBING MULTIVARIATE PROCESSES OVER TIME

One needs to be able to visualize the phenomenon under investigation. Psychologists typically do not make as much use of graphs as they should (a strong case for the use of graphs in regression models was made by Gelman & Hill, 2007). They are familiar with graphs to diagnose models (such as using residual plots in regression analyses to detect outliers, to check the equality of variance assumption, and to check the normality assumption). Psychologists also use graphs to plot means and trends over time and to highlight details in multivariate time series.

In multivariate time series with k variables in t time points, one observes a data matrix for a single subject structured as

$$\begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1t} \\ y_{21} & y_{22} & \cdots & y_{2t} \\ \vdots & \vdots & \vdots & \vdots \\ y_{k1} & y_{k2} & \cdots & y_{kt} \end{bmatrix}, \tag{12.1}$$

where each row denotes a different measured variable and each column a different time. In general, there may be missing cells in this matrix (e.g., the participant decided not to respond to a variable at a particular time, the researcher chose to omit the variable at a particular time, or some event occurred prohibiting an observation).

The standard plot most people construct has time on the horizontal axis and the dependent variable on the vertical axis, thus displaying the dependent variable as a function of time. Curves can be plotted for individual subjects to illustrate trajectories. Psychologists tend to take each row of the data matrix (a single variable for a single subject over *t* time points) and plot each row against time. Recent advances in latent growth curve analyses wrap statistical theory around the fundamental properties of such a plot (e.g., random effects on intercept and slope terms to handle heterogeneity).

A different type of plot shows multiple variables changing in time on the same plot. That is, multiple rows in the previously described matrix are plotted on the same graph. One way to accomplish this is by plotting variables against variables (as opposed to variables against time) with time as an implicit variable. For instance, Figure 12.1 shows a plot for an individual subject's meals throughout a particular day. The complete variation of the percentage of the meal's carbohydrate, fat, and protein content (which sum to 100%) creates a triangle on which each vertex corresponds to a meal with 100% of that component. For instance, the point (0%, 100%) corresponds to a meal (unrealistic as it may be) that is 100% fat. The other two vertices denote meals that consist of 100% of the other two components. A point in the interior of the triangle corresponds to a meal that has some combination of all three components. To use a concrete example, a meal that is ⅓, ⅓, and ⅓ for each of the components is depicted as the point (33%, 33%) in the triangle. The three meal components sum to 100%, so only two axes are needed to display all three components (the third component is implicit in the graph). I chose to make carbohydrate the implicit component in the graph.

The plot can display more information. For instance, the size of the point can be related to the number of calories, so together the location of the point in the triangle and the size of the point inform the viewer about the key elements of the meal (composition and calories). Time can be depicted as the trajectory of points within the triangle. In this way the meals (including time) can be represented in the triangle. On a computer screen, the graph can be animated so that the trajectory over multiple days can be visualized relatively easily. My collaborators Grazyna Wieczorkowska and Malgorzata Siarkiewicz and I have used this representation to understand actual diets of research participants over several weeks. We have also used color to code points to reflect a sixth variable such as self-reported hunger (the other five variables are percentage of carbohydrate, protein, and fat of the diet; the number of calories; and time). By studying how the points move around the triangle and lagging the hunger variable various hours prior to and after the meal, we can examine, for instance, how hungry a participant feels after a predominantly high-protein meal with low calories compared with a high-calorie predominantly

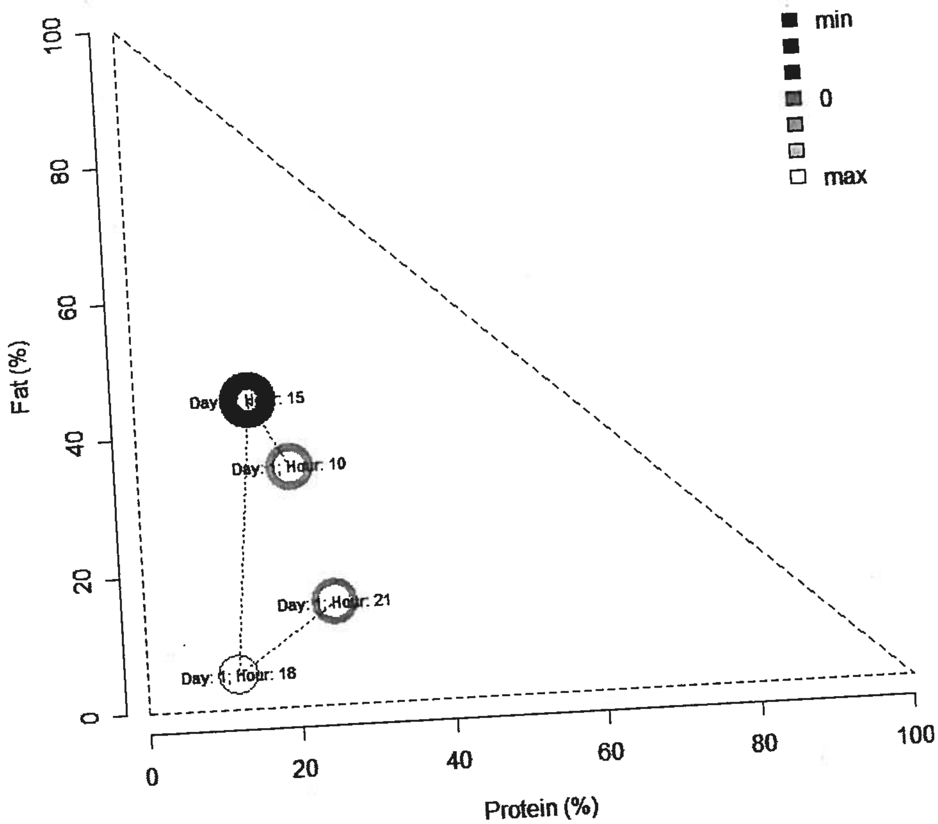

Figure 12.1. Triangle plot depicting four time-stamped meals in one day; the percentage of fat, carbohydrate, and protein for each meal; the number of calories (related to the size of the circle); and the hunger (related to the gray scale of the point) for one subject.

high-carbohydrate meal. Such relationships between multiple variables over time would be relatively difficult to detect using the standard graph with time on the horizontal axis and different curves for each variable. Of course, there remains the problem of adding a statistical conclusion such as a hypothesis test to this graph, but the value of visualizing a dynamic process over multiple variables cannot be underestimated for developing intuition about the processes that underlie the phenomenon. As one adopts more complicated statistical models that allow for individual differences (i.e., different trajectories for each subject), it becomes important to visualize such data and represent the patterns descriptively to achieve a deeper understanding of the underlying processes. A well-designed plot can complement a table of variance component estimates from a multilevel statistics package, making it much easier to understand the finding.

The diet example involves a plot with time as an implicit attribute that is depicted by the movement of the points. A related plot has been used in the study of social engagement between parent and child (Hollenstein, Granic, Stoolmiller, & Snyder, 2004). There are different types of plots that are also useful with multivariate dynamic data, such as plots that illustrate the relation between a variable and its first derivative (e.g., Boker & McArdle, 1995) and plots of parameters, which can facilitate model comparison (e.g., Pitt, Kim, Navarro, & Myung, 2006).

Statistical and Software Implementations

The computer package R has several plotting commands for visualizing trajectories. The lattice package in R produces elegant graphs using modern principles of graph design. The diet plot in Figure 12.1 was generated with an R function that I wrote. A similar triangle plot can be found in the R package plotrix. MlWin, a commercial package for multilevel modeling, has capabilities for plotting trajectories in the context of multilevel models. Other types of useful graphs are vector field graphs that can be computed in packages such as Maple and Mathematica or specialized programs (e.g., Boker & McArdle, 1995).

Researchers sometimes counter with, "Looking at graphs is a great idea, but how do I convey them in a paper?" Obviously, it is not possible to include every graph in a paper, and animated graphs cannot be depicted on the printed page. Key findings, though, need a corresponding table or graph. Consider how much more informative an analysis of variance (ANOVA) can be when accompanied by a table or graph of means. As researchers move into fancier statistical techniques, including growth curve analysis, multilevel models, and dynamic processes, it becomes necessary to develop corresponding methods of displaying descriptive information. Omitting those descriptive features from such fancy analyses detracts from the overall informativeness of the analysis in much the same way as omitting the table of means detracts from the informativeness of an ANOVA source table.

HETEROGENEITY

People differ. Many psychologists give lip service to individual differences; a few psychologists spend a major part of their careers understanding how people differ. Heterogeneity is one of the observations that led to the development of the transactional model. Why is it that not all kids with brain damage develop problems later in childhood? Differences in people are now seen as important and as providing clues about key elements of the underlying process. Psychologists need to use statistical analyses that (a) make it straightforward to estimate and describe how people differ; (b) do not make unrealistic assumptions that all people have the same degree of responsiveness to a manipulation; (c) make it easy to find the predictors of such individual differences; and, if needed, (d) allow variability to be a predictor of other variables.

One freeing aspect of modern statistics is that it is no longer necessary to make simplifying assumptions that all participants within a treatment group are the same or respond to treatment the same way. It is relatively easy to extend the standard statistical models so that heterogeneity is permitted in the parameters—a statistical way of saying that there are individual differences. Coupled with a multilevel model approach, or a Bayesian approach to statistical estimation and inference, it is fairly routine to proceed with statistical models that encompass heterogeneity.

The idea underlying a latent growth curve is that there is a common curve that characterizes a general trend, but there is variability across individuals. Each person can be endowed with his or her own curve, modeled in an efficient way, on which interindividual differences are examined simultaneously with intraindividual differences. This follows from new developments in random-effects analysis, multilevel modeling, and also structural equation modeling. The curve is decomposed into key components such as a slope and intercept in the case of a straight line trajectory, and those components are treated as random effects that permit the modeling of heterogeneity. To me, this is the major benefit that random-effects models offer—they allow one to model heterogeneity. As a side benefit, they also allow a more solid generalization to the population (which is the aspect that many in the field believe is the key benefit of random-effects models).

Latent class models allow one to find groups of individuals who respond similarly. Intuitively, if we take points in parameter space, then individuals with similar parameters form a cluster. Latent class analysis and its extensions deal with the problem of finding such clusters of parameters, such as which subjects have similar slopes and intercepts.

Rather than finding clusters, latent growth curve analysis allows each subject to have his or her set of parameters. Intuitively, one can view latent growth curve analysis as a latent class with N clusters (i.e., one cluster for each subject).

Sameroff and MacKenzie (2003) pointed out that although there may be variability in the individual observations and traits, there may be *constants* in the types of processes that maintain the relation between the individuals and the context. One of the obvious constants is that the person is part of the context. I am impressed with the politeness of many undergraduates who hold the door open or hold the elevator for me as I approach. This could be a general trait of undergraduates or it could be that the graying, middle-aged man who looks like a professor is a common element in those social interactions between me and the undergraduates.

Statistical and Software Implementations

Perhaps the simplest model that includes a form of heterogeneity is the simple paired *t* test. Indeed, every repeated measures ANOVA includes heterogeneity. I illustrate with the simple paired *t* test. The standard parameterization for this model is

$$Y_{ij} = \mu + \alpha_j + \pi_i + \varepsilon_{ij}, \tag{12.2}$$

where each observation Y_{ij} for subject i at time j is modeled as a function of four terms: a constant μ for all subjects at all times, a fixed-effects time parameter α that codes the main effect for the difference between Time 1 and Time 2, a random-effects subject parameter π that codes the main effect for subject, and the usual error term ε. The π terms model individual differences and form the key aspect of what people talk about when they say that a repeated-measures ANOVA can be more powerful than a between-subjects ANOVA because it controls for individual differences. Unfortunately, the standard application of a repeated-measures ANOVA treats individual differences as something to control to reduce the error term. This is not a limitation of the statistical technique but a constraint imposed by how the model is typically used and how it is interpreted. It is possible, as one sees in multilevel models, to embrace individual differences as something to understand, model, and predict.

Latent class analysis can be implemented in various statistical programs. Nagin (1999) has SAS macros for latent class analysis of growth curves. The package flexmix in R allows for the fitting of latent classes in general regression contexts. Latent growth curves can be implemented in standard multilevel programs (e.g., HLM, MlWin) as well as structural equation programs (e.g., LISREL, EQS, AMOS). The statistical package Mplus provides much flexibility for incorporating latent class in standard multilevel and structural equation models. Gibbons and Hedeker (see http://tigger.uic.edu/~hedeker/mix.html) have developed specialized programs for random-effects models for advanced regression such as survival analysis and ordinal regression.

The Bayesian approach provides a natural way to implement multilevel models. Once the analyst assumes a distribution over parameters, then the analyst is in the domain of multiple levels. Bayesian implementations, however, still require some programming on the part of the user for all but the simplest analysis problems. A good Bayesian software program is WinBUGS, and there are several packages in R that implement Bayesian approaches (e.g., MCMCpack). The package MLWin provides a simple Bayesian estimation approach to multilevel models.

Trajectories need not be straight lines. There may very well be important psychological information in the nonlinearity. At what age does a process begin? At what age does the process accelerate or produce a dramatic change in another variable? At what age does the curve hit asymptote? Once one can parameterize such questions, one can ask second-order questions such as what variable predicts the point at which acceleration begins and what processes control the time at which the asymptote occurs? In other words, once one operationalizes a psychological property as a parameter and treats it as a random variable to allow for heterogeneity, then one can predict that parameter (as in any multilevel model). The additional concept here over the previous section of this chapter is that I am now dealing with parameters from a nonlinear representation.

Most psychological researchers who model nonlinearities restrict themselves to polynomials. The usual statistical advice is as follows: If something is not a straight line, try a quadratic. If that does not work, try a cubic, and so on, until you get a good fit. This is a mindless way to model curvature because it merely models the number of bends in the curve—no bends is linear, one bend is quadratic, two bends is cubic, and so on, with a *bend* being an inflection point in the curve or a point having a first derivation equal to zero.

A natural extension of polynomials is to model nonlinearity directly through nonlinear functions. There is much work in mathematical psychology in this regard, for example, in estimating models of reaction time distributions, decision making, and performance (e.g., Busey & Loftus, 1994; Gonzalez & Wu, 1999; Rouder, Lu, Speckman, Sun, & Jiang, 2005; Wu & Gonzalez, 1996). The model by Busey and Loftus (1994) is relatively simple to illustrate without getting into the experimental details. The problem is digit recall when strings of four digits are presented for very brief durations. The performance curves (chance corrected) fit quite well with a nonlinear function of the following form:

$$p(t) = \begin{cases} 0 & (t < L) \\ Y\left(1 - e^{-(t-L)/c}\right) & (t \geq L) \end{cases} \tag{12.3}$$

where p is the proportion correct, t is the exposure duration of the stimulus, c is the exponential growth constant, L is the maximum duration that gives chance-level performance, and Y is asymptotic performance (which Busey & Loftus, 1994, set to 1 in their article).

The interesting point about this functional form is that the parameters have psychological meaning. Specifically, L is interpreted as the duration at which the curve begins to move away from chance performance. In other words, the value of L is the time at which the process begins. The parameter c indicates the rate of change; the parameter Y indexes the asymptotic level of performance. It is possible to allow for heterogeneity by incorporating a random-effects approach to the estimation, thus allowing each subject to have his or her own values for the three parameters L, c, and Y. In this way, the curves are modeled with parameters that provide relevant psychological information. Contrast this approach with the more typical polynomial regression approach that merely indicates whether trajectories are linear, quadratic, cubic, or some higher order (though compare with Cudeck & du Toit, 2002, who reparametrized the quadratic into more interpretable and psychologically relevant parameters).

There is a different sense of nonlinearity that is also relevant to the transactional model. This form is related to dynamic systems theory and involves, for example, models that are sensitive to starting configurations and deal with nonlinear differential equations. There has been some attempt to use dynamic systems theory in developmental psychology but a complete review of this approach is beyond the scope of this chapter (see, e.g., Granic & Hollenstein, 2003).

Statistical and Software Implementations

Most statistical packages allow nonlinear estimation techniques, including the popular commercial packages SPSS and SAS. There are plenty of new developments underway in the statistical community, including generalized nonlinear regression that works for data that follow generalized exponential distributions such as binomial, Poisson, gamma, and negative binomial. Within the R program, two relevant packages are lmer and nlme.

There are also nonparametric techniques such as splines that can be used to model nonlinearities. There have been new developments within the Bayesian approach to include a random-effects approach to splines (i.e., each subject is allowed to have a different number of knots as well as different knot locations). An excellent example of this work is found in Kim, Menzefricke, and Feinberg (2007). In this way it is possible to have interpretable parameters (knots) of subject trajectories, allow for nonlinearities through splines, and allow for heterogeneity through random effects in the parameters.

MULTIPLE INDIVIDUALS

People influence each other. Together, people create contexts that are emergent and influence the behavior, thoughts, and feelings of individuals in the group. There are processes related to both interdependence and similarity that can occur with multiple interacting individuals. My collaborator Dale

Griffin and I have written about these different approaches to modeling dyadic and group data (Gonzalez & Griffin, 2001). Interdependence and similarity challenge the usual statistical assumption of independence by introducing correlations across people. Statistical models of social interaction include interdependence and similarity as psychological parameters that are modeled rather than treated as nuisance variables that need to be eliminated or controlled. For a recent book-length treatment, see Kenny, Kashy, and Cook (2006).

For a long time, interindependence of data was treated as a nuisance that needed to be eliminated or corrected. The focus had been on the nasty effect violating independence had on standard errors and tests of significance. The newer models change the emphasis of interindependence from one of nuisance to showcasing the interdependence as a relevant psychological property that needs to be captured, modeled, and understood (Gonzalez & Griffin, 2000).

One notion of interdependence can be captured using the actor–partner model (e.g., Cook & Kenny, 2005). The actor–partner model involves an extension of the standard cross-lagged regression path model to a situation of multiple variables over multiple people. Figure 12.2 depicts a simple model for distinguishable dyad members. The stability coefficients (horizontal paths *a* and *d*) reflect the relation between two variables for the same person, partialing for the cross-path (i.e., in the case of the dyad, for the other person's predictor variable). Likewise, the cross-paths (paths *b* and *c*) represent the influence of one person's predictor on the other's criterion variable, partialing

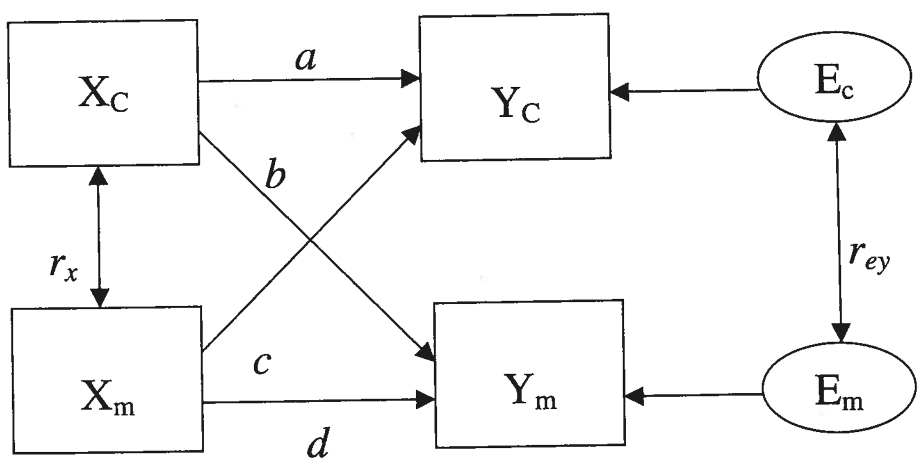

Figure 12.2. Actor–partner model. The focus is on partialing out one's own influence on different variables from the influence the partner has on one's own variables. Horizontal paths *a* and *d* reflect the relation between two variables for the same person, partialing for the cross-path. The cross-paths *b* and *c* represent the influence of one person's predictor on the other's criterion variable, partialing for the stability coefficient.

for the stability coefficient. A simple example is that a predictor variable is measured for both the mother and the child (X_m and X_c). A second variable is also measured for both the mother and the child (Y_m and Y_c). Errors are correlated because individuals are members of the same dyad or group. The model is modified slightly when the two individuals are exchangeable, such as in same-sex couples or same-sex siblings (D. Griffin & Gonzalez, 1995), and can be extended to groups (Gonzalez & Griffin, 2001).

A different type of interdependence is based on similarity. The latent variable model captures similarity, or shared variance, at the dyad level as distinct from unique variance at the individual level. The latent variable model allows for correlations across the dyad-level latent variables (group-level correlation) as well as correlations across the unique variance for the same individuals (individual-level correlation). As shown in Figure 12.3, in the case of two variables for each dyad member, one can model the shared variance on each person as well as each dyad. As with the actor–partner model, modifications to exchangeable cases and groups have been made (Gonzalez & Griffin, 2001; D. Griffin & Gonzalez, 1995).

Statistical and Software Implementations

Models for nonindependence across multiple individuals can be implemented in multilevel models and in structural equation models as well as in simple Pearson correlations once the data are organized in an appropriate manner (e.g., Gonzalez & Griffin, 1999; D. Griffin & Gonzalez, 1995; Olsen

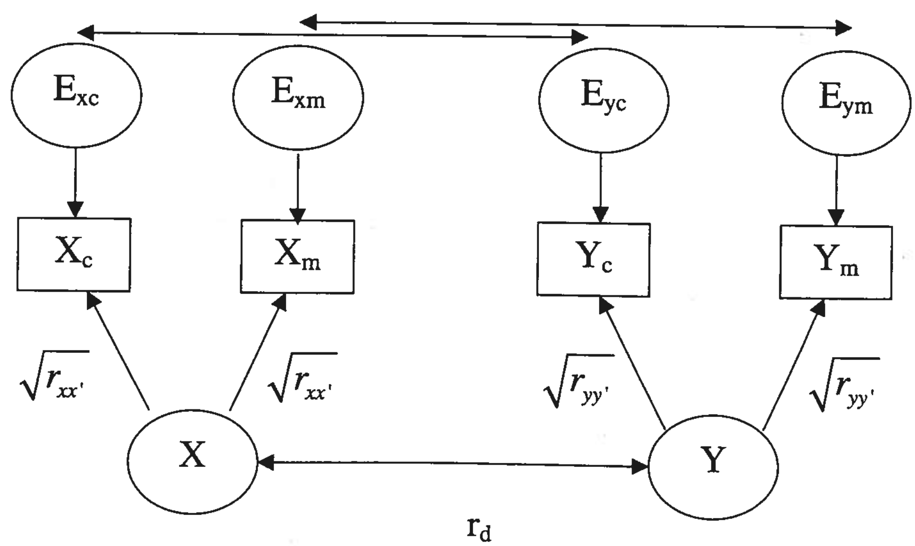

Figure 12.3. Latent variable model. The focus is on (dis)similarity within dyad members.

& Kenny, 2006; Woody & Sadler, 2005). One can take a multilevel approach that individuals are nested within the dyad (or group). Thus, interdependence can be captured and modeled through the nesting procedure. One implementation of this approach assigns a different linear equation to each person and estimates covariances of parameters across equations, that is, the covariance of the same parameter over two people (Barnett, Marshall, Raudenbush, & Brennan, 1993). Or, equivalently, one may take a structural equation modeling approach and add another layer of latent variables to represent the group as shared variance from the individuals who compose the group (e.g., Gonzalez & Griffin, 1999; Kenny & la Voie, 1985). There are additional complications in implementing analyses in statistical packages, such as whether the dyad members are exchangeable or distinguishable. The extension to longitudinal dyadic or group data analysis necessary for testing transactional models is still in its infancy, though for preliminary suggestions see (Gonzalez & Griffin, 2002). W. Griffin (2000) provided a coordination index for multiple individuals along with ideas for visualization and animation of the process. Another interesting extension is the added richness that can emerge from data that have a round-robin structure (e.g., Cook & Kenny, 2004). Much progress is needed though to capture the interdependent processes that are posited in the transactional model—multiple people across multiple variables with bidirectional, reciprocal relations.

ENDOGENEITY

The property of endogeneity may be one of the most important components of the transactional model. Usually, an *endogenous variable* is defined as a variable that is modeled as inside the system being studied. An endogenous variable can be observed or latent; it is predicted by other variables, and it can also predict other variables. By contrast, an exogenous variable is a variable outside the system being studied and so cannot be modeled as a dependent variable within the system.

The transactional model posits that multiple people influence each other across multiple variables over time. Hence, one individual influences another and vice versa. This leads to problems of bidirectionality as discussed by Sameroff and MacKenzie (2003). There are also corollary processes to endogeneity. There are processes that are emergent within the system. *Emergent* properties are those that are not necessarily observable at, say, the individual level but are observable at a higher level of analysis such as the group level. The hydrogen atom has particular characteristics, as does the oxygen atom, but the H_2O molecule has emergent properties that are not easy to predict from only the properties of the individual atoms. Psychological examples of emergent properties are constructs such as norms and groupthink, which can be modeled as group-level processes that are not so easy to understand from merely knowing the properties of individual group members.

Economics has been dealing with endogeneity for a long time. Theoretical relations between supply and demand are endogenous and bidirectional. Demand changes supply; supply changes demand. In economics, such processes are modeled through simultaneous equations. Demand may be influenced by supply along with a set of predictor variables; supply may be influenced by demand along with a set of (not necessarily overlapping) predictor variables. A body of econometrics is devoted to these kinds of bidirectional models (e.g., Haavelmo, 1943; Yang, Chen, & Allenby, 2003).

Endogeneity characterizes the idea that although the system influences me, I also influence the system because I am part of the system. That is, I am part of and define the very system that is influencing me. This is a difficult concept for researchers brought up in the sophomoric tradition of an independent variable influencing a dependent variable. As Sameroff and MacKenzie (2003) pointed out, one needs to understand both the parents' role in the child's behavior as well as the child's role in eliciting the parents' behavior (see also chap. 11, this volume). The physicist Robert Savit and I, in collaborative research, modeled the decision-making behavior of individuals in a group in which the outcomes given to each group member depended on the group decision (Savit, Koelle, Treynor, & Gonzalez, 2004). The outcome is created by the decisions of each group member. Many aspects of social cognition, for example, that borrow ideas blindly from cognitive psychology to model social psychological processes miss the key idea that in social interaction the social stimuli that make up the cognitive machinery "look back" and interact with the social perceiver. Such an interaction with stimuli and subject is not usually possible in the standard cognitive study, and attempts by researchers to apply such models from cognitive psychology to social interaction when the possibilities for endogeneity are high are likely to fail in modeling complicated social interaction (Ickes & Gonzalez, 1994).

Statistical and Software Implementations

Structural equation modeling can handle endogeneity and also the simultaneous equation problem (e.g., supply and demand). One merely sets up a linear equation for demand and a linear equation for supply (with supply as one of the predictors for demand and demand as one of the predictors for supply). Each equation has its own error term, and the error terms are correlated because the two equations are estimated from the same data. Details of the implementation can be found, for instance, in the documentation to PROC CALIS, which is the structural equation modeling procedure built into SAS. In this way bidirectionality can be incorporated into the structural equation framework. There are identification difficulties one may encounter, so bidirectionality is not as straightforward as merely adding two arrows in opposite directions to the usual "square–circle–arrow" structural equation graph. Some of these difficulties are discussed in path analysis and structural equation modeling textbooks such as Kenny (1979).

DYNAMIC PROCESSES

People change. It is obvious that time is a necessary aspect of change. To understand a dynamic process, though, one typically needs more than two time points. As Rogosa, Brandt, and Zimowski (1983) pointed out in their defense of difference scores to measure change, the difference of two time points can be interpreted as a slope. But to have a rich data source to test dynamic processes it is necessary to have more than two time points.

There has been recent interest in nonlinear dynamical systems (e.g., Durlauf & Young, 2001; Gottman, Murray, Swanson, Tyson, & Swanson, 2002; Nowak & Vallacher, 1998). These analytic tools promise to provide much insight into dynamic processes. However, a major chunk of the work has been isolated to computer simulation and has focused on studying how the parameters of the model in different combinations produce different observable behavior. Much knowledge can be gained from this approach if used properly, but it is not the standard data-fitting exercise. Indeed, I am aware of very few cases in which model fitting in the sense of using data to find the best fitting set of parameters in a dynamic system is a tractable problem. In many cases, the parameters themselves are highly intercorrelated, making a general estimation routine intractable.

Statistical and Software Implementations

Time-series analysis is a standard analytic approach that psychologists use with repeated measures. A major focus of this approach is the autocorrelation due to repeated measurement. In some cases this autocorrelation is treated as something that needs to be corrected, whereas in other cases the temporal correlations are the exciting aspects of the data to mine. As mentioned in an earlier section, Boker and McArdle (1995) provided one attempt to use data to analyze graphically the properties of dynamic systems. Developing statistical analyses of dynamic systems that include data fitting and parameter estimation is still, in my opinion, an open area of research. Boker and Nesselroade (2002) provided insights into dynamical systems and discussed the challenges in implementing statistical estimation with few time observations and in the presence of measurement error.

PATH DEPENDENCE

It matters where I came from. The property of endogeneity reviewed in the previous section produces a further nuance: Not only do variables influence each other, but there are processes that constrain future responses and outcomes. If a child throws a temper tantrum, the parent has various ways to respond, but the act of the temper tantrum constrains the parent's potential responses. Likewise, once the parent selects a response to the temper tantrum, that response constrains or affords subsequent behaviors of the child. Such a process is represented in Figure 12.4. The child performs an action *a*. The parent has several responses and chooses one (depicted by the solid line segment). An optimal response for that situation is denoted with an asterisk. Following the solid line path from left to right (time), one sees that the child has several actions at *g*, not all of which may be optimal. If the child's choice depends only on the previous action *f* by the parent, without consideration of how the situation arrived at *f* (e.g., either through *e* or *e**), then the system is said to be path independent (i.e., satisfies the Markov property). However, if the child's choice depends on the particular path in getting to *f*, then the system is said to be path dependent. Heckman's (2005) framework of causality could handle such processes through selection in the sense that parent's

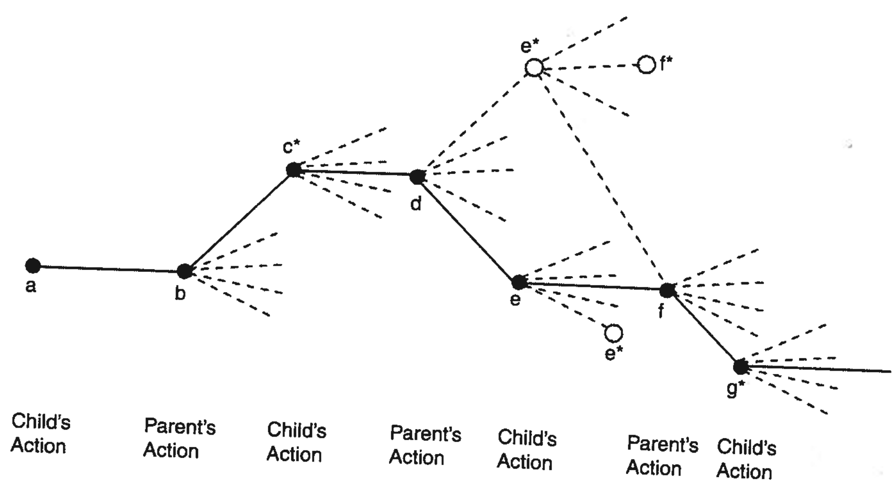

Figure 12.4. Sequence of child–parent behaviors in which each behavior provides constraints for the next turn. An asterisk denotes an optimal response.

choice assigns constraints to child's behavior, child's behavior assigns constraints to parent's behavior, and so on.

Contemporary researchers in computer science and neuroscience have rediscovered concepts from the learning literature (e.g., Hebbian learning) and have developed new models called, broadly, *reinforcement learning models* (Sutton & Barto, 1998). Most of these models assume the Markov property of path independence, but these models show promise for allowing the modeling of path-dependent processes. I am aware of some applications of reinforcement learning models in economic behavior and decision making (Erev & Roth, 1998; Salmon, 2001). There is opportunity to develop statistical models that permit testing of path-dependent transactional processes.

Statistical and Software Implementations

A very promising statistical approach in the spirit of such models that permit path dependence is the state space approach developed by Granic and Hollenstein and colleagues (e.g., Granic & Hollenstein, 2003; Granic, Hollenstein, Dishion, & Patterson, 2003; Granic & Lamey, 2002; Hollenstein et al., 2004). The extension of traditional models based on conditional probabilities to allow for path dependence is an exciting open area of research that could use various new developments in complex systems. Gardner (1990) presented a Markov model for sequential categorical data that allows for heterogeneity. W. Griffin and Gardner (1989) provided a framework for path independent models of duration data in social interaction and pointed to the difficulty of adding heterogeneity to such models within a classical statistical approach.

CONCLUSION

In this chapter, I have presented a brief conversation about interesting issues that a transactional perspective to developmental psychology poses for statistical models. There are many details I did not elaborate and many issues I could not discuss. Sameroff and MacKenzie (2003), for example, discussed examples of the transactional model involving experimental manipulations, quasi-experimental designs, naturalistic observations, and intervention studies. There have been many exciting developments in the statistics community around the problems of inferring causality in cases in which randomization is not possible (e.g., Heckman, 2005; Rubin, 2005). The general approach is to model the observation and its corresponding counterfactual, such as what the observation would have been had the case been in another condition. This approach has provided several new advances: new ways of dealing with covariates such as propensity scores, new ways of dealing with differential dropout rates, new ways of handling matching designs, and new ways of modeling missing data. It may provide a useful framework for developing some of the extensions that are needed to implement the transactional model.

There has been a long-standing appreciation of the power of models in our understanding of developmental processes (Sameroff & Chandler, 1975; Sameroff & MacKenzie, 2003; Sigel & Parke, 1987). Likewise, there are many new developments in the statistics and biostatistics literature for general models of repeated measurements that have direct relevance to the transactional model (e.g., Lindsay, 2004; Vonesh & Chinchilli, 1997). But there is much left to develop. This chapter has met its goal if it has alerted developmental psychologists to some of these new statistical tools and has made it clear that for these statistical tools to be useful one needs to be clear about the features of the psychological model. There is a transactional model of sorts between the theory one wants to test and the statistical procedures one uses to test that theory.

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