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Growth rate of large-scale structure as a powerful probe of dark energy

Asantha Cooray
Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125, USA

Dragan Huterer
Department of Physics, Case Western Reserve University, Cleveland, Ohio 44106, USA

Daniel Baumann
DAMTP, Centre for Mathematical Sciences, University of Cambridge, Cambridge CB3 0WA, United Kingdom

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The redshift evolution of the growth rate of the gravitational potential, $d(D/a)/dz$, is an excellent discriminator of dark energy parameters and, in principle, more powerful than standard classical tests of cosmology. This evolution is directly observable through the integrated Sachs-Wolfe effect in cosmic microwave background (CMB) anisotropies. We consider the prospects of measuring the growth rate via a novel method employed through measurements of CMB polarization towards galaxy clusters. The potentially achievable errors on dark energy parameters are comparable and fully complementary to those expected from other upcoming tests of dark energy, making this test a highly promising tool of precision cosmology.

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One of the key issues in modern cosmology is developing efficient and complementary methods to measure cosmological parameters and cosmological functions. In particular, much interest has been devoted to developing methods to constrain the properties of the mysterious dark energy component that causes the recently discovered accelerated expansion of the Universe [1]. To this end, it has been pointed out that type Ia supernovae, number counts, and weak gravitational lensing are all very promising probes of the dark energy equation of state w and its energy density relative to critical $\Omega_{\rm DE}$ (see [2] and references therein), and that a number of other methods are likely to contribute useful information.

These cosmological tests probe various fundamental quantities. For example, type Ia supernovae effectively measure the luminosity distance, number counts are sensitive to a combination of the volume element and the growth of density perturbations, while cosmic microwave background (CMB) anisotropy effectively determines the distance to the surface of last scattering. While these tests are well understood and pursued in various observational programs, the information one can extract from these measurements is limited by the presence of fundamental degeneracies of cosmological parameters that enter the observable quantity in question.

It is typically advantageous when the measurements involve not the quantity itself but rather its derivatives with respect to time or redshift, since in that case the dependence on the equation of state $w(z)$ is more direct. For example, the Hubble parameter $H(z)$ is more sensitive to the equation of state than the comoving distance $r(z)$ since $r(z) = \int dz/H(z)$, but the latter has the advantage of being readily and accurately measurable. In this respect, the linear growth factor of density perturbations provides important information since it is a function of the Hubble parameter and the equation of state of dark energy w (see below).

As we will show, the *rate of evolution* of the growth factor with redshift is a tremendous tool for measuring dark energy parameters. We will further suggest the integrated Sachs-Wolfe (ISW [3]) effect as a probe for this purpose and propose polarization measurements of CMB anisotropy towards galaxy clusters. The latter provides an indirect method to extract the temperature quadrupole associated with the ISW effect as a function of the cluster redshift, with a reduction in the cosmic variance which plagues large scale temperature anisotropy measurements. To make this study practical, we consider the prospects of upcoming arcminute scale CMB polarization observations with instruments such as the South Pole Telescope (SPT) and the planned CMBPol satellite mission.

To begin, we review aspects related to the growth of large scale structure. In linear theory, all Fourier modes of the density perturbation, $\delta(\equiv \delta\rho_{\rm M}/\rho_{\rm M})$, grow at the same rate: $\delta_k(a) = D(a)\,\delta_k(a=1)$, where $D(a)$ is the growth factor normalized to unity today and $a=(1+z)^{-1}$ is the scale factor. In the matter-dominated era $D(a)=a$, while in the presence of a smooth dark energy component perturbation growth slows and $D(a)$ increases less rapidly with a. In general, the growth function can be computed by solving the linear perturbation equation $\ddot{\delta}_k + 2(\dot{a}/a)\,\dot{\delta}_k - 4\pi G\rho_{\rm M}\delta_k = 0$ where the dot is the derivative with respect to physical time.

Defining the growth suppression rate (growth rate relative to that in a flat, matter-dominated universe) as $g(a) \equiv D(a)/a$, and still allowing for a general $w(a)$, one can write

$$2\frac{d^2 g}{d\ln a^2} + [5 - 3w(a)\Omega_{\rm DE}(a)]\frac{dg}{d\ln a} + 3[1-w(a)]\Omega_{\rm DE}(a)g = 0, \tag{1}$$

where $\Omega_{\rm DE}(a)$ is the fractional dark energy density at the scale factor a. Here we only consider a flat universe, as indicated by recent CMB results (in principle, curvature also induces a late-time ISW effect and may be included if desired). For constant w, the solution is given in terms of the hypergeometric function [4], while to compute $g(a)$ and/or $D(a)$ for a nonconstant $w(a)$ one can either solve Eq. (1) numerically or use analytic approximations [5].

It has long been known that the growth function strongly depends on Ω_M, the fractional density in matter, and w. Also, the strength of several cosmological tests, such as number counts [6], clustering measured in redshift slices [7] and weak lensing [8] comes primarily from their dependence on the growth function $D(z)$. On the other hand, it has been known that redshift or time derivatives of distance are more directly related to dark energy parameters; in particular, the equation of state $w(z)$ is directly related to the first and second derivatives of distance with respect to redshift [9]. Unfortunately, the derivatives are not directly measured but are obtained by taking numerical derivatives of noisy data, which significantly increases the error in the reconstructed $w(z)$.

It is interesting to examine the sensitivity of the *rate* of change of the growth suppression factor to Ω_M and w. To do this, we first consider constant w, and then a two-parameter description of time-varying w. For the latter we do not choose the commonly used $w(z)=w_0+w'z$ [10], but rather $w(z)=w_0+w_a z/(1+z)$ [11] which is bounded at high redshift and facilitates the integration of Eq. (1). (For w' aficionados, we mention that the error in w_a is roughly twice the error in w'.) Figure 1 shows the error bars in the Ω_M-w plane (top) and w_0-w_a plane (bottom) using various classical tests assuming a fiducial model of $\Omega_M=0.3$ and $w=-1$; we use the same fiducial model throughout the paper. The calculation uses the Fisher matrix formalism and assumes 10% measurements in a given quantity at each interval of $\Delta z=0.1$ in redshift between $z=0.1$ and $z=2$. We show a variety of quantities, including the distance $r(z)$, the volume element $r^2(z)/H(z)$ and the growth factor $D(z)$, which are the most commonly considered probes of dark energy. We also consider $dD/d\eta$ (η is conformal time) and dg/dz. As emphasized in Ref. [12], $dD/d\eta$, which is measured by large-scale velocities, is mostly sensitive to Ω_M and not w. What Fig. 1 illustrates is that dg/dz is much more powerful than other probes due to the specific way the degeneracy is broken. For example, for the same relative accuracy in observations, dg/dz is about 15 times stronger than the comoving distance $r(z)$. Of course, this comparison is not necessarily fair, since an enormous amount of work has gone into developing methods to determine distances, which are now expected to be measurable to an accuracy of about 1% (per interval of 0.1 in redshift) by SNAP [13], making them the most direct probes of the cosmological expansion history, while not much attention has been devoted to the more esoteric quantity dg/dz. In the remainder of this paper we show that there indeed exists a very promising cosmological test which is sensitive to dg/dz.

The above discussion indicates that it would be ideal to have a cosmological probe of the evolution of growth suppression, dg/dz. It turns out that just such a probe exists in a universe that is not matter-dominated at late times. The dark energy domination causes the time-variation of the gravitational potential, which in turn contributes to CMB anisotropies through the ISW effect [3]. The resulting temperature fluctuation is given by

$$\Delta T^{\rm ISW}(\hat{\mathbf{n}})=-2\int_0^{r_{\rm rec}} dr'\,\frac{d\Phi(r')}{dr'}, \qquad (2)$$

where $r_{\rm rec}$ is the radial comoving distance to last scattering with $z_{\rm rec}=1100$. From Poisson's equation, $\nabla^2\Phi=3/2H_0^2\Omega_M(\delta/a)$, it follows that the gravitational potential Φ is proportional to the growth suppression g. The ISW

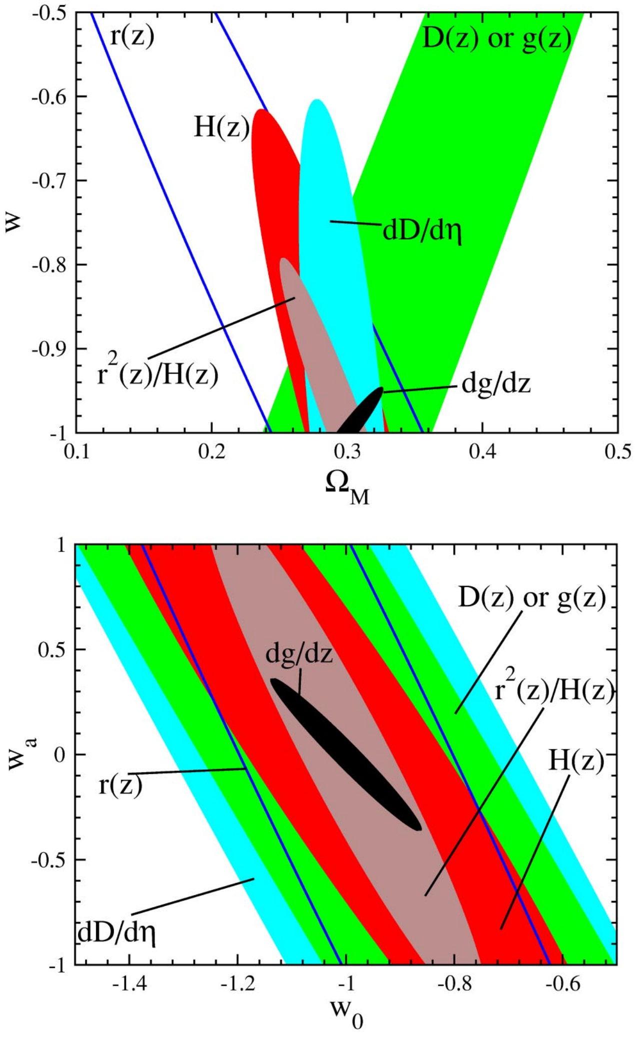

FIG. 1. Errors in the Ω_M-w plane (top panel) and the w_0-w_a plane [bottom panel, with a prior on matter density, $\sigma(\Omega_M)$ of 0.01] assuming a 10% measurement of a given quantity at redshifts $z=[0.1,0.2,\ldots,2.0]$. We show errors for distance $r(z)$, Hubble parameter $H(z)$, growth factor $D(z)$, differential volume element $r^2(z)/H(z)$, $dD/d\eta$ (η is the conformal time), and rate of growth suppression, dg/dz. Note that the latter, dg/dz, is by far the most sensitive to Ω_M and w. For example, a 10% accuracy measurement of dg/dz is comparable and complementary to $<1\%$ measurement of distance.

effect therefore gives a direct measure of the integral of dg/dr (or dg/dz) computed over some effective time (or redshift) interval.

While the ISW effect determined at the present time can be used as a probe of dark energy [14], its contribution to CMB temperature fluctuations is dwarfed by the primordial anisotropy contribution at last scattering. Though the cross-correlation between the large scale structure and CMB anisotropy fluctuations has been considered as a method to extract the ISW contribution [15], such correlations are affected by the dominant noise contribution related to primary anisotropies [16].

There is another way of extracting information captured in the ISW effect: through the measurement of CMB polarization towards galaxy clusters [17]. The polarization signal is generated by rescattering of the temperature quadrupole seen by free electrons in the cluster frame [18]. Provided that the optical depth to scattering in individual clusters is determined *a priori* by other methods, such as the Sunyaev-Zel'dovich (SZ [19]) effect, one can measure the quadrupole at the cluster location with a reduction in cosmic variance [20]. Note that the quadrupole measured from a cluster at high redshift is not the same quadrupole as one observes today due to the difference in the projected length scales. Since the ISW effect contributes a significant fraction of the quadrupolar anisotropy at late times, cluster polarization provides an indirect probe of dark energy. Because clusters can be selected over a wide range in redshift, the polarization signal can be measured as a function of redshift and inverted to reconstruct the evolution of the ISW quadrupole [17].

The anisotropy quadrupole, $C_2(z)$, has two contributions: one at the surface of last scattering due to the Sachs-Wolfe (SW) effect, $C_2^{\rm SW}(z)$, and another at late times due to the ISW effect, $C_2^{\rm ISW}(z)$. We write these two contributions to the power spectrum, projected to a redshift z, respectively as

$$C_2^{\rm SW}(z)=\frac{4\pi}{9}\int_0^\infty \frac{dk}{k}\Delta^2_{\Phi\Phi}(k,r_{\rm rec})j_2^2[k(r_{\rm rec}-r)],$$

$$C_2^{\rm ISW}(z)=16\pi\int_0^\infty \frac{dk}{k}\Delta^2_{\Phi\Phi}(k,r_{\rm rec}) \times\left[\int_r^{r_{rec}} dr'\frac{1}{g(z_{\rm rec})}\frac{d}{dr'}g(z')j_2[k(r'-r)]\right]^2. \quad (3)$$

Here r is the radial comoving distance out to redshift z and $\Delta^2_{\Phi\Phi}(k,r_{\rm rec})[\equiv k^3P_{\Phi\Phi}(k,r_{\rm rec})/2\pi^2]$ is the logarithmic power spectrum of fluctuations in the potential field at the last scattering surface. We will concentrate on the dark energy properties, whose effects are dominant at low redshifts, and assume that the parameters that define the power spectrum, such as the normalization, spectral tilt, and physical matter and baryon densities $\Omega_{\rm M}h^2$ and $\Omega_{\rm B}h^2$, are known to the accuracy expected from the Planck mission with polarization information [22]. Given these priors, the SW contribution is then known to a few percent accuracy. Also note that, conveniently, only the large scales in the power spectrum contribute to the ISW effect, so that we do not need to consider thorny issues related to small-scale nonlinear structures and additional parameters such as the neutrino mass. There is one complication related to the fact that for models with $w \ll -1$, the dark energy component is expected to cluster such that the quadrupole amplitude is suppressed. The dark energy clustering also leads to a scale dependent growth function instead of the scale independent one considered here. For now, we ignore this subtlety since we focus on models where $w \sim -1$. Moreover, we do not expect this issue to be important since by renormalizing the quadrupole, as a function of redshift, to its value today, we extract information from the quadrupole evolution and not its absolute amplitude.

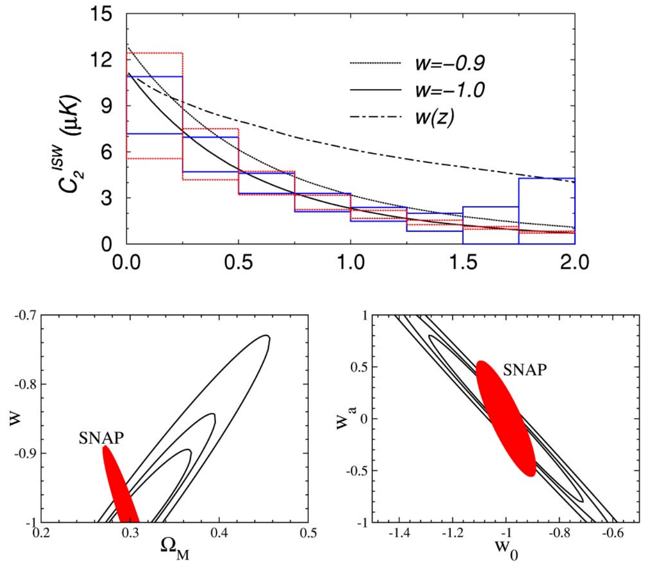

FIG. 2. Top panel: the projected ISW quadrupole as a function of redshift. The solid error bars assume a reconstruction with clusters down to 10^{14} $M_\odot$ in an area of 10^4 deg^2 with an instrumental noise of 0.1 μK. The dotted lines show the cosmic-variance for an all-sky reconstruction computed from the number of independent volumes sampled by clusters at each redshift bin [20]. The dot-dashed curve is a late-time transition model for the dark energy studied by Ref. [21] with $w(z)=w_f/\{1+\exp[(z-z_t)/\Delta]\}$. We illustrate the redshift dependence of the quadrupole, normalized to the quadrupole-today, with parameters $w_f=-1$, $z_t=2$, and $\Delta=z_t/30$. Bottom left: Parameter errors from the projected ISW quadrupole measurements, assuming $w=$const. The small ellipse is for the case shown in the top panel (with cosmic variance added in quadrature), while the two larger ellipses assume a factor of 3 and 10 increase in the instrumental noise contribution, respectively. For comparison, we also show the constraints expected from SNAP. When the most optimistic polarization information is added, SNAP's constraints on w improve by a factor of 4. Bottom right: same, but for w_0 and w_a and assuming an additional prior $\sigma(\Omega_{\rm M})=0.01$.

The galaxy cluster polarization signal arises from the rescattering of the quadrupole which receives a contribution from $C_2^{\rm ISW}(z)$ at low redshifts. Reference [17] discussed how well this quadrupole can be measured as a function of redshift with Planck and a ground-based experiment with significant instrumental noise. In the top panel of Fig. 2 we show the projected ISW contribution to the temperature quadrupole as a function of redshift, and expected errors for a ground-based survey targeting clusters down to a mass limit of $5\times 10^{14} M_\odot$ in a total area of 10^4 deg^2 with an instrumental noise for polarization observations of 0.1 μK. Note that we use clusters only as tracers of the quadrupole; the proposed test is therefore insensitive to the exact number of clusters and its dependence on cosmology.

We assume four channels for these observations so that the ISW quadrupole can be separated from the contribution of the kinematic quadrupole. The latter has a distinct spectrum and the separation based on frequency information leads to an overall increase in noise by a factor of 2 to 3 depending on the exact frequencies of channels selected. Note that we have assumed an instrumental noise of 0.1 μK for these observations. While a polarization sensitive detector array on the SPT can be expected to reach noise levels of ~ 1 μK or less per pixel, we have assumed an order of magnitude reduction in noise, as expected from the planned CMBPol satellite mission. Since the expected noise level for arcminute scale polarization observations from such a mission is not currently defined, and to consider ground-based efforts such as the SPT, we have considered the range of values between 0.1 and 1 μK so as to obtain some guidance on how well cluster polarization measurements with noise in this range can be used to probe dark energy.

In addition to instrumental noise, the polarization measurements are subject to cosmic variance. This variance is determined by the number of independent volumes that last scattering spheres of individual clusters, in some redshift bin, occupy [20]. Dotted lines in the top panel of Fig. 2 show the cosmic variance contribution for an all-sky experiment. As one moves to higher redshift, the number of independent samplings of the local quadrupole increases, leading to a reduction in cosmic variance. The expected redshift distribution of clusters peaks at redshifts around 1–1.5 where it provides the best estimate of the local quadrupole, while errors increase at very low and high redshift due to the smaller number of clusters.

To consider how well these observations can be used to understand dark energy parameters, we again perform a Fisher matrix calculation. The bottom panels of Fig. 2 show how well $\Omega_{\rm M}$ and w (assuming a flat universe and constant w), and w_0 and w_a [assuming a two-parameter description of $w(z)$ as before and a prior on $\Omega_{\rm M}$ of 0.01] can be measured. While the errors are fairly large with a 1 μK noise level per pixel, improving this noise threshold to 0.1 μK leads to significant gains in the determination of $\Omega_{\rm M}$ and w. Note also that these errors roughly scale as the inverse square root of the area of sky covered, and with all-sky coverage the errors are expected to decrease by a factor of two. With an order of magnitude improvement in noise, the redshift evolution of the ISW effect extracted from polarization measurements becomes a powerful probe of dark energy providing significant estimates of parameters, comparable and complementary to type Ia supernovae.

To conclude, we have argued that the rate of evolution of the growth suppression factor, dg/dz, is a very powerful probe of dark energy. We have shown that the polarization signal from a large number of galaxy clusters is directly related to this quantity, and can be used to constrain dark energy parameters. In the next decade, the planned mission CMBPol is expected to reach a sensitivity of order 0.1 μK at arcminute resolution and have all-sky coverage, providing polarization measurements of a significant number ($\sim 10^4$) of clusters, from which the quadrupole can be reconstructed as a function of redshift. Although our study is preliminary, we have shown that this method can provide constraints on the dark energy equation of state and its time variation comparable and complementary to those from type Ia supernovae and other well-studied probes of dark energy. More importantly, this method is entirely different from most of the others both in its theoretical underpinnings and in the systematic errors expected. Combining this method with others opens the exciting possibility of significantly improving the constraints on w and helps usher a new era in our exploration of dark energy.

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