THE DEVELOPMENT OF STUDENTS' CONCEPTIONS OF SIZE

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The concepts of size and scale are important to both science and science learning. These concepts must be robust if students are to use them to connect their science learning, as reform documents suggest. Yet recent research shows that most people do not have a firm grasp on size and scale. Improved curriculum, instruction, and assessment of size and scale can be guided by a learning progression – a characterization of learners' successively more sophisticated ways of thinking about a topic over time. This empirical study uses interviews and card tasks with 7th graders through undergraduates to generate a preliminary learning progression describing how well and in what order students establish connections among four conceptions of size: ordering, grouping, number of times bigger one object is than another, and absolute size. Students' knowledge ranges from entirely disconnected to well connected, varying widely within pre-college grades. Science course and academic ability are significant predictors of connectedness; gender, race, and grade are not. The progression fits 46 of 48 students in one of two closely related trajectories. The connection between number of times bigger one object is than another, and absolute size, is the last to be made. Implications for practice are suggested.

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The Importance of Size and Scale

In Scientific Theory

Greek atomism was the most influential ancient theory "to account for the apparent order and regularity found in the world" (Berryman, 2004) without resorting to teleological or theological explanations (Berryman, 2005). Atomism was revived in the seventeenth century to explain the natural world, although there was no empirical evidence for the theory at that time and atoms lay "far beyond the domain of observation" (Chalmers, 2005). By the nineteenth century, "the fact that the properties of chemical compounds are due to an atomic structure that can be represented by a structural formulae [sic] was beyond dispute" (Chalmers, 2005). In the twentieth century, empirical investigation of the atomic nature of matter finally vindicated aspects of the ancient Greek theory. According to Thomas Kuhn,

> only the civilizations that descend from Hellenic Greece have possessed more than the most rudimentary science. The bulk of scientific knowledge is a product of Europe in the last four centuries. No other place and time has supported the very special communities from which scientific productivity comes. (1996/1962, p. 168)

Today, the central importance of atomic theory to science is well established. In Feynman's words:

> If, in some cataclysm, all scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words? I believe it is the *atomic hypothesis* (or atomic *fact*, or whatever you wish to call it) that *all things are made of atoms - little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another*. In that one sentence you will see an *enormous* amount of information about the world, if just a little imagination and thinking are applied. (1963, I, 1-2)

Just as the concept of the atom is central to science, size is an essential attribute of the atom. In Democritus' atomistic theory, the only perceptible qualities in atoms are size, shape, and perhaps weight (Berryman, 2004). When atomism was revived in the seventeenth century, atoms "were characterized in terms of just a few basic properties, their shape, size and motion." (Chalmers, 2005). Twentieth-century quantum theory problematized the very concept of the size of an atom. Central to quantum theory is the idea that electrons behave simultaneously like waves and particles. Waves continue indefinitely, thus having no clear size. The probabilistic nature of the size of an atom and other objects is a novel and essential feature of quantum theory. Thus, size is a central element of arguably the most important scientific theoretical construct – the atom.

In Scientific Practice

While theory is important in science, empirical investigation is also key, and here, too, size is important. Size and shape are directly observable properties of objects. The history of science is, to a large degree, the history of the development of tools that allow us to explore and observe objects too small, too large, or too distant to see with earlier tools. Kuhn notes that one mechanism for the resolution of scientific crises is for the problem to be set aside for future generations with the necessary tools to solve it (1996/1962).

In the application of scientific knowledge, scale is a paramount consideration. Objects or organisms of different sizes behave differently, even if scaled up faithfully. As Haldane (1928) points out, a thousand-yard vertical fall that barely shocks a mouse will kill a rat, break a man, and practically liquefy a horse. According to Bazant (2002),

> Scaling is the quintessential aspect of every physical theory. If scaling is not understood, the theory itself is not understood. So it is not surprising that the question of scaling has occupied a central position in many problems of physics and engineering…In solid mechanics, the scaling problem of main interest is the effect of the size of structure on its strength. This problem is very old…Its discussion started in the Renaissance. (p. 1)

In Scientific Disciplines

In specific disciplines of science, the importance of size and scale is clear, too. In astronomy, the distances and sizes of celestial objects are so great that they are nearly beyond human comprehension. In biology, the maximum size of a cell is limited partially by the rate at which diffusion can allow nutrients in and waste products out. In biochemistry, the proofreading mechanisms of translation, whereby the genetic code of DNA is used in the synthesis of proteins via RNA, depend largely on the size and shape of amino acid side chains. In chemistry, many phenomena can described and explained using macro-level variables such as pressure or temperature, as well as using atomic-level constructs such as elastic collisions between atoms or molecules, or the velocity of these particles.

As new fields of science and technology emerge, science instruction and curriculum materials need to change accordingly. One such emerging field is nanoscale science and technology (Gilbert, De Jong, Justi, Treagust, & Van Driel, 2002, p. 395). The nanoscale is defined by the size of the objects it studies, between one and 100 billionths of a meter in one or more dimensions. Objects at this scale behave differently than both the bulk (macro-level) materials we are accustomed to, and smaller, atomic-sized objects. The greatly increased surface area-to-volume ratio of nanoscale objects – a size-dependent quantity – is responsible for many of the interesting properties and behaviors of these objects. New forms of microscopy allow scientists and engineers to study and manipulate matter at this scale, with important applications for materials science, information technology, medicine, and consumer products. Nanotechnology is expected to generate one trillion dollars in yearly revenues within a decade (Roco, 2005). The US is presently a world leader in this emerging field. A "firm grasp on size and scale [is] a prerequisite for any further inquiry into nanoscale science and engineering" (Waldron, Sheppard, Spencer, & Batt, 2005, p. 375). Thus, one motive for researching student conceptions of size and scale is to inform the introduction of nanoscience topics to the curriculum, in order to promote the economic competitiveness of the US, and in order to help educate future citizens that are scientifically literate and able to work and make informed decisions in an increasingly high-tech world.

In Scientific Learning

Conceptually, size and scale are fundamental. They constitute a cognitive framework that helps us make sense of scientific and everyday phenomena. Scale is an important "common theme" that can link student understandings across topics, disciplines, and grade levels (American Association for the Advancement of Science [AAAS], 1993, Ch. 11). Common themes are ideas that "pervade science, mathematics, and technology and appear over and over again…They are ideas that transcend disciplinary boundaries and prove fruitful in explanation, in theory, in observation, and in design." (AAAS, 1989, cited in AAAS, 1993, Ch. 11). These common themes, including scale, are ways of thinking rather than content to be taught (AAAS, 1993, Ch. 11). Common themes can bring coherence and continuity to the American science curriculum, helping learners structure their knowledge (AAAS, 1993; Tretter, Jones, Andre, Negishi, & Minogue, 2006), consistent with a reform vision of education (as embodied by National Research Council [NRC], 1996, 1999; AAAS, 1993). Size is intimately related to scale: large variations in the magnitude of scientific variables such as size, distance, weight, and temperature are a "starting point for the idea of changes of scale" (AAAS, 1993, Ch. 15). (See Appendix A for definitions of size and scale-related concepts created by a team of expert scientists and educators.)

The National Science Education Standards (NSES) state that unifying concepts and processes "unify science disciplines and provide students with powerful ideas to help them understand the natural world." (NRC, 1996). These unifying concepts include measurement, which is closely related to size and scale.

The *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000/1989) identify measurement as one of six strands describing content students should learn in K-12th grade. These standards also emphasize the need for interconnections among math topics: "When students can connect mathematical ideas, their understanding is deeper and more lasting." (p. 63). The math standards also point to the need to connect mathematics with other fields of knowledge. In a similar vein, the *Benchmarks* identify mathematics as an essential tool of science (AAAS, 1993).

In summary, the main standards documents in science and mathematics point to the need for unifying concepts – such as scale and the related concepts of size and measurement – that can create connections in curriculum as well as in students' minds, creating a more robust type of knowledge. This idea is congruent with current conceptions of how people learn (e.g., NRC, 1999).

However, current curriculum and instruction may not be successfully addressing size and scale, for science education research has identified many areas of conceptual difficulty for learners that are related to size and scale. In Chemistry, research has documented "levels confusion" (Wilensky & Resnick, 1999), where students attribute macro-level changes to atoms or molecules. For instance, students may state that during a phase change, atoms melt, or during thermal expansion, atoms get bigger. In the context of geography, incomplete understanding of scale leads to confusion about localization on a map (Montello & Golledge, 1998). Recent research directly concerning students' and adults knowledge of the size of objects has similarly shown that a paucity of accurate knowledge (Tretter, 2004; Tretter, Jones, Andre, et al., 2006; Tretter, Jones, & Minogue, 2006; Waldron et al., 2005; Castellini et al., 2007; Waldron, Spencer, & Batt, 2006).

Teaching Size and Scale: The Need for a Learning Progression

The US science and math curriculum has been characterized as being a mile wide but an inch thick (Schmidt, McKnight, & Raizen, 1997). What typically results from such a curriculum is not robust, connected knowledge but a "splintered vision" (Schmidt et al., 1997). Focusing on size and scale, and other "common themes" (AAAS, 1993) or "unifying concepts and processes" (NRC, 1996) in science education, can help address the fragmentation of knowledge. By explicitly making salient the links between topics and disciplines over years of school instruction, curriculum and assessment is more likely to foster connections in the mind of the learner.

A coherent, multi-year instructional plan for size and scale will need to take into account the prior knowledge and preparedness of students at different points in time. Instruction cannot be guided solely by experts' determination of a content-oriented logical sequence, for experts are likely to have forgotten the conceptual challenges and difficulties they encountered as learners (NRC, 1999). Empirical research can guide the development of "learning progressions" (Smith, Wiser, Anderson, Krajcik, & Coppola, 2004; NRC, 2007), which are "descriptions of the successively more sophisticated ways of thinking about a topic that can follow one another as children learn about and investigate a topic over a broad span of time (e.g., 6 to 8 years)." (NRC, 2007, p. 214).

One of the goals of the National Science Foundation-funded National Center for Learning and Teaching Nanoscale Science and Engineering (NCLT) is to suggest ways of incorporating appropriate nanoscale science concepts into the 7-12th grade curriculum. A learning progression for size and scale can guide these efforts. However, there is little research into how people understand the concepts of size and scale (Tretter, Jones, Andre, et al., 2006, p. 283). The AAAS *Benchmarks* (1993) include supporting research literature for many topics, but not for the common theme of scale or the related concept of size. Recent research into size and scale (e.g., Tretter, 2004; Tretter, Jones, Andre, et al,. 2006; Tretter, Jones, & Minogue, 2006; Waldron et al., 2005; Castellini et al., 2007; Waldron et al., 2006), begins to address this research gap, but more work needs to be done. The current paper describes our efforts to augment the research base on size and scale, and proposes a learning progression for size, to be further elaborated by future analyses and studies.

Literature Review

Survey-Based Studies

Several recently-published papers present the findings of surveys focused entirely or partially on size and scale. The survey format allows for large numbers of participants but precludes clarifications about prompts and the extensive, individualized probing that the interview format affords.

Castellini and colleagues (2007) report on a survey of nearly 500 people of ages seven to 91, focused on understanding of nanotechnology but including some items on size and scale. Responding about the smallest object they could think of, more than two-thirds of the 2^{nd}-5^{th} graders mentioned a small but macroscopic object, that is, an object visible to the naked eye, or a nonsensical item. By middle school, fewer than 20% of 6^{th}-8^{th} graders mentioned macroscopic or nonsense items, and nearly 60% mentioned the atom. Surprisingly, early high school students (9^{th}-10^{th} graders) had poorer responses than the middle schoolers. Early high schoolers had half again more macroscopic or nonsense replies than middle school students; only around one-third mentioned the atom. Around 12% of early high school students mentioned sub-atomic particles, while next to no middle schoolers did. Older high school students had better responses, with around 14% macroscopic or nonsense answers, and over 30% mentioned subatomic particles. College-educated adults fared best of all groups, with only around 10% macroscopic/nonsense answers and over 40% responding with subatomic particles. High school-educated adults performed quite poorly: around 10% answered with a microscopic object, with the rest about equally divided between the atom and macroscopic/nonsense answers, and no mention of subatomic particles. Perhaps the most noteworthy finding are that most pre-middle school students have trouble thinking of objects too small to see with the unaided eye; a dip in performance from middle school to early high school; and the poor performance of high school-educated adults, better only than that of elementary students. Additional questions asked the respondents to rank by size four objects too small to be seen, and four small, visible objects. Only 7% correctly ordered the atom, water molecule, bacterium, and cell, while 45% correctly ordered housefly, dust, eyelash, and grain of salt. From our own experience interviewing students (see below), where we had to clarify respondents' doubts about a head of a pin, it seems possible that the difficulty in distinguishing some of the items (e.g., bacterium and cell) and the ambiguity of others (e.g., "dust", or the dimension to be considered for the eyelash), may have affected student performance on the tasks. The survey shows, however, that people are better at ordering macroscopic objects than objects too small to see.

Waldron and colleagues (2006) surveyed 1500 respondents from ages six to 74, also about nanotechnology and including some items on size and scale. This survey also asked respondents to name the smallest object they could think of. The age ranges and coding categories used for analysis differ from those of the Castellini survey (Castellini et al., 2007) , making direct comparison difficult, although the overall trends seem similar. The percentage of macroscopic responses descends from a high of over 70% for under-eight year olds, to less than 30% for adults 18-22 years old. Other adult age groups varied between 25 and 50% of macroscopic responses. Respondents were also asked to rank three objects by size: atom, molecule, and germ, as well as three units of measure: millimeter, micrometer, and nanometer. Across all ages, respondents were more successful at ordering units of measure than objects, although children under 11 had difficulty with both. Success rates are reported only for 11-13 year olds on the object-ordering task (15%, which is around chance for three items), and the authors note that adults had trouble with the task as well.

Tretter and colleagues (Tretter, Jones, Andre, et al., 2006) explored how people from grades 5 through doctoral students organize objects into categories by size. Participants from grade 5-9 were 76% white, with 10-35% qualifying for free or reduced lunch. Participants in the 12th grade were elite students, 63% of whom were white. The procedure included a survey instrument as well as interviews (discussed in the next section), in which the participants assigned 26 objects to predetermined classes, such as 'between 10 m and 100 m' or 'between 1000 and 1 million m'. These objects ranged from subatomic particles to galactic distances. That study analyzed the results from the survey in two ways, in both cases using aggregate data averaged over all the participants of a given educational grade or level. The first procedure paid attention to the size categories into which the participants had placed the objects, yielding an 'absolute' size. The second procedure focused on the order in which students had organized the objects, yielding the 'relative' ranking. Younger groups displayed lower accuracy and greater variance than older, more expert groups. The youngest students tended to rank small macroscopic objects as being smaller than atoms, viruses, and cells (in concordance with the survey studies mentioned above). Relative rankings were more accurate than absolute rankings across all groups. This means that participants found it easier to say that the distance from Los Angeles to New York is smaller than the distance from Earth to the moon, than to correctly assign these distances to a measurement range. This finding is consistent with many research studies showing that relative quantities are more accessible to learners than absolute (e.g., Vasilyeva & Huttenlocher, 2004; Bryant, 1974, cited in Markovits & Hershkowitz; Trend, 2001; Dahl, Anderson, & Libarkin, 2005; Graham, Ernhart, Craft & Berman, 1964).

Interview-based Studies

Tretter and colleagues (Tretter, Jones, Andre, et al., 2006) also conducted interviews to determine what types of experiences students drew upon in learning about size and scale. Experts referred to more formal, specific experiences in work or school, with generic descriptions of informal experiences more common as age decreased. These experiences fell along two dimensions: visual to kinesthetic, and holistic to sequential. This study also included a card sort task in which participants were asked to arrange 31 cards portraying objects again covering a huge range of sizes, into groups (as many as they decided were necessary), and later asked to explain the reasoning behind their groups. The data were again aggregated. All groups used the human body as a reference size, and older, more expert people categorized objects into larger numbers of groups; room size and field size were two other landmark objects that distinguished size ranges. Younger students tended to group small visible objects together with microscopic and nanoscopic objects. The authors suggest that the landmark objects (body, room field, etc.). "may serve as useful prototypes for students to use to anchor conceptions of spatial scale across a spectrum of sizes." (p. 210).

The same population as in the study above was interviewed and surveyed to determine accuracy of spatial scale conceptions (Tretter, Jones, & Minogue, 2006). The survey asked participants to name objects of sizes ranging from 1 nanometer to 1 billion meters, and from one-billionth to one billion times body length. The sizes differed by factors of 10 near body size, and by factors of a thousand at sizes farther from human size. Responses were coded as acceptable if they were within an order of magnitude of the correct size; other categories distinguished between answers 10-100, 100-1000, and over 1000 times too large or small. The study found that all age groups had good performance on a central range of sizes near human size, accuracy on large objects/distances declined smoothly as size increased, but accuracy dropped precipitously at the micron range. Younger students had a smaller range with good accuracy, and experts had better accuracy for nm-sized objects than for micron-sized ones. Except for experts, respondents tended to provide objects that were much too small when asked for large objects/distances, and objects too large when asked for very small items. Elementary students preferred thinking in meters due to the perception that body size changes; experts and seniors preferred the metric system due to familiarity with the units. Middle and high school students preferred thinking in terms of the body. Experts divided the scale of sizes into "worlds" distinguished by benchmark objects such as atom or Earth, or by units, or by tools used to visualize at that scale. These worlds were largely disconnected from each other and from everyday objects. Experts also used a "unitizing" strategy, redefining distances into more convenient units or in terms of benchmark objects.

Areas Unaddressed by the Research Base

The four papers outlined above include a variety of tasks assessing size and scale skills. Determining relative size, or ranking objects (or assigning them to ranked categories) by size corresponds to what in this paper is called *ordering*. Stating the smallest object one can think of implicitly requires ordering as well. Ordering skills are assessed in the studies by Castellini and colleagues (2007) and Waldron and colleagues (2006), where both surveys ask for the smallest object and include ranking tasks of three or four objects, which were either all macroscopic or submacroscopic (too small to see with the naked eye). The survey by Tretter and colleagues (Tretter, Jones, Andre, et al., 2006) has respondents order 26 objects, including macroscopic and submacroscopic objects, by placing them into ranked size categories. However, this task allowed respondents to place multiple objects into a single category, with no mechanism for tie-breaking. Thus, research about learners' ability to unambiguously order multiple objects including macroscopic and submacroscopic objects, would be informative.

Grouping was assessed by Tretter and colleagues (Tretter, Jones, & Minogue, 2006) using 31 cards depicting objects of a large range of sizes. Using two different tasks with different cards for grouping and ordering, and aggregating data, does not allow investigation of an interesting question: Do learners of different ages order and group objects in a consistent fashion? In other words, once a person has ordered the objects, does she make logical groups that maintain the original order, or does she place non-adjacently ranked objects into one group? An experimental design investigating grouping and ordering skills with the same objects, in linked tasks, might shed light not only on the two skills but on the degree of connection between the two that students display.

Absolute size tasks involve assigning a size to an object (using a number and measurement units, e.g., 1 nm), or conversely, coming up with objects of a given size. Both studies by Tretter and colleagues (Tretter, Jones, Andre, et al., 2006 and Tretter, Jones, & Minogue, 2006) assess absolute size, by placement of given objects into size ranges or asking for objects for given size ranges. Investigating what sizes learners assign to objects, rather than a size range, would seem a promising research direction. Another interesting research question would be to see if students in fact assign larger sizes to objects they had previously ranked larger.

A further way of thinking about size is not addressed by any of the above studies, namely, how many times larger or smaller one object is than another. As will be explained below, our study assesses respondent knowledge of size and scale through four distinct but linked tasks, each assessing one of these conceptions of size. The design allows us to investigate whether individual learners' responses are self-consistent and connected. The connection between conceptions of size is precisely the focus of this study.

Previous research did not examine differences in size and scale accuracy or performance by race, gender, or ability, only by age or grade range. Harding (1998), Keller (1987/1999), Lemke (1990) and others have argued that traditional methods of instruction of science tend to privilege middle-class, white, male, mainstream students. Thus, it is important to examine possible differences between demographic segments. In examining a racially and ethnically diverse, low- to mid-SES sample, our study begins to examine the question of cultural differences in cognition about size and scale.

The present study thus builds upon the research base in various ways: by studying a population with different demographics, focusing on individual rather than aggregate data, investigating an additional way of conceptualizing size and scale, and studying how students link different conceptions of size. By examining if, when, and how learners of different grades, gender, abilities, and race/ethnicities link and use different conceptions of size, our study begins to trace an empirically-derived learning progression for size and scale.

Theoretical Framework: Conceptions of Size

As mentioned above, four ways of thinking about size are studied. Particularly, the connection between pairs of conceptions is analyzed. The four conceptions, and their basis in the literature, are described below.

Ordering

Ordering involves creating a sequence of objects by size, from largest to smallest, e.g., $A < B < C < D < E < F < G < H < I < J$. The NCTM Principles and Standards state that young students should be able to "recognize the attributes of length, volume…compare and order objects according to these attributes" (NCTM, 2000/1989. Pre-K–2 Expectations - Measurement). The *Benchmarks for Science Literacy* (AAAS, 1993) similarly state that "By the end of the 2nd grade, students should be able to use whole numbers and simple, everyday fractions in ordering, counting, identifying, measuring, and describing things and experiences". (Habits of Mind, K-2, B). In a similar vein, Wiedtke (1990) states that, for young children, "Sorting activities can be used at the intuitive level to define length as a characteristic of an object." (p. 231), and notes that the ability to make comparisons is a prerequisite to measurement. Wiedtke suggests that this is developmentally the first size ability to develop:

"Many children may see no need to learn these [measurement] ideas. Measurement tasks may no be part of their out-of-the-classroom experiences…*big* suffices whenever comparisons are to be made" (1990, p. 230).

Grouping

This conception of size involves placing objects of similar size into groups, with successive groups containing objects of distinct size ranges: {A, B, C} < {D, E} < {F, G} < {H, I, J}. The Principles and Standards state that young children should be able to "sort and classify objects according to their attributes" (NCTM, 2000/1989. Pre-K–2 Expectations - Data Analysis & Probability). The *Benchmarks* state that "In the earliest grades, students make observations, collect and sort things…" (AAAS, 1993, Nature of Mathematics, K-2, B).

Number of Times Bigger or Smaller

This conception involves the comparison of two objects, making it a relative measure, yet it also uses numbers to quantify the degree to which one object is larger than another, e.g., "Object C is 1000 times smaller than object E." This size skill is related to measurement: "The measurement process is identical, in principle, for measuring any attribute: 'choose a unit, compare that unit to the object, and report the number of units.'" (NCTM, 2000/1989, p. 104). In this case, the unit is another object. Similarly, Wiedtke states that "It is helpful to think of something continuous as being made up of small, equal-sized discrete parts, or *units*, that can be put together to reconstruct the original quantity." (1990, p. 229), and suggests activities using body parts and non-standard units to help the child develop the notion of a unit. Experts employ this conception of size when they "unitize" (Tretter, Jones, & Minogue, 2006) – redefining sizes in terms of convenient units or benchmark objects.

Absolute Size

This way of thinking about size involves reporting the size of an object in terms of a conventionally defined unit, such as meters or inches, and a number, e.g., "Object B is 2.3 nm in length". Thus, absolute size is a special case of the relative but quantitative size conception identified above, number of times bigger or smaller, in which an object is compared to conventionally-defined lengths (for instance, a platinum-iridium alloy bar kept in Paris, or wavelengths of orange-red light, in a vacuum, produced by burning the element Kr-86, or strips of wood ultimately tracing their calibration to one of these standards). Rather than iteratively placing the unit end-to-end along the object to be measured, we can read a calibrated instrument that simplifies the counting for us (Wiedtke, 1990, p. 230).

Expert Knowledge of Size Involves the Four Conceptions

An expert's knowledge of the size of an object must include solid understandings in terms of the four conceptions described above. For instance, in characterizing the size of a red blood cell, an expert scientist would be expected to know not only that its diameter is around 7 μm (absolute size), but also be able to contextualize this size measurement. The expert should know that the diameter of a blood cell is about twenty times too small to resolve with the naked eye, about 10 times larger than the typical unicellular organism, and around 5 orders of magnitude or 100,000 times larger than an atom (number of times bigger or smaller). Given other objects, the expert should be able to say where the red blood cell fits in terms of size, for instance, ranking it larger than an atom, a water molecule, or a mitochondrion, but smaller than a dust mite or pollen (ordering). The expert should also be able to form logical groups of similarly-sized objects, in the example above atom and water molecule, mitochondrion and red blood cell, and dust mite and pollen (grouping). These groups are logical because the relative size differences between objects in a given group are much smaller than those between groups. If macroscopic objects are included, then another logical way to group by size might be to place all the submacroscopic objects into a single group, and the macroscopic objects in another. Additional facets of expert knowledge not examined in this paper might include the tools used to study each object (e.g., magnifying glass, optical microscope, electron microscope) and the size range as identified by the SI unit that most conveniently can be used to express its size (Tretter, Jones, & Minogue, 2006).

Connections Between Conceptions of Size and Scale

For a "common theme" (AAAS, 1993) such as scale to be used by learners to create more robust, connected knowledge, the construct itself must be robust and connected. For this study, we created four distinct tasks using the same set of cards depicting macroscopic and submacroscopic objects, each task aimed at one conception of size. Our tasks placed few constraints on the respondent compared to prior research. For instance, rather than separating macroscopic and submacroscopic objects to be ordered, we used both types in a single task that allowed for direct ordering (as compared to possible placement of multiple objects in a single size range). Rather than providing size ranges into which objects ought to be placed, we asked students to assign a size to each object. This approach, structured mainly by the respondent, allows unexpected answers to emerge (Ambert, Adler, Adler, & Detzner, 1995). For instance, a child who believes that objects can have sizes described by negative numbers, will not be able to resort to these in a task with preexisting size ranges (if the ranges don't include negative sizes), but will be able to do so if simply asked to provide a size for an object. By using the same cards for distinct tasks, we allow students to be inconsistent between answers, whereas prior research has had a single task for absolute size and ordering, and a separate task for grouping.

Ordering-Grouping.

If properly constructed, two tasks that probe a respondent's ability to order and group, respectively, can be used to measure three things: how well she orders, how well she groups, and whether her responses to the two tasks are connected, or logically consistent. To illustrate this point, consider a student who orders and groups six objects by size as shown below, where the size dimension of interest is clearly pointed out to her: the height of an average adult male human (1.8 m); the diameter of a baseball (7 cm); the height of a 4-story building (12 m); the length of a typical credit card (8 cm); the length of a typical school bus (10 m); and the height of a typical house door (2.4 m). (The sizes are provided to the reader for the sake of concreteness, but not given to the student.)

(1) baseball < credit card < human < door < bus < building.

{baseball, credit card} {human, door} {bus, building}.

The ordering is correct. This grouping places objects whose sizes are within around 20% of each other into the same group, and objects outside of that size differential in a different group, thus relating to the number of times bigger or smaller. This order also places objects with sizes in the tens of meters in one group, in the units of meters in another, and in fractions of meters in a third group, thus relating to absolute size.

Both ordering and grouping rely on having specific factual knowledge of or experience with the objects. However, the connection between ordering and grouping can be assessed in a manner that is nearly content-independent. Imagine that a second student orders and groups as follows:

(2) credit card < baseball < human < door < building < bus

{credit card, baseball} {human, door} {building, bus}

The student's ordering is wrong because the credit card (8 cm) and baseball (7 cm) are reversed, as are the building (12 m) and bus (10 m). The grouping is however still reasonable, placing similarly sized objects together, and separate from those of much larger or smaller size . Furthermore, the grouping respects the initial ordering, with one group for the two smallest objects, one for the two largest, and one for the two middle objects.

Now consider a third student who for some reason orders and groups like this:

(3) bus < baseball < credit card < door < building < human

{bus, baseball} {credit card, door, building} {human}

Clearly, both the ordering and grouping are wrong. However, the ordering and grouping are consistent, maintaining the order he established. Even though the ordering itself is wrong, the grouping and ordering are linked, as the grouping includes the two objects the student ranked smallest, the three objects ranked next largest, and the object ranked largest of all in a group of its own. Note that the consistency of grouping and ordering is independent of factual knowledge about the actual size of the objects. This consideration becomes important when asking respondents to work with objects they do not have direct experience with, such as submacroscopic objects.

An example of inconsistent ordering and grouping is shown below:

(4) baseball < credit card < human < door < bus < building

{baseball, credit card} {human, door, building} {bus}

Since the student ranked building larger than bus, and door smaller than bus, any group that contains door and building should logically include bus. Thus, the grouping does not respect the order imposed by the student and the answers are inconsistent.

It is this consistency between linked tasks that this paper investigates, a consistency that corresponds to connections between conceptions of size. Even though the order and groups established by student 3 are factually wrong, even outlandish, the grouping respects the order the student established. For the focus of this paper, this student's answers are 'better' than those of student 4, whose grouping is inconsistent with ordering even though the ordering is correct and the grouping nearly so.

Similar connections between other sets of two conceptions of size can be evaluated, as explained below.

Ordering – Number of Times Bigger or Smaller

The connection between ordering and the number of times bigger or smaller one object is compared to another can also be assessed independently of content knowledge about the actual size of objects. Continuing with the scenario above, consider a subset of the items: baseball, human, and school bus. The human (height: 1.8 m) is about 25 times bigger than the baseball (diameter: 7 cm), for the dimensions of interest. The school bus (length: 10 m) is around 140 times bigger than the baseball. These numbers can be determined based on an estimate of relative size, or from the absolute sizes of the objects. However, the relationship between these two ratios of sizes is determined by the ordering of the three objects: the bus is a larger number of times bigger compared to the baseball than the human is. A student who correctly ordered the objects and who estimates (incorrectly) that the human is 100 times bigger than a baseball, must say that the bus is *more than 100* times bigger than the baseball, to be consistent with his ordering.

(5) baseball < human < bus

100

>100

This consistency is independent of the actual accuracy of ordering or number of times bigger or smaller. Student 6 (below) has ordered the objects incorrectly and estimated unreasonable numbers of times bigger, yet has consistent ordering and number of times bigger:

(6) bus < baseball < human

2

3

Ordering-Absolute Size

Similarly, the absolute size of objects should reflect the order in which the objects were ranked. Student 7 below will have to assign a size larger than 50 cm for his answers to be consistent with the order.

(7) 1 cm 50 cm >50 cm
baseball < human < bus

The consistency of ordering and absolute size can be evaluated independently of the accuracy of ordering or of the sizes. Student 8 below has consistent ordering and absolute sizes despite errors in both ordering and sizes, because she has assigned larger sizes to objects ranked larger:

(8) 1 cm 200 m 3 km
bus < baseball < human

Number of Times Bigger or Smaller - Absolute Size

The fourth type of content-independent connection is between number of times bigger or smaller and absolute size. Both of these conceptions of size stem from the definition of measurement, differing only in terms of the unit used (another object, or a conventional measurement unit). Thus, the two are logically linked – given one size and the number of times bigger or smaller, the second size can be calculated unambiguously, a matter of ratios and proportions. Student 9's responses below are factually inaccurate, but consistent, as the sizes given for two objects (e.g., baseball 10 cm, human 1000 cm) reflect a hundred-fold size difference, consistent with the number of times bigger response.

(9) 10 cm 1000 cm 5000 cm
baseball < human < bus
100
500

In contrast, Student 10's responses below are not consistent, since 140 cm is not 50 times bigger than 7 cm, and 7 m is not 500 times bigger than 7 cm. Thus, even though these assigned sizes are more accurate than Student 9's above, the responses are disconnected.

(10) 7 cm 140 cm 7 m
baseball < human < bus
50
500

Figure 1 below shows the possible connections among pairs of conceptions of size. The connection between grouping and absolute size is not investigated in this paper (indicated by the arrow labeled V), as it depends on specific content knowledge about the size of the objects; neither is the connection between grouping and number of times bigger or smaller (the arrow labeled VI).

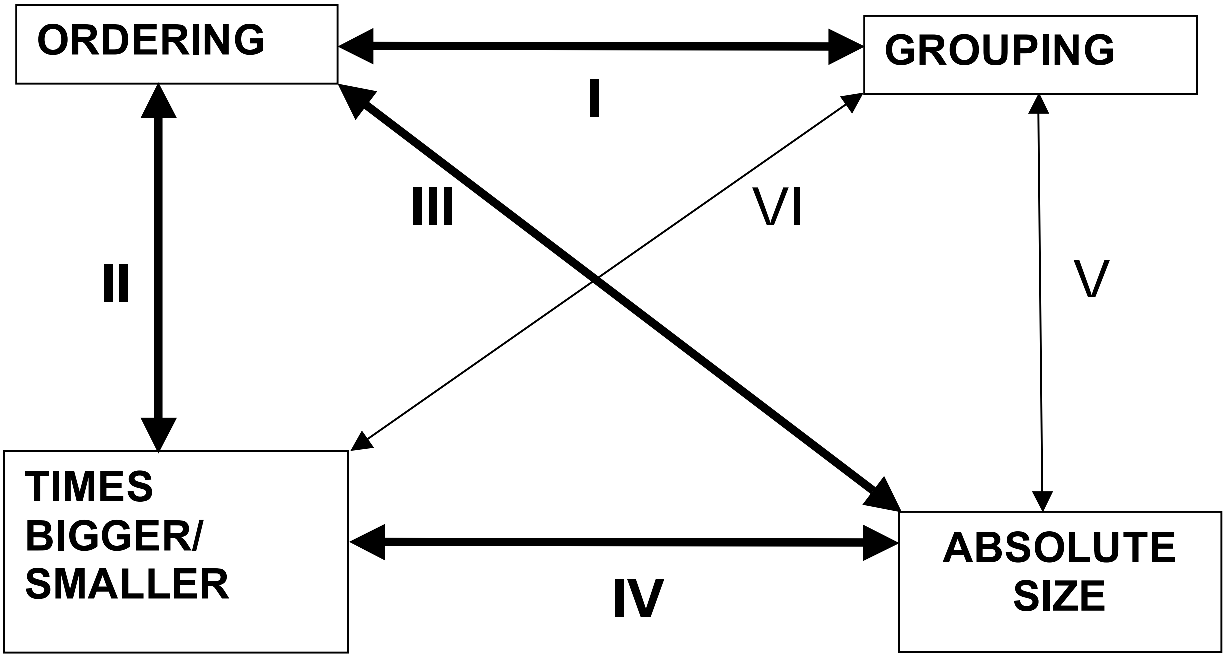


Figure 1. Connections between conceptions of size.

Development of Students' Conceptions of Size

The developmental trajectory that students might follow in establishing a robust conception of size is only partly addressed by the literature, leaving much to investigate empirically, particularly regarding the connection between conceptions of size.

Regarding individual conceptions of size, the NCTM *Principles and Standards* (2000/1989) suggest that ordering might be the first size skill children develop:

> Recognizing that objects have attributes that are measurable is the first step in the study of measurement. Children in prekindergarten through grade 2 begin by comparing and ordering objects using language such as *longer* and *shorter.* Length should be the focus in this grade band… (p. 45).

Classification into groups by a continuous variable requires some (possibly implicit) consideration of relative size differences, or absolute sizes, and should thus be more difficult than ordering. Number of times bigger or smaller is determined using the same procedure as absolute size, except that one object is compared to another. How many times bigger one object is than another thus requires quantitative thinking but does not resort to units of measurement. Thus, number of times bigger or smaller may be an important link from the earlier, more qualitative ways of conceptualizing size (grouping and ordering) to absolute size. In keeping with this idea, the NCTM document urges the use of invented units before introducing conventional units. Measuring one object in terms of another, as invented units do, is employing the times bigger or smaller conception of size.

As for connections between conceptions of size, it is not clear which set of conceptions (exemplified by the arrows in figure 1) should develop first. Perhaps ordering and grouping, being the earlier skills, are connected first. Number of times bigger or smaller, and absolute size, being more advanced skills, may be the most difficult to connect. The connection between these two conceptions of size is a matter of ratios and proportions, a notoriously difficult topic in the middle school math curriculum.

Research Questions

This paper seeks to answer the following research questions:

How well, when, and in what order do students through experts link the four conceptions of size? What differences do students of different races, genders, science classes, and abilities show in their connections between conceptions of size?

Methods

Participants

In this study, we individually interviewed and conducted card tasks with students in a diverse public middle school and a diverse public high school in a small, working class city near a mid-western college town. We interviewed and recorded 42 male and female middle and high school students of a mix of races/ethnicities and abilities, as shown in Tables 1, 2 and 3 below. We also interviewed six undergraduates at a selective, public, midwestern research university.

Table 1
Participants by Grade Level

Grade	7th	9th	10th	11th	Undergrad	Total
N	8	11	7	16	6	48

The ethnicity/race of the participants roughly mirror the demographic mix of the schools, including roughly 50% African Americans, with the balance European Americans, Middle Eastern Americans, Hispanic Americans, and some recent immigrants from around the world.

Interviews that were more than slightly incomplete due to time constraints, or in which the interview protocol was not followed, were not included in the analysis. We used stratified purposeful sampling (Patton, 2002) in order to obtain results that both generalize to some degree, and shed light on differences by gender, race/ethnicity, and academic ability level (as determined in holistic fashion by their current science teacher). We did not interview any 8th or 12th graders due to logistical constraints.

Table 2
Academic Ability and Race/Ethnicity by Grade, Pre-College Respondents

Student Grade	Ability	White non-Hispanic	Non-White, Hispanic, Other	Total N by Ability
7th	low	1	3	4
	mid	1		1
	high	3		3
7th total N by race/ethnicity		5	3	8
9th	low	2	4	6
	mid	1	1	2
	high		2	2
9th otal N by race/ethnicity		3	7	10
10th	low		1	1
	mid	3	1	4
	high	1	1	2
10th total N by race/ethnicity		4	3	7
11th	low		1	1
	mid	5	4	9
	high	4	2	6
11th total N by race/ethnicity		9	7	16

The pre-university sample studied came from schools with populations ranging from 57% to 64% African Americans or Hispanics. At the middle school, 67% qualify for free or reduced lunch, as do 41% at the high school.

We also interviewed 6 first- or second-year undergraduates at a major, midwestern research university, two of whom are majoring in a science discipline. Two experts were also interviewed; one has a PhD in theoretical physics and is a professor in Science Education among other areas, and the other has a PhD in chemistry and is a professor in the Chemistry Department, both at the same university as the undergraduates. The experts are not included in this analysis, but provided insight as to ideal performance on the tasks. Each participant was interviewed and audiotaped once. Some interviews consisted solely of size and scale, while others started with structure and properties of matter; the size and scale segment of the interviews generally lasted 15 minutes. In a few cases, a section on the motivational potential of different science topics and phenomena followed the size and scale interview. We interviewed fewer undergraduates than pre-college students, and even fewer experts, because we expect less variation in the knowledge of experts (Tretter, Jones, & Minogue, 2006).

Table 3
Academic Ability and Gender by Grade, Pre-College Respondents

Student Grade		male	Female	Total by ability
7th	low	1	3	4
	mid	1		1
	high	1	2	3
7th total N by gender		3	5	8
9th	low	3	3	6
	mid	2		2
	high	2	1	3
9th total N by gender		7	4	11
10th	low		1	1
	mid	1	3	4
	high		2	2
10th total N by gender		1	6	7
11th	low		1	1
	mid	6	3	9
	high	1	5	6
11th total N by gender		7	9	16

Instruments and Interviewers

We developed an interview protocol that was tested and refined iteratively over various cycles. All four interviewers read and discussed general guidelines for conducting interviews from the science education literature, and practiced the interview protocol on research associates and each other. Pilot testing on 3 middle school and 3 high school students led to revisions before starting data collection. Our interview protocol asks open-ended questions with precise wording, following Patton's (2002) standardized, open-ended format. The final protocol used for data collection is included as Appendix B. Several of the interview questions are not analyzed for this paper, but will be used for future analyses.

In order to test respondents on ordering, we asked them to arrange by size ten cards, depicting objects ranging in size from an atom to the Earth, and also including molecule, virus, mitochondrion, red blood cell, head of a pin, human, and mountain. We offered respondents a choice of two sets of ten cards with identical objects: one set with cartoon-like depictions, and one with more realistic photos/images (See Appendix C for the more commonly-chosen set of cards). The cards are all the same size and are labeled with the name of each object. Pilot testing revealed that students were often not certain what type of pin the cards referred to, so a straight pin was shown to them (this pin is roughly 2.5 cm long, with a head about 1 mm in diameter). After the respondent ordered the cards, the interviewer stated the order of the objects out loud, in order to have a record on the audio recording, or wrote down the order.

The second task tested grouping, and used the same ten cards. Participant were asked to group the cards by size, creating as many groups as made sense to them. There was no constraint on the number of objects per group. Students were prompted for their reasoning, and asked to label the groups they created. The parallel design of the ordering and grouping tasks allows us to test for consistency in the thinking of the respondents.

The third task assesses the number of times bigger or smaller size conception. It utilizes five of the cards from the previous card sort, ideally atom, cell, pin head, human, and Earth. If these five cards were not correctly ordered relative to each other in the card sort, alternate cards which were ordered correctly were used, as long as two objects were ranked smaller than a pin head and two objects larger (this constraint is due to the next task, described below). If the respondent's ordering did not meet this constraint, then this times bigger task was not undertaken. The respondents were asked to say how many times bigger or smaller each object is, compared to the pin head. For clarity, a physical pin was shown to the respondent, and the width or diameter of the head pointed out. Additionally, the dimension of interest was pointed out by the interviewer (diameters, except in the case of the human, were the height of an average adult was used). The interviewer recorded the answers on an answer sheet, so the student would be able to recall their answers for the next question (see Appendix D).

The final task involved the same five cards, but asked the participant to assign absolute measurements to four of the cards. The size (diameter) of a head of a pin was provided (1 mm). The rationale behind using a small macroscopic object as the central reference point is that its size is bound to be familiar to respondents through their use of rulers (Wiedtke, 1990, pp. 237-238). The answer recording sheet was given to the respondent to record answers on; thus, the respondent had access to her previous answers concerning how many times bigger the objects were compared to the pin head. The five-card tasks test both for performance on times bigger and absolute conceptions of size, and for the degree of connection between the two. The students were asked whether and how the numbers in the times bigger or smaller task informed their answers to the absolute size task. This question explicitly addresses the degree of conceptual connection between times bigger or smaller and absolute conceptions of size, and is designed to identify students who have the conceptual knowledge that the two sets of numbers are logically linked, but may lack the procedural knowledge to calculate absolute sizes from the number of times bigger one object is than another.

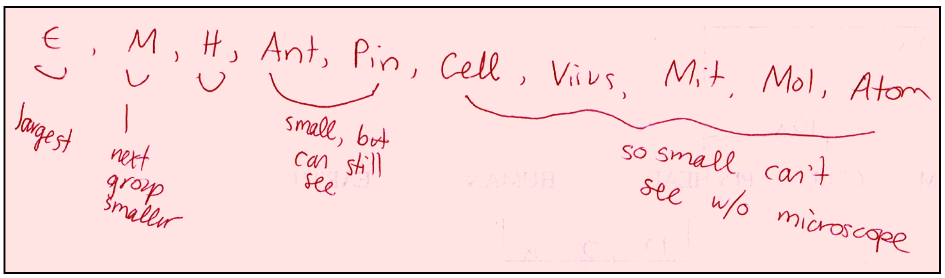
Coding

The principal coder (the first author) wrote summaries of each interview, including the ordering and grouping data extracted from the recordings, along with paraphrased evidence for coding, as well as verbatim transcription of interesting or representative segments. Number of times bigger or smaller, and absolute size data were recorded on the answer sheet during the interview (see Appendix D). Some of these transcriptions are excerpted in the results section below, and a sample summary page included as Appendix E. A second coder scored 10 % of the data directly from the answer sheet and recordings. Interrater reliability greater than 90% was achieved after training and discussion plus one round of practice coding, as well as clarifications and minor revisions to the coding rubric (attached as Appendix F).

The coding rubric that we developed was informed both by our theoretical framework positing four conceptions of size, and by our emergent understanding as we conducted and listened to our interviews. The connections labeled I-III in Figure 1 above (I: order-group, II: order-times bigger or smaller, III: order-absolute size) were coded in the same fashion: 0 for the respondent's inability to complete one or both of the tasks (and thus no basis for judging the connection between the two), 1 for inconsistent responses, and 2 for consistent responses. Examples are provided below.

Coding for Order-Group Consistency

The ordering and grouping tasks both used all ten cards. Figure 2 below provides an example of connected responses by a 7th grade student. Even though the order of virus and mitochondrion are reversed, the groups respect the order the student generated, so the answer was coded a 2 for consistent responses. The student responses are transcribed below the artifact for clarity.

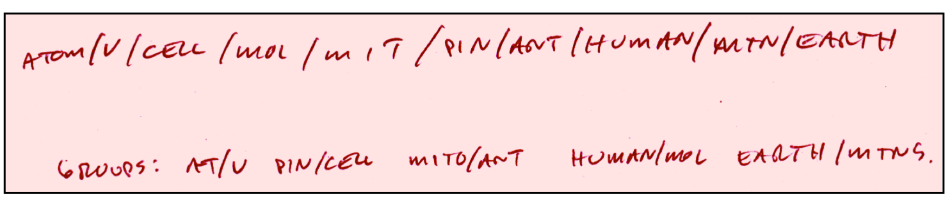


#1007, 7th grade. Ordering and grouping consistent. Coded: 2.

Order: Earth, mountain, human, ant, pinhead, cell, virus, mitochondrion, molecule atom.
Groups: {Earth} {mountain} {human} {ant, pinhead} {cell, virus, mitochondrion, molecule, atom}

Figure 2. Example of consistent order-group responses, coded 2 (consistent).

Figure 3 below provides an example of disconnected ordering and grouping. Note how some groups include objects which were not adjacently ranked, but the intermediate objects are not placed in that group (e.g., pinhead is ranked larger than mitochondrion and smaller than ant, but is absent from the pin + ant group).



#1001, 7th grade. Ordering and grouping inconsistent. Coded: 1.

Order: atom, virus, cell, molecule, mitochondrion, pinhead, ant, human, mountain, earth
Groups: {atom, virus} {pin, cell} {mitochondrion, ant} {human, molecule} {earth, mountains}

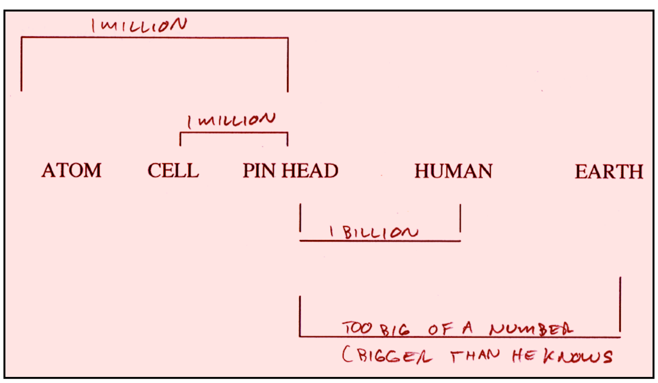
Figure 3. Example of inconsistent order-group responses, coded 1 (inconsistent).

The unorthodox and inconsistent response by #1001 was probed; the following transcription sheds some light on this respondent's thinking:

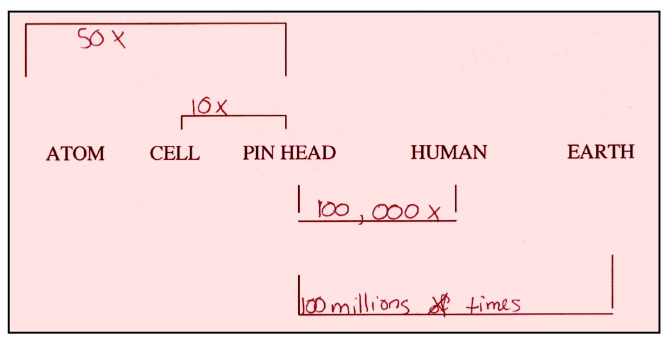
Interviewer (I): You grouped human and molecule together. Tell me about them. Why are, why do they go together? (pause 20 sec)
I: What are you thinking? (pause 10 s) You have a group that's the two smallest and a group that's the two biggest, and here you have human and molecule. Can you tell me more about them? (pause 6 s)
Respondent (R): I think that it's possible that they can be not the same exact size but (pause 8 sec)
I: So you can have groups having objects that are not the same size but…(4s)
R: Kind of similar
I: Kind of similar. And they can still go together?
R: Mm hmm. (#1001, 7th grade)

Coding for Order-Number of Times Bigger or Smaller Consistency

The task for number of times bigger or smaller used a subset of five cards from the ordering and grouping tasks. These five were atom, cell, pinhead, human, and Earth, unless the student reversed the order of two or more objects. In this case, correctly ordered alternate cards were used, as long as two objects were ranked smaller than a pin head and two objects larger. If the respondent did not rank two items as smaller (or larger) than the head of the pin, then this task was coded a 0 for inability to order in such a way that this task could be carried out. Figure 4 below shows two examples of student responses. The respondent on the left used the same number of times smaller than a pinhead for both atom and cell, despite ranking the atom smaller than the cell, constituting an inconsistent response. The respondent on the right had the atom a larger number of times smaller than the pinhead as compared to the cell. Even though her factors are far from accurate, they are consistent with the size order of the objects, as are the numbers for human and earth. The response is coded a 2 (consistent).



#1008, 7th grade. Ordering and number of times bigger or smaller inconsistent. Coded: 1



#1007, 7th grade. Ordering and number of times bigger or smaller consistent. Coded: 2.

Figure 4. Examples of inconsistent (left) and consistent (right) responses for number of times bigger or smaller and ordering.

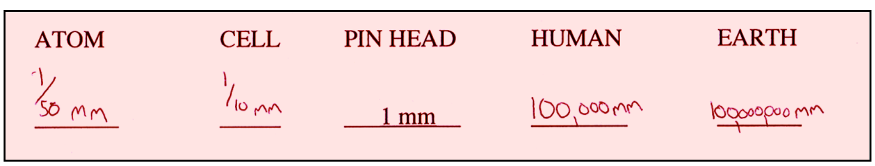
The following excerpt shows that some students had trouble with the concept of times smaller or bigger. In this case, the student seems to confuse the concept of "times smaller" with a "size smaller".

> I: How many times smaller is the atom than the pinhead?
> R: One size smaller.
> I: One?
> R: You want to know why? No?
> I: Yes. No, I don't, I understand. Don't you think…yeah, OK, one time is what, same size or what?
> R: Like, like, between, you see the size of this, maybe go down another size, might come smaller than that.
> I: So kind of just one, next.
> (#0094, 11th grade)

Other students simply refused to guess how many times bigger or smaller objects were compared to the pinhead. These responses were coded 0.

Coding for Order-Absolute Size Consistency

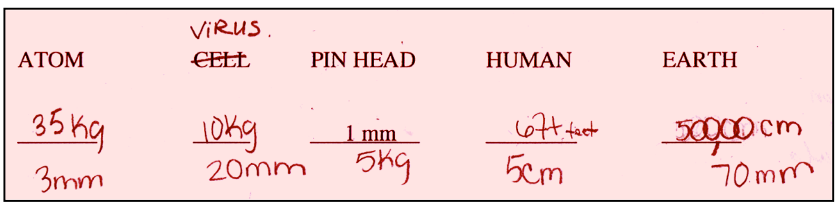
The two responses shown below in figures 5 and 6 provide examples of codes 1 and 2 for this category. Student #1007 (figure 5) has sizes that go from smallest to largest for the corresponding small to large objects, and was coded a 2, even though the actual sizes are far from correct.



#1007, 7th grade. Consistent sizes and order. Coded: 2

Figure 5. Example of consistent response for order and absolute size.

On the other hand, Student #0093 (figure 6) wrote sizes (in mm) for the submacroscopic objects that were larger than the pinhead, which she had previously ranked larger than atom or virus. (The substitution of virus for cell shows that she had previously ranked cell smaller than an atom.)



#0093, 11th grade. Inconsistent sizes and order. Coded: 1

Figure 6. Example of inconsistent response for order and absolute size.

Several students had trouble expressing sizes smaller than 1mm, as the following transcript excerpts show:

> R: What about if I put 0.5 mm?
> I: OK. Fine. How about atom?
> R: It's about the same.
> I: So you think virus and atom is the same. OK.
> R: It's about the same. Maybe a little smaller but I don't know what to put for smaller… Would it be like 3 or something? No, that'd be bigger. (#1004, 7th grade)

> R: I don't know the rest [sizes for cell, atom]. I can't even guess.
> I: OK. Umm, do you think you could use these numbers here along with the measurement of the pin head to think about some of these sizes? (90 s pause)
> I: You said for instance that the cell was 5 or 6 times smaller than the pin head.
> R: (sighs). Yeah. I don't know how to put that, though. (#0033, 9th grade)

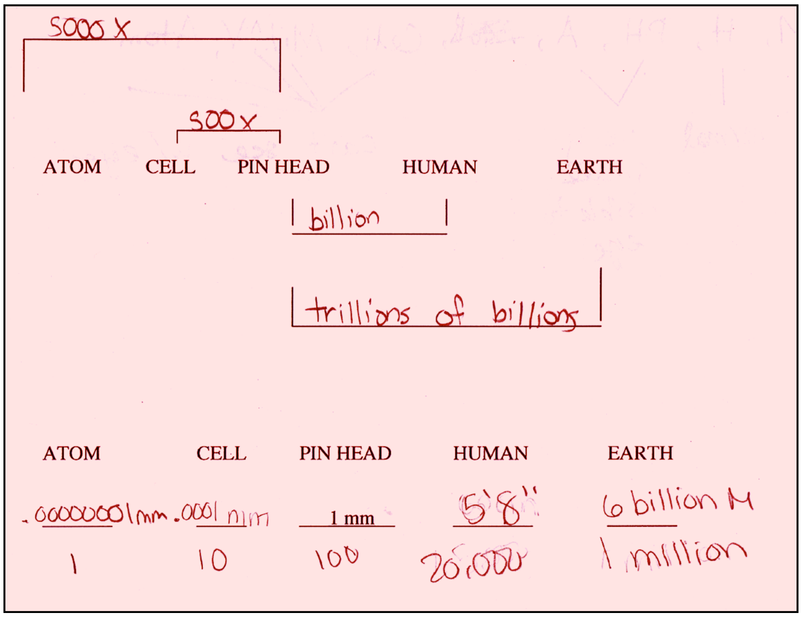
Coding for Number of Times Bigger or Smaller-Absolute Size Consistency

The coding scheme for the fourth connection of interest, that between number of times bigger or smaller-absolute size, is different from the other coding schemes. We created two coding subcategories, one (IVA in the coding scheme, Appendix F) for the perceived relationship between the two sets of numbers (which involves only conceptual understanding), and one (IVB) for numerical consistency between the number of times bigger/smaller and the absolute sizes assigned (which additionally involves procedural mathematical skill). Each of these subcategories consisted of four possible levels, as further explained below.

The two subcategories included a code 0 for those who could not assign times bigger or smaller, or absolute sizes, and a code 1, for students who did not see a relationship, as in the previous coding schemes. Code 2 was used for partially correct answers, and 3 for more fully correct responses. For coding scheme IVA, code 2 was used for those responses that demonstrated a weak link between the two sets of numbers, responses that said that one set of numbers gave an indication of the other set, or that they were "kind of" related. Code 3 was reserved for responses that clearly stated that the two were strongly related. Examples follow the description of the final coding subcategory.

For coding scheme IVB, responses which were consistent between absolute sizes and number of times bigger or smaller for 0 or 1 object (out of four) were coded a 1; those consistent for 2 or 3, a code of 2; and those consistent for all objects, a 3. The code of 2 was generated to capture students who generally recognized the relationship between the two conceptions of size, but who were distracted by knowing the height of an adult human in English units and didn't relate it to the given SI size of the pin head.

The example in figure 7 below shows a 10^{th} grader's response. The transcript excerpt shows that the student does not believe the times bigger or smaller numbers and the absolute sizes are related, at a conceptual level. Thus, his response was coded a 1 for IVA, conceptual connection. The numbers for times bigger or smaller are not connected. For instance, the cell is said to be 500 times smaller than the pinhead but the size assigned to the cell is 10,000 times smaller than the 1 mm pinhead, and similarly with the other numbers. This response was coded a 1 for IVB, procedural connection.



I: Did you use these numbers here to think about those numbers there?
R: No.
I: Do you think they're related?
R: Yes…NO…Because this is their actual size and this is them comparing them to other objects.
(Code: 1)
#0080, 10th grade

Figure 7. Example of disconnected responses for conceptual and procedural connections between number of times bigger or smaller and absolute size.

In contrast, the student in the example below (figure 8) had more connected responses. The response was coded 2 for IVB, because the atom's size and number of times smaller are inconsistent. We surmise that the student intended to relate the two numbers but employed a faulty procedure to obtain the reciprocal of 5,000,000. However, the other three numbers are consistent, and the student stated that the two sets of numbers were strongly related. This student in fact corrected the number of times bigger a human is than a pin after assigning sizes, in order to make the two sets of numbers consistent.

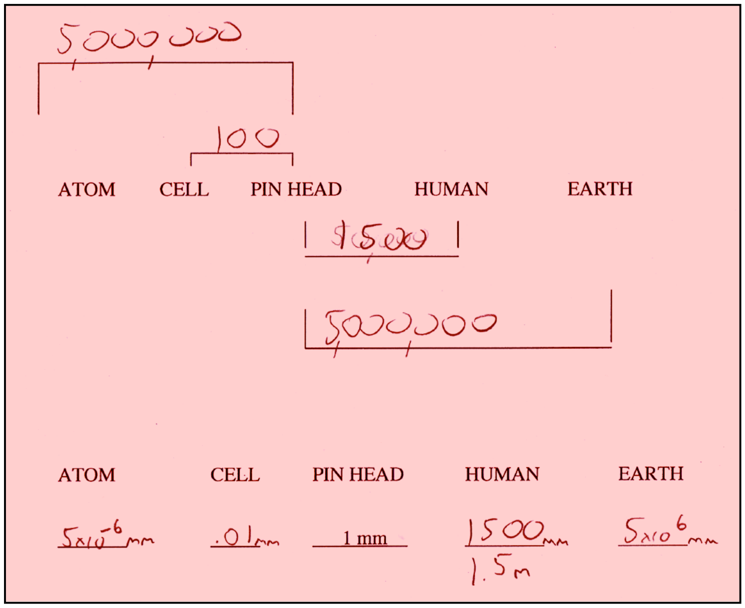


Figure 8. Example of more connected responses for conceptual and procedural connections between number of times bigger or smaller and absolute size.

Data Analysis

The coded data were examined in relation to the two research questions in separate procedures. The question of how well learners link the conceptions of size was addressed through calculating and examining the percentage of respondents who obtained the top score for each coding scheme. To determine the order in which learners tend to make connections, a tentative learning progression was generated by hypothesizing that students learn first the connection that is most commonly made, and the last connection learned is the one which least students make. The data were then inspected to see how many students actually followed this proposed progression.

In order to determine what difference science course, academic ability, gender, and race/ethnicity made in establishing connections between conceptions of size, a multiple regression analysis was carried out. The outcome variable used is a measure of student connectedness of size and scale, equal to the number of top codes each student received on their responses, and thus ranging from 0 to 5. Science course was coded 7 for 7th grade science, 9 for integrated physical and Earth science (which students typically take in 9th grade), 10 for biology, 11 for chemistry, and 12 for physics. College students were coded 13 regardless of their actual science course background. Academic ability was determined in holistic fashion by the science teacher, on a scale of 1 to 4 with half points. All college students were given the highest ability score, given their admission to a selective university. Race/ethnicity was coded 0 for White (non-Hispanic), and 1 for any other race or ethnicity.

Results and Analysis

In our sample, very few respondents had fully connected conceptions of size, particularly among pre-college students. The types of connection were not equally common, as Table 4 below shows.

Table 4
Percentage of Respondents Earning the top Code on Each Category

I: Order-group	II: Order-Times bigger/smaller	III: Order-absolute size	IVA: Times bigger or smaller-absolute size connection (conceptual)	IVB: Times bigger or smaller-absolute size consistency (procedural)
83% (N=48)	75% (N=48)	64% (N=47)	36% (N= 42)	13% (N=47)

Order-group consistency was the most common (83%), but still not universal, as 17% of students made groups with non-contiguously ranked objects (see figure 3 above for an example). Next most frequent was consistency of ordering and the number of times bigger or smaller (75%). One-quarter of the respondents had inconsistent answers. In some cases, this was due to the respondent refusing to even hazard a guess. In other cases, inconsistent answers were due to assigning the same factor to two objects, e.g., both cell and atom were considered to be 1000 times smaller than a pin head, despite having previously established an order in which the atom was smaller (see figure 4 above). Order-absolute size consistency was next most common (64%). Over one-third of respondents were unable to assign sizes that increased along with the ranked position of the objects (see figures 5 and 6 above).

The times bigger or smaller-absolute size connection was least common (see figures 7 and 8 above). Nearly two-thirds of the students interviewed did not perceive a logical, necessary connection between the sizes of two objects and the number of times bigger one object is than another. Only 13% of students were able to generate sizes consistent with the number of times bigger or smaller they had previously assigned. Four of the six are undergraduates, reducing the percentage of pre-college students to under 5%.

The differing percentages of students answering in consistent and connected fashion suggested a learning progression. All but two of the 48 students can be placed on the trajectory presented below in Figure 9 below. This diagram shows that students first make the connection between ordering and grouping or the connection between ordering and number of times bigger or smaller (figure 4), in either order, although seven of nine students who had made only one connection made the ordering-grouping connection.

The next connection to be made is order-absolute size. Of the 30 students who had connected order-absolute size, all had connected order-group conceptions and all but 1 had connected order and number of times bigger or smaller. This suggests that making the connections between ordering and times bigger, and between ordering and grouping, might be necessary conditions for the ordering-absolute size connection. This cannot be affirmed with confidence before longitudinal case studies establish the mechanism whereby one type of connection is necessary for another, more advanced one. On the other hand, of the 33 students who had established both of the early connections (order-times bigger, order-group), 4 had not made the connection between ordering and absolute size. Thus, having those two early connections is not sufficient to establish the order-size connection.

Similarly, of the 15 students who had made the conceptual connection between the absolute size of two objects, and the number of times bigger one is than the other, all had made the two early connections and all but one had made the order-size connection. However, of the 29 students who had established the three first connections, only 15 had made the conceptual connection between size and times bigger or smaller. This suggests that the three connections displayed to the left of the size-times bigger or smaller conceptual connection are necessary but not sufficient conditions for, or at least developmentally earlier than, the size-times conceptual connection.

Finally, of the six students who had procedural connection between size and times bigger or smaller, all had established the other four connections, though of the 15 who had made the first four connections, only 6 made the size-times bigger or smaller connection.

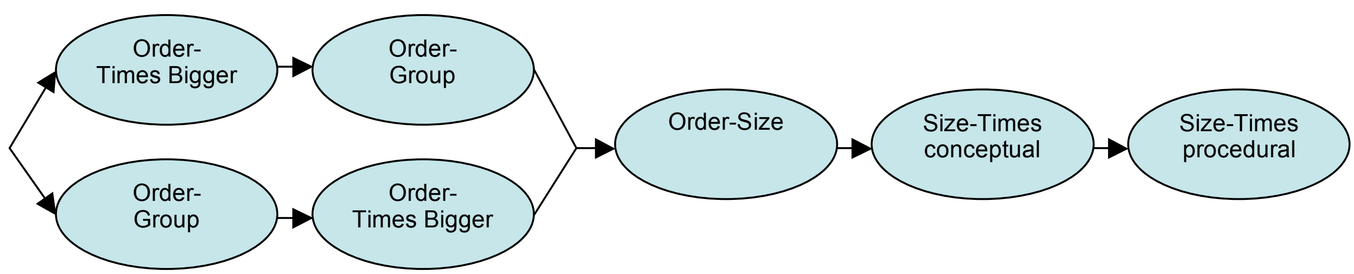


Figure 9. Learning progression for size

This learning progression needs further elaboration, as it does not as yet contemplate the role of content knowledge in building a robust conception of size. In addition, undergraduates at a selective university are mainly at the extreme right of this progression. There are many more, complex and useful size and scale skills that are not examined in this study. For instance, the *Benchmarks* (AAAS, 1993) for scale for 9^{th}-12^{th} grades suggest that students should know that "Because different properties are not affected to the same degree by changes in scale, large changes in scale typically change the way that things work in physical, biological, or social systems." (Common Themes, D – Scale, 9-12). Clearly, the skills required to understand such disproportionate changes are more advanced and difficult than the ones assessed here. Equally clearly, students who do not have a connected and robust construct of size will not likely be able to grasp this benchmark.

Our second research question has to do with factors that might be related to a student's developmental stage regarding his or her conceptions of size. We ran a multiple regression using the number of top codes as outcome variable, and grade, science class, gender, race, and ability, for all students in the study (N=48), using pairwise deletion. Despite the low number of cases, the overall model is statistically significant and explains nearly 50% of the variance. Gender, race, and grade were not significant; student academic ability relative to peers in the class, and the science class the student was taking at the time of the interview are statistically significant. This shows that science instruction impacts size and scale skills; and that gender and race are not significant predictors when controlling for science course and academic ability. The possible relationship between ability as determined holistically by the teacher, and race and gender, is not further explored due to the non-random and thus non-representative sampling strategy used to select participants for this study. The finding that ability relative to peers in the classroom is significant shows that science instruction alone does not account for differences in size and scale skills. The question of when students establish the connections is not addressed, since grade is not a significant predictor of size skills in the sample studied.

A second multiple regression (not shown) with only ability and science course as predictors is statistically significant, with a similar adjusted R^2. In both regressions, science course is a stronger influence, as demonstrated by the magnitude of the standardized coefficients.

Table 5
Summary of Multiple Linear Regression Analysis for Variables Predicting Connection Score.

Variable	B	SE B	Β
Grade	-.509	.347	-.587
Race	-.226	.384	-.073
Science course	.845	.320	1.041*
Ability	.553	.208	.325*
Gender	-.333	.354	-.107

Note: adjusted $R^2 = .467$
*p<.05

Implications and Future Directions

The surprising lack of congruence between ordering and grouping in a few younger students, and the widespread lack of coordination between times bigger and absolute size tasks, point to the possibility that a robust understanding of size and scale comes from the gradual linking of several strands, each representing qualitatively different ways of conceptualizing size. To be successful, nanoscale curricular activities for K-12 will need to attend to building fundamental concepts of size.

Understanding the relationship between the sizes of two objects and how many times bigger one is than another is very difficult for the middle and high school students we interviewed. Successful students and experts recall or estimate an absolute size for both objects, then divide to get number of times smaller or bigger. Clearly, this requires understanding that the two sets of numbers are related. Thus, it is necessary to introduce or reinforce classroom activities that highlight the logical connection between relative and absolute size. The conceptual connection between times bigger or smaller and absolute size is a prerequisite for the procedural connection and is itself difficult for the middle and high school students interviewed. Such instructional activities would presumably improve students' conceptual understanding of ratios and proportions, a traditionally difficult but important middle school mathematics topic. For some students, however, earlier connections between conceptions of size need to be made before such activities would be useful. Increasing the quantity and quality of elementary and middle school instructional activities, aimed at making the connections explicit through hands-on exploration, should help build a solid foundation for middle and high school science and math classes that deal with size and scale.

The role of content knowledge – knowing that a mitochondrion is part of a cell and therefore smaller than a cell; that atoms are too small to see with an optical microscope; that atoms compose molecules and thus must be smaller – in establishing the necessary connections, has yet to be explored. We plan next to analyze our data set to characterize how students order, how they group, what sizes they assign to submacroscopic and macroscopic objects, what they know about objects too small to see and the measurement units with which to express their size. Then we will incorporate these findings into our preliminary learning progression for size skills.

Our data set, along with similar interviews collected in rural and suburban middle-class, predominantly White middle and high schools yet to be analyzed, will afford further analyses of the differences, if any, by gender and race that may inform classroom practice regarding size and scale.

In conclusion, this study generated a preliminary learning progression for size, based on the connections between content-independent size skills. Further analysis will allow us to determine ways in which content knowledge and individual skills such as ordering and grouping interact with the progression suggested in this paper. A more complete learning progression will be a useful guide in developing instruction, curriculum, assessment, and policy for the important themes of size and scale.

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Appendix A: Draft SRI/NCLT Workshop Definitions of Size and Scale.

Definition of Size and Scale

The task force on size and scale at the Big Ideas in Nanoscience meeting organized by SRI and the National Center for Learning and Teaching Nanoscale Science and Engineering (NCLT) June 14-16, 2006, developed the following definitions:

- **Size** refers primarily to physical magnitude, including the distance between 2 objects. Learners initially tend to consider one dimension of objects, e.g. length. Later, learners may consider area and volume as well. Size involves comparing an object to other object(s), which may include agree-upon benchmark objects. The term "size"ù may refer to the magnitude of variables not related to physical magnitude, such as time, forces, etc.; however, only size as physical magnitude is considered in this document. There are various ways of conceptualizing size at the nanoscale, including ordering objects by size (relative conceptualization), organizing objects of similar size into groups of different size ranges (size category conceptualization), comparing objects in terms of how many times bigger or smaller one object is than another (quantitative relative conceptualization), and as an absolute size assigning a number and a measurement unit (NOTE: this particular conceptualization of size actually falls under the "scale" discussion below).
- **Scale** involves comparing an object to a defined reference standard (e.g. a meter). Note that for our definition of scale, the reference standard is not an object such as an atom or a human body; the reference standard refers to a more abstract reference point represented by a number and a measurement unit. This differs from size in that the comparisons are made to abstract measurement units which constitue a well-defined scale rather than to objects. The word scale can also refer to regions of size which help us understand and organize the world into component parts (e.g., the microscale or nanoscale). These scales represent different "worlds"ù, where different forces are dominant and different models are applicable, and where different tools are used to sense different characteristics.
- **Size vs. Scale** Size is usually a necessary component of scale, but scale is not necessarily a component of size. Absolute size, which includes a measurement unit, does involve scale. Size compares objects, possibly without reference to a numerical measure of the object; scale compares an object to a reference standard (number plus unit), thus anchoring its location along the continuum of size. A scale "world" refers to objects within a range of sizes in the reference standard. Scale MUST be quantitative; size may be qualitative or quantitative. However, the "quantitative" nature of size is subtly different from that of scale. For size, the quantitative nature refers to "relative quantitative" (such as 100 times smaller than a cell) whereas for scale the quantitative nature refers to a more abstract conception of a number paired with a measurement unit. Thus "size quantitative" is still anchored by reference to an object whereas "scale quantitative" is anchored to an abstract number/unit that exists on a continuum essentially independent of physical reality (e.g. a number line).
- **Scaling** means predicting phenomena of a system at one size based on the phenomena at another size. "Phenomena" here includes properties, behaviors, and dominant effects existing in the given system. Scaling is not always continuous, even within a scale "world". Even within regions in which a model applies, properties that depend on volume scale differently than other properties that depend on surface area or length. (NCLT/SRI, 2006)

More recent NCLT workshops have pointed out some problems and omissions with the above definitions, so these may be subject to elaboration and reformulation. However, it is worth noting that the experts in this work group agreed upon the four conceptions of size employed in the present study.

Appendix B: Interview Protocol

SIZE AND SCALE INTERVIEW

Bold: Directions or Introduction. Normal: Questions Prompts

Hello. My name is ____ and I'm here from the University of Michigan's School of Education. I want to ask you some questions about the size of things. This interview will be completely confidential, and will not affect your class grade in any way. Your teacher will not hear what you say. Your answers will help us design better science education materials for high school. '**I want to ask you some questions about the size of things. Think of some very small things you know of.** ***(Pause a few seconds.)***
What is the very smallest thing you can think of?

IF RESPONSE IS MACRO	IF AMBIGUOUS ("nucleus/particle")
Can you think of something too small to see with the naked eye?	What do you mean by that? Could you be more clear? What else do you know of that is too small to see with naked eye?

What type of measurement units would you use to express the size of that object? (If necessary, prompt by saying that the width of the table could be expressed in centimeters)
Which is bigger, a bacteria or a water molecule?
Why do you think that?

OK. Take a look at these two sets of cards. (*lay out cards in two separate sets).* **I'd like you to put them in order by the size of the objects, from largest to smallest. You can pick either one of the sets of cards – they both have the same objects. (Demonstrate the size of the head of a pin at this point).**

(Record order in which the cards were placed. Code: Earth=E, mountain=MTN, human=H, ant=ANT, pin head=P, red blood cell=C, mitochondria=O for organelle, virus=V, molecule=MOL, atom=AT. Abbreviations: OK= all correct. MACRO=pin to Earth correct.)
Could you please tell me why you ordered these cards (*the micro and nano cards*) the way you did? *(Select the micro- and nano- cards in pairs.)*

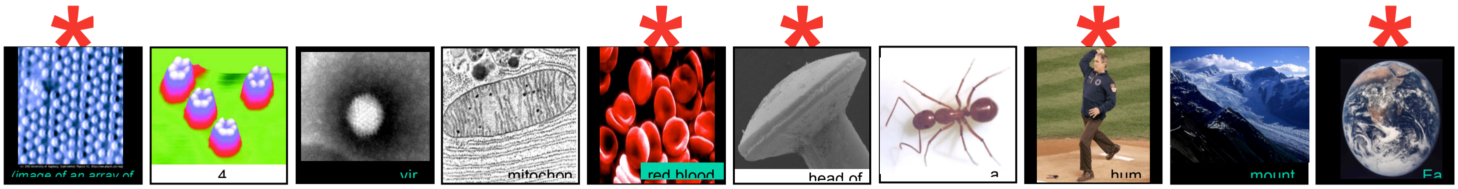
Why did you choose this set of cards?
Could you please place the cards into groups of objects of similar size? Make as many groups as you think makes sense. (Wait for task). Can you tell me how you decided to group these cards together? (Repeat for tape recorder how many groups and what cards in each). What do they have in common? What would you call this group? (Repeat for each group)

Interviewer selects five cards from task 3. These will be atom, cell, pin head, human, and Earth if they are ordered correctly. If atom and cell are out of order, select one of those, and choose another card in the correct order.
OK. Here are five of the cards you ordered. I want you to think about the length of these objects. For the pin head, think how wide it is ***(Trace width with finger)*****. For the person, the height. (*****Trace*****). For the Earth, atom, and cell, the diameter. (*****Trace*****). (*****record all answers on***

worksheet) How many times larger is the human than the pin head*?* How many times larger is Earth than the pin head? How many times smaller is the (red blood cell) than the pin head? How many times smaller is the (atom) than the pin head?
OK, a pin head is about 1 mm wide. That's a little less than 1/16th of an inch Would you write down the size of the other objects? *(Pass the student pen and worksheet, and offer scratch paper. Remind student to specify units, if necessary. Ask them to use metric system.)* Did you use these numbers here (indicate the relative sizes recorded on sheet) to think about the sizes of the objects? (if yes, ask how; if no, ask if student if s/he thinks the two sets of numbers are related) If you can think of other ways to express the sizes using different units or different ways of writing the numbers would you please write them below too?

Appendix C: Size and Scale Cards Used in Interview

Note: starred cards are used in the times bigger or smaller and absolute size tasks.



Appendix D: Recording Sheet for Size and Scale Questions 4 and 5

ATOM CELL PIN HEAD HUMAN EARTH

ATOM	CELL	PIN HEAD	HUMAN	EARTH
________	______	___1 mm__	_________	________

Appendix E: Sample Summary of Interview

0099 S&S Summary
6/2/06 – CD
S&S starts at 18:00
Smallest: electron.
Unit: nm. "Nanomicrometer"
Bacteria is bigger because you can see bacteria in water. But there might be bacteria smaller than a molecule of water. Molecule of water is made of two atoms of hydrogen and one of oxygen. There might be bacteria smaller than that.
Photos because recognized Bush.
Atom/mitochondria/virus/molecule/cell/pinhead/ant/human/mountain/earth
(24:00) Blue, green groups: visible to naked eye. Thought about placing them all together, but pin, ant, small so in their own group. Red group: building blocks. Yellow group: made up of red group, but not visible to naked eye.
Red blood cells are made of molecules, viruses are made of atoms, mitochondria is in the nucleus of a cell (or atom?). Viruses have mitochondria. Cells are made up of atoms.

(25:00) I: How many times bigger is human than head of a pin?
R: Oh, seeing as that's like a millimeter, and so, like 6 feet tall, is how many meters?, is two meters, so a millimeter is a thousandth – is it OK if I write something?
I: Sure – could you do it on the, here [back of the paper].
R: Hopefully a good estimate. So, 6 feet equals two meters, so if that's one millimeter (asked to speak into recorder). (26:00) So I'm saying 6 feet equals about two meters, and so then, how many millimeters are in a meter. There's a hundred centimeters, er, thirty centimeters in a foot, so thirty times ten, so there's 300 mm in a foot, I think. So then 300 times 6. So then 1800 millimeters in six feet…OK, so assuming all this math is right at [early] in the morning, I say it's like 1800 times. [Asked about Earth to pin] Oh my God (laughs) How many miles is the diameter of the Earth? [NOTE: see how immediately goes to absolute size to calculate ratio, here and above] Are you allowed to tell me how many miles the diameter of the Earth is? I: No. R: OK. Well, what I would do was, I'd think of how many mm in a foot, then I'd just find out how many miles in the diameter, find it out. I'm a math person. You just want an estimate…
I: Do you have any idea what the diameter of the earth might be?
R: We always have the conversion factors in our textbook and we just open it to do all our math problems, so I kind of mindless calculating. (28:00) So let's see, from here to Boston is around 3000 miles [NOTE: actual distance is about ¼ that]. Let's say it's like 10,000 miles (NOTE: 7926 mi is actual value), but I have no idea…so it's got to be like millions of times…
I: If you had a calculator, could you do it?
R: The diameter of the earth? Oh yeah, like the diameter of the Earth? Yeah.
I: Well, I can be your calculator if you need a little bit of help! You've got 10,000 miles. What do you know about miles?
R: There's 5280, or 60? 5260, 80…(NOTE: correct number is 5280)
I: So how many feet is 10,000 miles then? What operation would you do?
R: OK, so this is how many feet per mile, so then times 10,000.
I: OK, and then how would you get to, you've got feet now.

R: OK, feet, and you want it in mm…you multiply by 300. Assuming that's right, but I don't know about that.
I: (mumbles numbers) About 15 billion then.
(Meanwhile, R is calculating with pencil and paper. She uses units at several steps, including mm/ft in a conversion factor)

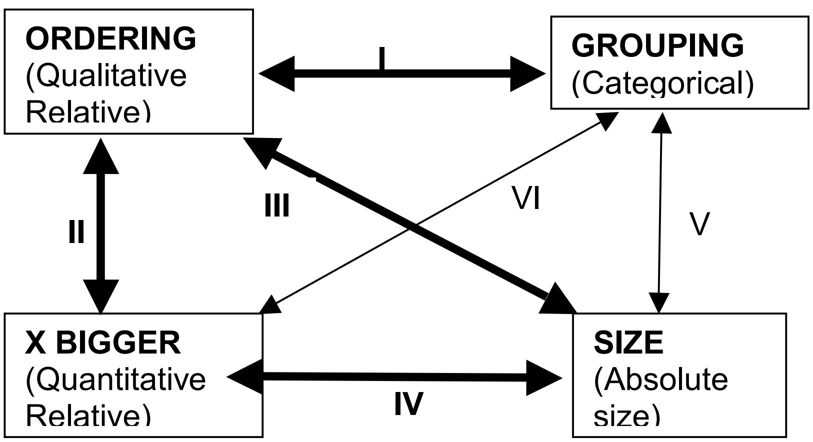
I helps her notice that there is one zero missing.(31:10)

I: Use numbers here to think of numbers here?
R: Yes. Because it's the conversion factor. I mean, it's the same thing…
I: So this number is dictated by the number here?
R: Yes. Correspondingly. Like compared to this, it's basically writing the same thing.
I: If a person (who did it wrong said they're different?)
R: Once you give it a number, it corresponds, it comes out to be the same thing…
I: Do you think there was a time, when you were smaller, in middle school or elementary, when you didn't know numbers like this had to be connected to numbers like that, or do you think you ALWAYS knew how to do this?
R: Well, probably, every skill is leaned, but I can't remember a specific time like learning it.
I: It seems obvious now.
R: Yeah.

Appendix F: Coding Rubric

DRAFT Coherence of Size and Scale Ideas coding rubric – Cesar Delgado, U. Michigan
Revised Jan. 18, 2007 after 1st round of IRR.
Revised Jan. 19 to have roman numerals match developmental stage (switched II and III, IV A and IVB).
Revised Jan. 21 with various small clarifications from recoding 7th and 9th graders.

This coding checks for robustness of size and scale ideas by gauging whether respondents correctly link four conceptualizations of size:
A) Ordering (or qualitative relative, or seriation) – ordering objects by size
B) Grouping (or categorical, or classification) – putting objects of similar size into groups
C) Times bigger (or ratio of sizes, or quantitative relative) – how many times bigger one object is than another
D) Absolute size – the actual size of an object, including a number and units of length



I) Ordering-Grouping Consistency
Respondent orders 10 objects by size, then groups them. Groups should contain adjacently-ranked objects.
Codes:
0: Can't group, or makes 10 groups of 1, or 1 group of 10
1: Groups incorrectly, given the order they established e.g. {A, B, D}, {C, E}
2: Groups correctly, given the order they established.
9: Did not finish this task – missing data

Examples:
1) atom/virus/cell/molecule/mitochondria/pin/ant/human/mountain/earth (1001)
Cards were ordered as shown above. Groups shown by colors. Red and purple groups are consistent with order, but the rest are not. Pin should be in the green group given that the objects bigger and smaller than it are in the green group. Code: 1.
2) cell/atom/molecule/virus/mitochondria/pin/ant/human/mountain/earth (1004)

Groups are consistent with the order. Even though order has some mistakes, the grouping is consistent, so code: 2.

II) Order-Times Bigger/Smaller Consistency
The number of times larger or smaller objects are than the head of a pin is assigned by respondent; this number should be greater for objects ranked largest and smallest than for intermediate objects.
Codes:
0: Did not assign times larger/smaller numbers
1: Times larger/smaller of any pair of objects is inconsistent with the order (a factor ≥ assigned to an intermediate object than to largest or smallest object).
2: Times larger/smaller of all objects are consistent with the order
9: Did not finish this task – missing data

1) ATOM	CELL	PIN HEAD	HUMAN	EARTH(1001)
3000	100		1000	5 000 000

Respondent has a larger number of times smaller for atom (3000) than cell (100), and larger number of times bigger for earth (5 million) than for human (1000), so it is consistent. Code 2

2) ATOM	CELL	PIN HEAD	HUMAN	EARTH(none)
thousands	millions		billions	more than billions

(Fictitious) respondent said that a cell is millions of times smaller than a pin, and atom is thousands of times smaller than a pin, yet ordered atom as smaller than a cell. Thus, is inconsistent. Code 1. (For codes of 1, the case where student states the same factor for atom and cell despite ranking atom smaller than cell)
NOTE: If pinhead is ranked smallest or second smallest object (see #1014), then this task and III, IV A, IV B can't be evaluated; code as 0.

III) Size-Order Consistency[1]
The size of objects previously ordered is assigned by respondent. Sizes assigned to larger or smaller objects should likewise be smaller or larger.
Codes:
0: Did not assign sizes (includes unitless numbers if they were prompted to provide units but didn't/couldn't, and negative numbers).
1: Sizes of any pair of objects are inconsistent with the order (a size ≥ assigned to an object ranked smaller in the ordering task)
2: Sizes of all pairs of objects are consistent with the order
9: Did not finish this task – missing data

Examples:

1) ATOM	CELL	PIN HEAD	HUMAN	EARTH	
1 mm	1.2 mm	1 mm	5'8	700 cm	(1001)

Respondent assigned sizes that were larger or equal to pin head to atom and cell, despite having ordered atoms and cells as smaller than pin head. Code: 1 (even though atom-cell, atom-human,

[1] The coherence of ordering and ratio cannot be determined because students were asked "how many times bigger (or smaller)" one object is than another; thus, students did not have the opportunity to be inconsistent here.

atom-earth, pinhead-human, pin head-earth, and human-earth have sizes that reflect the size ranking)

2) ATOM	CELL	PIN HEAD	HUMAN	EARTH	
0.00001 mm	0.5 mm	1 mm	149,000 mm	1 billion mm	(1005)

Respondent assigned sizes that went from smallest to biggest in the same order as s/he had ordered them. Despite having most of the actual sizes wrong, the sizes are consistent with the order, thus Code: 2

IV A) Times bigger-Absolute Size Connection

Students have said how many times bigger (or smaller) object A is than a pinhead. Given the size of the pinhead (1 mm diameter), do they realize that they can use the times bigger number to calculate the size of object A? Students might realize that the two types of size are logically and necessarily connected, but be unable to represent the sizes implied by their times bigger data (e.g., not know how to write a number 100x smaller than 1 mm). This category and category IV B distinguish between conceptual and procedural knowledge. Students' ideas usually surface when asked how the times bigger numbers influenced their answers to absolute size.
NOTE: occasionally the interview departs from the interview protocol and extensively guides, prods, scaffolds the student until he or she finally recognizes that the numbers ought to be connected. Code the student response at the point he or she is at corresponding to the interview protocol plus a small amount of probing or clarification.

Codes:
0: Respondent was unable to assign times bigger or absolute sizes, despite having enough time (students unable to finish due to time should be coded "9" for missing data)
1: Respondent states that there is no relationship between times bigger and size
2: Respondent states that there is a weak link between times bigger and size ("it gives some indication of the size") OR that there is a link – but does not attempt to write down numbers reflecting that linkage.
3: Respondent states that the two are (strongly) related (includes students who use absolute sizes to go to times bigger but do not explicitly address the relationship).
9: Did not finish this task – missing data

Examples:
1) I: Did you use the numbers up here to think about the numbers down here? (1001)
R: Uh uhh (no).
I: No. How do you think these numbers up here are related to these numbers down here, if they're related at all? What do you think?
R: A cell is kind of.. (7 s)
I: Like for instance, you said that a cell was 100 times smaller than a pin head. How would that number, 100, would that number, would that affect what you write down here, or is it like not very related?
R: Not very related.
Code: 1

2) I: "So when you write these numbers, did you look at these ones?" (1006)
R: Not really.
I: "Do you think these number and these numbers is relate?
R: Is what?
I: Kind of related each other
R: Yes.
I: But when you write down these one you didn't think about these numbers?
R: I didn't think about them, but they would relate to each other.
Code: 2 – if student had said, "Oh, wait, so then I need to change my answers" then it would be a 3 (and might change IV B if they do it successfully).

3) I: "Did you use the numbers here to think about the numbers down here?" (1009)
R: "Uhh uhh [no]."
I: how related?
R: "If the atom is 500 times smaller than the pin, then I would know that the atom down here would be smaller than the pin. That's 1 mm so it'd have to be smaller than 1 mm, by a lot."
I: "So if it's 500 times smaller, then it'd be 1/16 of a mm?"
R: "Uh huh.[yes]"
Code: 2 – times smaller gives a rough indication: "by a lot".

IV B) Times bigger-Absolute Size Consistency
Given how many times bigger one object is than another, and the size of one, finding the size of the second object is "simply" a matter of proportions or scaling. The ratio of absolute sizes is the same number as how many times bigger one object is than another.
Codes:
0: Was unable to assign times bigger or absolute sizes, despite having enough time (students unable to finish due to time should be coded "9" for missing data)
1: Times bigger and ratio of absolute sizes are consistent for 1 or no objects.
2: Times bigger and ratio of absolute sizes are consistent for 2-3 objects[2]
3: Times bigger and ratio of absolute sizes are consistent for all objects
9: Did not finish this task – missing data
Note: If student uses more than one representation for actual sizes (e.g., both metric and English units), then score according to the single system with best performance (e.g., only metric, or only English, whichever one includes more accurately connected numbers). Do NOT consider the accurate numbers from more than one system at a time, as this would unfairly favor those who provide many representations.

[2] This category aims to capture respondents who basically know that the two conceptualizations are linked but may be distracted by using English units for the size of a human.

Examples:

3000 5 000 000 (1001)

100 1000

1) ATOM CELL PIN HEAD HUMAN EARTH

<u>1 mm</u> <u>1.2 mm</u> <u>1 mm</u> <u>5'8</u> <u>700 cm</u>

Code: 1, as not consistent for any object.

50 100 million (1007)

10 100 000

2) ATOM CELL PIN HEAD HUMAN EARTH

<u>1/50 mm</u> <u>1/10 mm</u> <u>1 mm</u> <u>100 000mm</u> <u>100 000 000 mm</u>

Code: 3, as sizes and how many times bigger are consistent for every object (even though the actual sizes are all wrong).