The Astrophysical Journal, 615:595–602, 2004 November 10

REDSHIFT ACCURACY REQUIREMENTS FOR FUTURE SUPERNOVA AND NUMBER COUNT SURVEYS

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Received 2004 February 13; accepted 2004 July 5

ABSTRACT

We investigate the redshift accuracy of Type Ia supernova and cluster number count surveys required for the redshift uncertainties not to contribute appreciably to the dark energy parameter error budget. For the *Supernova/Acceleration Probe* experiment, we find that without the assistance of ground-based measurements individual supernova redshifts would need to be determined to about 0.002 or better, a challenging but feasible requirement for a low-resolution spectrograph. However, we find that accurate redshifts for $z < 0.1$ supernovae obtained with ground-based experiments are sufficient to protect the results against even relatively large redshift errors at high z. For the future cluster number count surveys such as with the South Pole Telescope, *Planck*, or *DUET*, we find that the purely statistical error in the photometric redshift is less important and that the irreducible systematic bias in redshift drives the requirements. The redshift bias must be kept below 0.001–0.005 per redshift bin (which is determined by the filter set), depending on the sky coverage and details of the definition of the minimal mass of the survey. Furthermore, we find that X-ray surveys have a more stringent required redshift accuracy than Sunyaev-Zeldovich (SZ) effect surveys since they use a shorter lever arm in redshift; conversely, SZ surveys benefit from their high-redshift reach only as long as some redshift information is available for distant ($z \gtrsim 1$) clusters.

Subject headings: cosmological parameters — cosmology: theory — large-scale structure of universe — supernovae: general

1. INTRODUCTION

Two of the most promising methods of measuring cosmological parameters, in particular those describing dark energy, are distance measurements of Type Ia supernovae (SNe Ia) and number counts of clusters of galaxies in the universe. SNe Ia have provided original direct evidence for dark energy (Riess et al. 1998; Perlmutter et al. 1999; for earlier, indirect evidence, see Krauss & Turner [1995] or Ostriker & Steinhardt [1995]) and are currently the strongest direct probe of the expansion history of the universe (Tonry et al. 2003; Knop et al. 2003). Their principal strengths are the simplicity of relating the observable, which is essentially the luminosity distance, to cosmological parameters and the fact that each SN redshift-magnitude pair provides a distinct measurement of a combination of those parameters. Number counts, on the other hand, use the fact that galaxy clusters are the largest collapsed structures in the universe that have undergone a relatively small amount of postprocessing. Their distribution in redshift can be reliably calculated in a given cosmological model. The evolution of cluster abundance is principally sensitive to the comoving volume and growth of density perturbations (Haiman et al. 2001), and this cosmological probe is expected to reach its full potential with upcoming and future wide-field surveys.

Rapid improvement in the accuracy of measuring cosmological parameters implies that various systematic uncertainties, previously ignored, now have to be controlled and understood quantitatively. In the case of SN measurements, an example is provided by the proposed *Supernova/Acceleration Probe* (*SNAP*) satellite (Aldering et al. 2004), whose goals for measuring the equation of state of dark energy w and its variation with redshift dw/dz drive requirements on the systematic control that are considerably more stringent than those attainable with current surveys. Similarly, the principal systematic difficulty in cluster counts is in establishing the relation between observable quantities (X-ray temperature or Sunyaev-Zeldovich [SZ] flux decrement) and the cluster's mass, which is necessary for comparison with theory. The mass-temperature relation, for example, is known to have a considerable scatter and is currently poorly determined, with fairly large intrinsic statistical errors and considerable systematic disagreements between different authors (see, e.g., Fig. 2 in Huterer & White 2002). The cleanest way to include the mass-observable relation might be to determine it from the survey itself (this is known as "self-calibration"; Levine et al. 2002; Majumdar & Mohr 2003, 2004; Hu 2003; Lima & Hu 2004), but this will almost certainly lead to degradations in parameter accuracies. Future surveys will require a careful accounting of all systematic errors, theoretical and observational.

In this paper we concentrate on one of the most basic ingredients of SN and cluster count measurements: the determination of redshift. In the case of SNe Ia, spectroscopic observations are necessary to identify the SN type, and redshift is then supplied for free. Recently completed and ongoing surveys have sufficiently poor magnitude uncertainty, relatively low statistics, and relatively weak control on known systematic errors that the spectroscopic redshift error is small enough for the redshifts to be considered perfectly known. However, as we see below, future SN observations will require such accurate redshifts that even the spectroscopic accuracy is not a priori guaranteed to be sufficient.

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In the cluster count case, the situation is even more interesting, as spectroscopic observations will not be possible for all clusters, which may number in the tens of thousands. One will therefore rely on photometric redshifts. Although photometric redshifts are already impressively accurate (e.g.. Fernández-Soto et al. 2002; Csabai et al. 2003; Collister & Lahav 2004; Vanzella et al. 2004), we shall find that their bias (the difference between the mean photometric value and the true value at any redshift) needs to be kept exceedingly small for the redshift error not to contribute appreciably to the total error budget. Our analysis is timely, as follow-up surveys to obtain cluster redshifts, such as that at Cerro-Tololo Inter-American Observatory, are about to get underway. Our analysis also complements recent analyses of the effect of systematic errors on future SN Ia measurements (Kim et al. 2004; Frieman et al. 2003) and a variety of related analyses regarding the cluster number count surveys (e.g., Bartlett 2000; Holder & Carlstrom 2001; White et al. 2002a, 2002b; Benson et al. 2002; White 2003).

The paper is organized as follows. In § 2 we outline the procedure to include the redshift uncertainty in the standard Fisher matrix parameter estimation. In § 3 we discuss the redshift requirements for future SN surveys, while in § 4 we do the same for future cluster count surveys. We conclude in § 5. Our fiducial model is a flat universe with matter energy density relative to the critical value $\Omega_m = 0.3$ and the equation of state of dark energy $w = -1$. Other cosmological parameters, necessary for the cluster abundance calculation, are discussed in § 4.

2. METHODOLOGY

Let us assume we have an observable $O(z)$ from which we want to determine P cosmological parameters $\theta_1, \ldots, \theta_P$. Since the number of observed objects is large (thousands for future SN Ia surveys and of order 10,000 for cluster surveys), we bin observations in Q redshift bins centered at z_k ($1 \le k \le Q$), each with width Δz_k.

To include the redshift uncertainty, we treat the bin centers $z_1, \ldots, z_Q$ as Q additional nuisance parameters $\theta_{P+1}, \ldots, \theta_{P+Q}$. Variation of any of these parameters moves the location of the whole corresponding redshift bin. We use the Fisher matrix formalism to estimate all $P + Q$ parameters simultaneously, and we give priors to redshift parameters that represent how accurately they are independently determined.

The $(P+Q) \times (P+Q)$ Fisher matrix is given by

$$F_{ij} = \sum_{k=1}^{Q} \frac{N_k}{\sigma_O(z_k)^2} \frac{\partial O(z_k)}{\partial \theta_i} \frac{\partial O(z_k)}{\partial \theta_j}, \qquad (1)$$

where N_k is the number of objects in the kth bin. The observable $O(z_k)$ is the mean apparent magnitude of an SN $m(z_k)$ in a given redshift bin, or else the number of clusters $N(z_k)$ in a bin. Note that the redshift parameter representing the ith bin affects the observable $O(z_k)$ only if $i = k$. Therefore,

$$\frac{\partial O(z_k)}{\partial \theta_{P+i}} = \frac{\partial O(z_i)}{\partial \theta_{P+i}} \delta_{ik} \quad \text{for} \quad i \in [1, \ldots, Q], \qquad (2)$$

and the expression for the Fisher matrix simplifies accordingly for the redshift parameter terms.

We assume the error in the observable $O(z_k)$ is Gaussian-distributed with standard deviation $\sigma_O(z_k)$. With that assumption, any Gaussian prior imposed on the individual object, $\sigma_{\rm pr}$, will be equivalent to imposing a prior of $\sigma_{\rm pr}/\sqrt{N_k}$ on the observable representing the kth redshift bin since there are N_k objects in this bin. Note that the number of redshift bins, Q, needs to be large enough to retain the shape information of the function $O(z)$; we use steps of 0.02 for both SN Ia and number counts and have checked that a higher number of bins leads to negligible changes in all our results. Note too that these bins are used for computational accuracy; they should not be confused with the *physical* redshift bins, which are determined by the filter set of the experiment and which we later discuss. Finally, we ignore the cosmic variance contribution to the error in number count surveys, since it has been shown that cosmic variance becomes small for the high-redshift cluster surveys with relatively high mass thresholds (Hu & Kravtsov 2003), which is the case we study in this paper.

While our Fisher matrix formalism assumes the redshift errors to be Gaussian, it is conceivable that the errors have significant non-Gaussian tails and/or pronounced skewness. This may especially be true for the photometric redshift errors, for which a small fraction of redshifts may have a large deviation from their true value. In this situation our formalism is still appropriate: by the central limit theorem (and as confirmed with Monte Carlo methods) the central (mean) value of any given redshift bin, z_i, is guaranteed to have Gaussian error dispersion (whose center may be shifted from the true value of z_i) *even if individual objects in this bin have errors that are strongly non-Gaussian*. This fully justifies our use of Gaussian priors on z_i, and the same argument applies to other observables we consider, the mean SN magnitude and number of clusters in a redshift bin. To *determine* the resulting rms of z_i and its bias (shift relative to its true value), however, one needs to know a priori the redshift distribution of objects and their measurement errors. While detailed modeling of the photometric redshift error (which takes into account various types of galaxies and their properties at any given redshift) requires a Monte Carlo approach that is outside the scope of this work, here we explore results for a range of widths of the Gaussian distribution of the redshift bin centers, z_i.

3. TYPE Ia SUPERNOVAE

3.1. *Redshift Dependence of Supernova Measurements*

The measurement of the cosmological parameters using calibrated candles requires both the magnitude and redshift of the object in question. In SN studies the redshift measurement is typically taken from the spectrum of the host galaxy, either from sharp emission lines or from the 4000 Å break. Up to the present, the magnitude error of high-redshift SNe has dwarfed the redshift measurement error. As we enter an era of high-precision SN cosmology with significant improvement in statistical and systematic uncertainties, we need to explore the effects of redshift measurement error on the determination of cosmological parameters. Cosmological observations of high-redshift SNe are generally made in the observer X and Y bands, which roughly correspond to supernova-frame B and V bands.

Observed SN Ia magnitudes are modeled as

$$\begin{aligned} m_X = {} & M_B - \alpha(s-1) + A_B(s, z) \\ & + K_{BX}(s, z) + \mu(z; \theta_i). \end{aligned} \qquad (3)$$

The peak absolute magnitude of an SN $M_B - \alpha(s-1)$ is a function of the "stretch" s of its light-curve shape; SNe with higher stretch are intrinsically brighter (Perlmutter et al. 1997). The K-correction K_{BX}, a function of redshift and stretch, accounts for differences in the spectral energy distribution (SED) transmitted through the B and X bands for low- and high-redshift SNe, respectively. The extinction from the host galaxy is given by $A_B = R_B E(B - V)$, where the color excess is

$$E(B - V) = \{[m_X - K_{BX}(s, z)] - [m_Y - K_{VY}(s, z)]\} - (B - V)_{\rm exp}(s), \quad (4)$$

where $(B - V)_{\rm exp}(s)$ is the expected SN color, which is a function of stretch. The distance modulus μ is a function of redshift and the cosmological parameters. Gravitational lensing magnification is not considered here since its effect on the inferred magnitudes of distant SNe is not sensitive to small variations in redshift.

The effect of redshift error on the estimated distance modulus is straightforward: a positive redshift error, dz, incorrectly gives an inflated distance to the SN. In addition, the measurement of stretch itself depends on z; the stretch is obtained from the observed width W of a light curve using the formula $s = W/(1 + z)$, so that an error in z propagates as $ds = -s\,dz/(1 + z)$. An overestimate of redshift gives an underestimate of the stretch factor and, therefore, an overestimate of the expected SN magnitude.

The extinction terms and K-correction depend on both redshift and stretch. Expected observed colors with a fixed pair of filters near the rest-frame B and V are bluer for slightly higher redshift SNe. An overestimated redshift thus gives an overestimated extinction determination. In contrast, the simultaneously underestimated stretch determination overestimates the intrinsic redness of the SN, underestimating the extinction. In addition, stretch-dependent SEDs and redshift errors introduce K-correction errors whose behavior depends on the specific redshift and filters involved.

We propagate redshift errors into errors in the expected observed peak magnitude for a canonical SN search. We adopt a filter set consisting of redshifted B filters such that the nth filter has throughput $F_n(\lambda) = B(\lambda/1.16^n)$, where λ is the wavelength. We adopt the empirically derived $\alpha = 1.9$ stretch-magnitude relation found by Perlmutter et al. (1997). The stretch-dependent SN color is given as $B - V = -0.19(s - 1) - 0.05$. We use the K-correction methodology given in Nugent et al. (2002); the observer filters with effective wavelengths closest to $4400(1 + z)$ and $5500(1 + z)$ are associated with rest-frame B and V, respectively. The host-galaxy extinction is assumed to obey the standard $R_B = 4.1$ dust model of Cardelli et al. (1989).

Figure 1 shows individual contributions to the derivative of magnitude with respect to redshift, dm/dz, as well as their sum for an $s = 1$ SN. Note that the total error is dominated by the distance modulus at $z \lesssim 0.4$, while at larger redshifts the K-correction and extinction errors become increasingly dominant. The K-correction and extinction errors are discontinuous and periodic in redshift as different observer filters are traversed. The distance modulus error, being proportional to the relative error in luminosity distance, $d(d_L)/d_L$, goes to infinity as the redshift goes to zero. This is simple to understand, as in the $z \to 0$ limit, $d(d_L)/dz$ approaches a constant, while d_L itself goes to zero. Therefore, low-redshift SNe have the most need for accurate redshifts; we discuss this further below. Note that the use of dm/dz to assess the effect of redshift errors is only approximate since the K-corrections can be highly nonlinear; however, the approximation is good for the span of redshift errors that we consider.

3.2. *Results*

We explore the effects of redshift error in the measurement of dark energy parameters based on a SN search such as that

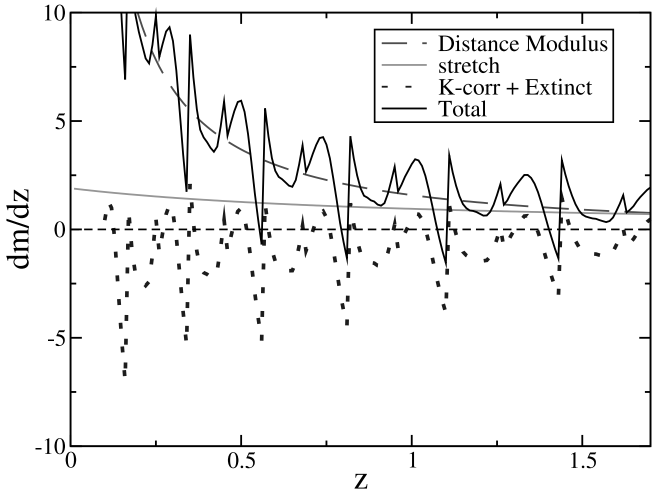

FIG. 1.—Differential effect on the expected SN peak magnitude with variation in redshift, or dm/dz, as a function of redshift. Also shown are the contributions from individual terms: the distance modulus, stretch, K-correction, and extinction.

with the proposed *SNAP* space telescope (Aldering et al. 2004). As we discuss below, more powerful experiments require more stringent control of redshift errors; therefore, our requirements for *SNAP* will be more than sufficient for other ground- and space-based surveys in the next 5–10 years. We assume an SN distribution with around 2700 SNe distributed between $z = 0.1$ and 1.7, together with 300 additional low-redshift ($z \approx 0.05$) SNe from the ground-based SN Factory (Aldering et al. 2002). The fiducial magnitude error per SN, the quadratic sum of measurement error and intrinsic SN magnitude dispersion, is 0.15 mag. (An analysis of the effect of redshift-dependent magnitude uncertainties will be discussed in L. M. Krauss et al. 2004, in preparation.) We consider the degradation, due to imperfectly known redshifts, of the accuracy in the equation of state of dark energy σ_w, where w is assumed constant. The uncertainty σ_w is computed by marginalizing over the matter density Ω_m, the overall offset in the magnitude-redshift diagram, M, and the redshift parameters z_i ($i \in [1, \ldots, Q]$). We also consider the degradation in the accuracy in measuring the redshift evolution in the equation of state, dw/dz, where $w(z) = w_0 + z(dw/dz)$; in this case we further marginalize over w_0 and add a Gaussian prior of 0.01 to Ω_m to allow comparisons with other analyses in which this procedure has become standard.

Figure 2 shows the degradation in σ_w (*top*) and dw/dz (*bottom*) as a function of the redshift error per SN. The solid line shows the case in which the redshift error is constant for all SNe, whereas the dashed line represents an uncertainty growing as $dz \propto (1 + z)$. The two cases are qualitatively similar and show that, for example, a redshift error of 0.005 per SN leads to a 25% increase in σ_w and a 7% increase in $\sigma_{dw/dz}$.

However, these results assume that the error in redshift, absolute or fractional, is the same at all redshifts. In reality, the SN Factory spectra will have a fixed high resolution. The accuracy at low redshift will be limited by peculiar velocities of galaxies, which are of order 400 km s^{-1}, corresponding to the redshift accuracy floor of about 0.0013. With this in mind, we assume that redshifts at $z < 0.1$ have a fixed accuracy of 0.0015 per SN (the case shown with the flatter pair of lines in Fig. 2), while those at $z > 0.1$ have the accuracy shown.

The degradation in w or dw/dz is now smaller than about 10%, even for errors of 0.02 for SNe at $z > 0.1$! Therefore,

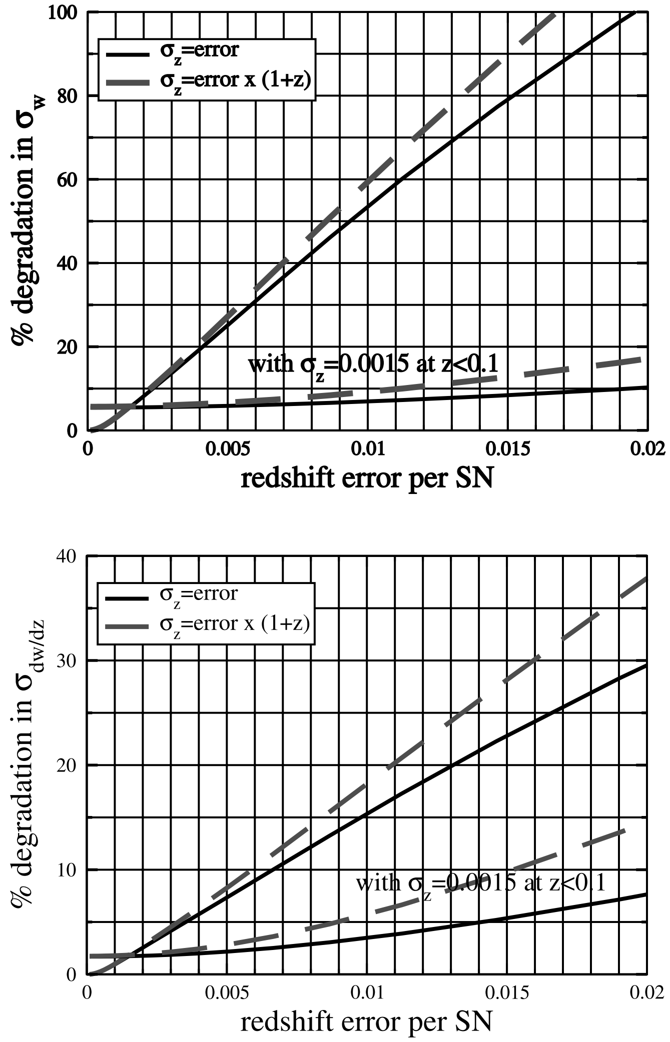

FIG. 2.—Degradation in the measurement accuracy of the equation of state w (*top*) and, alternatively, its rate of change with redshift dw/dz (*bottom*), as a function of redshift error per SN. The solid line shows the case for which the redshift error is constant for all SNe, while the dashed line shows the case for which the error (*abscissa*) is multiplied by $1 + z$. The flatter pair of curves in each panel corresponds to the case in which $z < 0.1$ SNe were assumed to have a fixed redshift error of 0.0015 per SN.

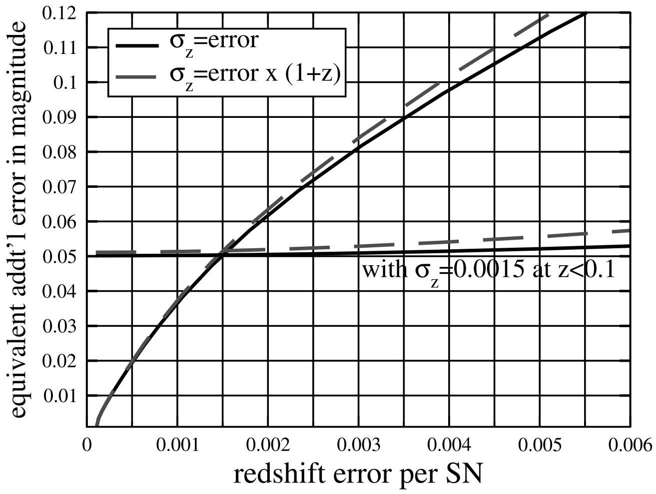

FIG. 3.—Redshift error increases the degradation in σ_w as some equivalent *additional* magnitude error (*ordinate*). The solid line shows the case for which the redshift error is constant for all SNe, while the dashed line shows the case for which the error (*abscissa*) is multiplied by $1 + z$. As in Fig. 2, the flatter pair of curves corresponds to the case in which $z < 0.1$ SNe were assumed to have a fixed redshift error of 0.0015 per SN.

accurate redshifts of low-redshift SNe prevent larger errors at higher redshifts. This conclusion is easy to understand: the fixed error in redshift roughly corresponds to the fixed error in distance to an SN, while the total SN magnitude error increases linearly with distance. Therefore, the redshift error contributes a larger percent of the total error budget for low-redshift SNe. Furthermore, low-redshift SNe are crucial for parameter determination, and their omission (or inclusion, but with a large redshift error) would lead to nearly a factor-of-2 degradation in the constraints on w and dw/dz (e.g., Huterer & Turner 2001). Therefore, we conclude that, provided that redshifts of low-redshift SNe are measured with high accuracy, measurements of w and dw/dz are weakly sensitive to the redshift errors of high-redshift SNe.

For convenient reference, we associate redshift errors with the magnitude error that gives an equivalent uncertainty in w (see Fig. 3). For example, a 0.005 redshift error introduces an uncertainty equivalent to the additional 0.1 intrinsic magnitude dispersion per SN. As before, the flatter pair of lines in the same figure shows the effect of accurate redshifts for $z < 0.1$ SNe, in which case the overall redshift uncertainty contributes little ($\lesssim$0.02 mag in the range of redshift errors shown) to the total error budget.

The spectroscopic follow-up of high-redshift SNe (or, more appropriately, their host galaxies) from surveys must be carefully considered. Ground-based spectroscopy associated with current high-redshift SNe searches have more than sufficient resolution to measure redshifts to $\lambda/\delta\lambda = 200$. (Subpixel interpolation gives a wavelength resolution several times better than the instrumental resolution R.) Very wide field SNe searches that discover thousands or more SNe may depend on photometric redshifts as an alternative to spectroscopic follow-up. Photometric redshift determination is currently limited by a statistical accuracy floor of a few percent (see § 4); in this case the effect of redshift error can be comparable in size to the intrinsic corrected-magnitude dispersion of SNe Ia of $\sim$0.1 mag, although we have just shown that accurate redshifts at $z < 0.1$ largely reduce the overall redshift contribution to the error budget. Of course, the two more serious problems are identification of SNe Ia and control of systematic errors, both of which are very difficult without their spectra.

In the case of a 2 m space telescope (such as *SNAP*) observing $z \sim 1.7$ SNe, the Poisson noise from the zodiacal background and source can be low. Considering the noise properties of HgCdTe detectors, signal-to-noise ratio arguments push for a low-resolution spectrograph to avoid a detector noise–limited instrument. If the on-board spectrograph is to provide SN redshifts, the competing needs for low and high resolution must be considered in the design.

We end with several comments. First, we do not consider peculiar velocities since they have negligible effect on high-redshift SNe. Second, our SN calculations involve statistical magnitude errors only. Adding the irreducible systematic error in each redshift bin, as is done in Frieman et al. (2003), for example, is straightforward and leads to a slight weakening of the required redshift accuracy. This is easy to understand, since the fiducial parameter accuracy, such as σ_w, slightly weakens in the presence of systematic errors, and the redshifts do not need to be known quite as accurately as in a perfect world without systematic errors.

Finally, we comment on the possibility of using the *photometric* redshifts for nonlocal ($z > 0.1$) SNe. In this approach one has to consider the irreducible redshift errors within coarse redshift bins; this point is discussed in detail in § 4.1. In the SN survey considered here, if we assume a correlation length of 0.15 in redshift, approximately 240 SNe would fall in each coarse redshift bin. A redshift uncertainty per SN of $\delta z_{\rm SN}$ can then be viewed as equivalent to a redshift uncertainty per bin of $\delta z_{\rm bin} = \delta z_{\rm SN}/(240)^{1/2}$. Using this relation, Figure 2 can provide a first estimate of the degradation in the measurement of w and dw/dz from these irreducible photometric errors. For example, a redshift bias per bin smaller than 0.001 would be necessary to ensure that the degradation in the measurement of w and dw/dz is less than 5%. Clearly, photometric redshifts would need to be exceedingly accurate not to degrade the dark energy constraints from SNe Ia.

4. CLUSTER NUMBER COUNT SURVEYS

4.1. *Fiducial Surveys and Assumptions*

Clusters can be found using their X-ray flux, through their SZ temperature decrement, or through deflection of light from background galaxies caused by weak gravitational lensing by the cluster. Current or upcoming surveys specifically suited to find clusters include the *XMM* Serendipitous Cluster Survey (Romer et al. 2001), the *XMM* Large Scale Structure survey (Pierre et al. 2003), the ACBAR,[5] the Sunyaev-Zeldovich Array,[6] the APEX SZ survey,[7] and the Atacama Cosmology Cluster Telescope.[8] Cosmology will realize its full potential with future wide-field surveys, such as the South Pole Telescope (SPT),[9] the space-based mission *Planck*,[10] the proposed space-based mission *DUET*, and, in the next decade, *SNAP*.

Cluster redshifts are required to use clusters as a probe of cosmology, yet the large number of clusters expected in the aforementioned upcoming surveys (thousands to tens of thousands) makes it impractical to obtain their redshifts spectroscopically. Therefore, cluster abundance studies must rely on the photometric redshifts. Fortunately, cluster photometric redshifts are currently measured with very good accuracy (e.g., Bahcall et al. 2003), chiefly because the photometric z-values of individual galaxies in the cluster can be averaged, leading to statistical errors in cluster redshifts of about 0.02. However, goals for the cluster abundance surveys are set high—to measure the equation of state of dark energy w to an accuracy of 5%–10% and the power spectrum normalization σ_8 to about 1%—and it is worthwhile to study the photometric redshift accuracy required to achieve this goal. Previous studies of the cluster abundance (Haiman et al. 2001; Levine et al. 2002; Battye & Weller 2003; Hu & Kravtsov 2003; Majumdar & Mohr 2003; Molnar et al. 2004) have explored the efficacy of cluster number counts as a probe of cosmology, but all assumed perfect knowledge of redshifts.

We adopt the fiducial cosmological model, which accords with recent results from various recent experiments (Riess et al. 1998; Perlmutter et al. 1999; Freedman et al. 2001; Percival et al. 2001; Pierpaoli et al. 2003; Tegmark et al. 2004; Spergel et al. 2003; Knop et al. 2003; Jarvis et al. 2003; Riess et al. 2004; Massey et al. 2004). We assume a flat universe with matter energy density relative to the critical value $\Omega_m = 0.3$, a dark energy equation of state $w = -1$, and power spectrum normalization $\sigma_8 = 0.9$. We use spectral index and physical matter and baryon energy densities with mean values $n = 0.97$, $\Omega_m h^2 = 0.140$, and $\Omega_B h^2 = 0.023$, respectively. We add a fairly conservative prior of 5% to each of these parameters; we checked that our results are insensitive to this prior. The parameters to which cluster surveys are most sensitive are Ω_m, w, and σ_8, and we do not give any priors to these parameters. The fiducial surveys we consider determine Ω_m and σ_8 to an accuracy of about 0.01 and w to about 0.02–0.12. We are, however, interested only in the *degradation* of these accuracies due to uncertain knowledge of the redshifts, and this fact makes our results less dependent on the details of the survey. Finally, for this analysis we assume that the "mass-observable" (i.e, mass-temperature or mass–X-ray flux) relation is perfectly known. We have checked that leaving the normalization of this relation as a free parameter to be determined by the data can strongly degrade the fiducial parameter constraints, but it affects the sensitivity to the knowledge of redshifts, which we explore here, much more weakly.

To compute the comoving number of clusters, we use the Jenkins et al. (2001) mass function. The required input is the linear power spectrum; for $w = -1$ models, we use the formulae of Eisenstein & Hu (1999) that were fitted to the numerical data produced by CMBFAST (Seljak & Zaldarriaga 1996). We generalize the formulae to $w \neq -1$ by appropriately modifying the growth function of density perturbations. The total number of objects in any redshift interval centered at z with width Δz is

$$N(z,\, \Delta z) = \Omega_{\rm sur} \int_{z-\Delta z/2}^{z+\Delta z/2} n(z,\, M_{\rm min}(z)) \frac{dV(z)}{d\Omega\, dz}\, dz, \qquad (5)$$

where $\Omega_{\rm sur}$ is the total solid angle covered by the survey, $n(z, M_{\rm min})$ is the comoving density of clusters more massive than $M_{\rm min}$, and $dV/(d\Omega dz)$ is the comoving volume element.

An incorrect determination of individual cluster redshifts leads to an incorrect central value of the redshift bin to which these clusters are assigned (see below for the definition of redshift bins). This in turn leads to evaluating the theoretically expected number of clusters $N(z, \Delta z)$ at an incorrect central redshift z, thus biasing the inferred cosmological parameters. Here we represent the uncertainty in the central value of the redshift bin as

$$\sigma_{z,\,\rm bin} = \sqrt{\frac{\sigma_{\rm clus}^2}{N(z,\, \Delta z)} + \sigma_{\rm sys}^2}; \qquad (6)$$

that is, the redshift error is the sum of the purely statistical (random Gaussian) error per cluster $\sigma_{\rm clus}$ and an irreducible systematic error $\sigma_{\rm sys}$. The source of the irreducible error could be, for example, a systematic offset in the photometric error determination that affects all clusters in that bin equally. We assume that the irreducible error is uncorrelated between bins. The Fisher matrix is constructed as in equation (1), with $O(z) \equiv N(z, \Delta z)$.

We assume three representative fiducial surveys: (1) the SPT, a 4000 deg^2 survey that will detect clusters through their SZ signature, (2) the *Planck* mission, which we consider to be a 28,000 deg^2 SZ survey, and (3) *DUET*, a planned X-ray space mission with coverage of 10,000 deg^2. For SPT and *Planck* we use the mass–SZ flux relation from Majumdar & Mohr (2004), while for *DUET* we assume the Majumdar-Mohr mass–X-ray

[5] See http://cosmology.berkeley.edu/group/swlh/acbar.
[6] See http://astro.uchicago.edu/sza.
[7] See http://bolo.berkeley.edu/apexsz.
[8] See http://www.hep.upenn.edu/~angelica/act/index.html.
[9] See http://astro.uchicago.edu/spt.
[10] See http://astro.estec.esa.nl/Planck.

flux relation. We normalize these analytical mass-observable relations so that all three of these surveys would produce between 18,000 and 25,000 clusters for our fiducial cosmology; we have checked that different normalizations do not change our results dramatically. Nevertheless, the choice of the mass-observable relation is important since $M_{\rm min}$ depends on cosmological parameters and modifies the error budget. To present a range of possibilities, we further consider the SPT survey with fixed limiting masses of 10^{14}, 2×10^{14}, and $5\times10^{14}\ h^{-1}\ M_\odot$; in our fiducial cosmology these three possibilities lead to about 95,000, 20,000, and 1700 clusters, respectively.

Redshifts for the clusters are provided by optical and near-infrared photometric follow-up. These photometric redshifts are calibrated using supplemental spectroscopic observations of a galaxy subset. We assume the Sloan Digital Sky Survey (SDSS) passbands; they have been extensively used for photometric redshift measurements of moderate-redshift galaxy clusters (Bahcall et al. 2003). The SDSS photometric system (Fukugita et al. 1996) is composed of five bands: u' peaks at 3500 Å with FWHM of 600 Å, g' peaks at 4800 Å with FWHM of 1400 Å, r' peaks at 6250 Å with FWHM of 1400 Å, i' peaks at 7700 Å with FWHM of 1500 Å, and z' peaks at 9100 Å with FWHM of 1200 Å. The redshift bins are defined by where the 4000 Å line enters and leaves these bands, and the bin width is typically 0.1–0.2 in redshift. At redshifts greater than about 1.2, the 4000 Å line leaves the observable bands and enters the infrared, which makes obtaining the photometric redshifts at higher redshift much more difficult. To represent the situation in a few years, we assume redshift bin widths of 0.5 at $z > 1.2$, keeping the error per bin the same; this is roughly equivalent to doubling the error per redshift interval found at $z < 1.2$. In general, modeling the detailed accuracy at high redshifts is not crucial as long as *some* redshift information is available there, and we further quantitatively discuss this in § 4.2.

4.2. *Results*

The *purely statistical* error in photometric redshifts, $\sigma_{\rm clus}$, is largely irrelevant for cosmological constraints, as can be seen from equation (6), since the large number of clusters (in all redshift bins except for those at the highest redshifts) makes the error per bin very small. Therefore, the current statistical error with scatter of about $\sigma_{\rm clus} = 0.02$ in redshift contributes negligibly, by itself, to the total error budget, and we assume this statistical error for each individual cluster. However, we find that the results are sensitive to *uncorrelated irreducible systematic* errors in redshift bins, $\sigma_{\rm sys}$. The measurement of photometric redshift primarily relies on the position of the 4000 Å break. For each redshift there is a corresponding filter (or overlapping filter pair) that is sensitive at $4000(1+z)$ Å. Galaxies at similar redshift and whose redshift determinations rely primarily on the same filters are thus susceptible to common systematic errors. These errors can arise because of improper modeling of the filters, photometric calibration uncertainty, or statistical errors in the redshift calibration process.

Figure 4 shows the degradation in the accuracy in w as a function of the irreducible error. We show the degradation in w as a representative example and have checked that degradations in Ω_m and σ_8 are comparable and that their range of sensitivities is spanned by the various curves shown in this figure. We see that the X-ray survey is most sensitive to this redshift error; this is not surprising as the X-ray survey runs out of clusters at $z \gtrsim 1.0$ (most of them are actually at $z \lesssim 0.6$) and hence uses a shorter lever arm in measuring the redshift than SZ surveys. The differences in various curves show the

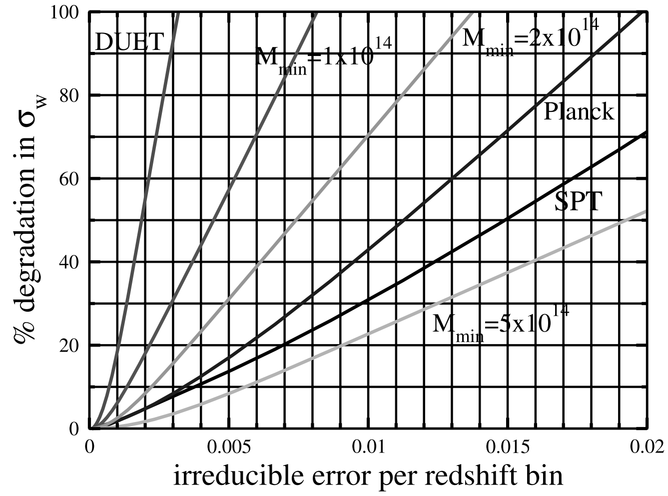

Fig. 4.—Degradation in the measurement accuracy of the equation of state as a function of the *irreducible* redshift error per bin. The SPT and *Planck* curves use the mass–SZ flux relation to compute $M_{\rm min}$, while the *DUET* curve uses the mass–X-ray flux relation. The three other curves show the SPT survey with fixed $M_{\rm min}$ in units of $h^{-1}\ M_\odot$.

dependence of the sensitivity of σ_w on details of the survey and, in particular, on the definition of the minimal cluster mass. The irreducible error per bin has to be kept below 0.001–0.005, depending on the survey details, for it not to contribute more than ~10% to the error in w and other cosmological parameters. Furthermore, we have explored a range of fiducial surveys, varying the parameter set, the sky coverage, and the details of the mass-observable relation, and have found that surveys with less power to measure cosmological parameters typically have weaker requirements on the redshift accuracy. While we have shown a range of possibilities in Figure 4, we note that the exact requirements on the redshift accuracy for any given survey will be known only after the survey in question has started its operation and the accuracy of cluster mass determination from the observed flux or temperature becomes known.

We mentioned earlier that the future accuracy of photometric redshifts at $z \gtrsim 1$ is uncertain. We explore the accuracy in measuring w on redshift information at $z > z_*$, where we let z_* vary between 0.8 and 3.0. Figure 5 shows that not obtaining redshifts for clusters at such redshifts can significantly degrade the performance of SZ surveys (X-ray surveys do not have this problem, since they have very few clusters at $z \gtrsim 1$). Redshift information is assumed to be perfect for $z < z_*$, while for $z > z_*$ we alternatively assume no redshift information (*solid lines*), irreducible errors of 0.05 per redshift bin width of $\Delta z = 0.1$ (*dashed lines*), and 0.02 per redshift bin (*dotted lines*). We show cases for the flux-limited SPT and for the same experiment with fixed $M_{\rm min}$. This figure shows that, while missing information at $z \gtrsim 2$ can be tolerated, having some redshift information in the region $1 \lesssim z \lesssim 2$ is very important. While photometric redshifts in this intermediate interval are difficult to obtain because of the lack of prominent features, we see that relatively modest information (redshift bias accurate to about 0.05 *per bin*) is sufficient to recover most of the information obtainable with perfect redshift accuracy. For example, the Dark Energy Camera[11] and VISTA[12] surveys are expected to extend the photometric redshift range, with roughly uniform accuracy, out to $z = 4$. We have checked that using this stronger

[11] See http://home.fnal.gov/~annis/astrophys/deCam.
[12] See http://www.vista.ac.uk.

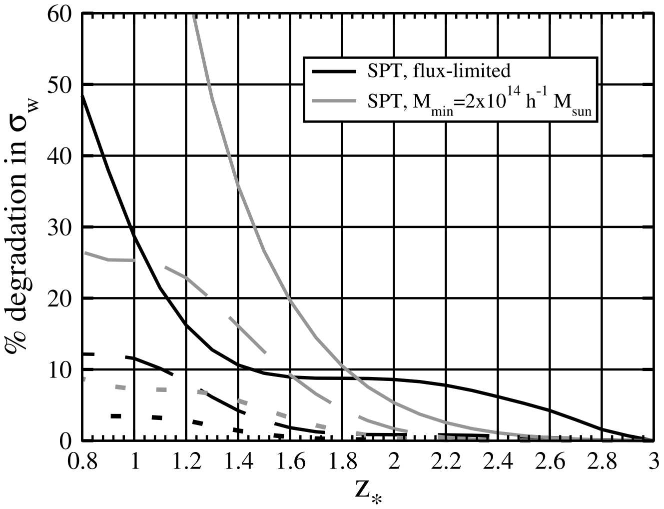

FIG. 5.—Degradation in the measurement accuracy of the equation of state as a function of the redshift z_* beyond which photometric redshift information is poor or nonexistent. Redshift information is assumed to be perfect at $z < z_*$, while at $z > z_*$ we alternatively assume no redshift information (*solid lines*), irreducible errors of 0.05 per redshift bin width of $\Delta z = 0.1$ (*dashed lines*), and 0.02 per redshift bin (*dotted lines*). We show the cases for the flux-limited SPT and for the SPT with fixed M_{min}. Note that an X-ray survey would show no degradation in σ_w at all, since essentially all its clusters are at redshifts $\lesssim$1.

assumption (instead of assuming degraded redshifts at $z > 1.2$) changes our results on the required redshift accuracy in Figure 4 only slightly, by less than 20% even for surveys extending to high redshift, such as the SPT. Finally, we also checked that the error degradations in Ω_m and σ_8 are very similar to that in w.

Note that our cluster count requirements were based on the degradation in the accuracy of measuring the equation of state w, which, in our fiducial surveys, is measured to accuracy $\sigma_w =$ 0.02–0.12. Weaker constraints on w, due to the inclusion of systematic errors, new parameters (such as the mass-observable normalization and slope), or else smaller sky coverage of the survey, will lead to *weaker* redshift requirements. Nevertheless, precision measurement of dark energy parameters is one of the principal goals of future number count surveys, and it is expected that they will be powerful enough to complement concurrent SN surveys. Therefore, redshift control less stringent than that advocated here would weaken the power of number count surveys to probe dark energy.

5. CONCLUSIONS

We considered how inexact redshifts affect future SN Ia and number count surveys. We treated the redshifts as additional parameters to which we assigned priors equal to their assumed measurement accuracy. Requiring that the redshift uncertainty not contribute more than ~10% to the error budget in cosmological parameters, we imposed requirements on the redshift accuracy.

For a future survey that studies ~3000 SNe out to $z = 1.7$ (e.g., the *SNAP* space telescope), we find that, with accurate redshift measurements of $dz \simeq 0.0015$ for $z < 0.1$ SNe, fairly poor redshift measurements can be tolerated at higher redshifts. Without this accurate measurement at low redshift, however, a fairly precise redshift measurement of $dz \lesssim 0.002$ would be required over the full redshift range.

Photometric redshifts are probably not an option, since spectral information is necessary to identify the SN type and control a variety of systematic errors. Spectroscopy can be provided using subpixel interpolation of galaxy data from an onboard low-dispersion $R \sim 100$ spectrograph (which is designed to measure broad SN features). Supplemental high-resolution ground-based observations using 10 m class telescopes, adaptive optics, and OH suppression can provide precise redshifts as necessary and cross-check the redshifts from the low-dispersion spectrograph. We thus conclude that redshift uncertainty will not significantly contribute to the error budget in the accurate measurement of dark energy parameters that *SNAP* can deliver.

For future wide-field cluster count surveys, such as SPT, *Planck*, or *DUET*, we find that the purely statistical errors are largely irrelevant as long as they are reasonably small (error of $\lesssim$0.02 per cluster), since they will average out because of the large number of clusters around any given redshift. However, the irreducible systematic error that does not decrease with increasing number of clusters drives the redshift requirements. This irreducible redshift-independent error has to be kept below 0.001–0.005 per redshift bin. The widths of the redshift bins are determined by how the redshift signature (i.e., the 4000 Å break line) goes through the filter set of the redshift follow-up experiment, and here for illustration we assumed filters from the SDSS. We found that the typical required redshift accuracy is more stringent for X-ray surveys since they have few clusters at $z \gtrsim 1$ and therefore use a shorter lever arm in measuring the redshift. SZ surveys benefit from their longer lever arm but, of course, only if their high-redshift clusters have decent redshift information. Obtaining redshifts for high-redshift clusters, therefore, should be an important goal of any redshift follow-up survey. While the photometric accuracy at redshifts greater than unity is highly uncertain at present, our analysis indicates that the lack of redshift information at $z \gtrsim 2$ does not significantly degrade the cosmological constraints, while at redshifts $1 \lesssim z \lesssim 2$ crude photometric information is sufficient to assure a small degradation in constraints on w (see Fig. 5). With the current rate of progress in photometric redshift techniques, this should be a feasible goal within the next few years.

We thank Jim Bartlett, Josh Frieman, Adrian Lee, and Tim McKay for useful discussions. We particularly thank Eric Linder for pointing out the importance of accurate redshifts for low-redshift SNe and Jim Annis and Martin White for comments on an early draft of the paper. D. H. and L. M. K. are supported by the Department of Energy grant to Case Western Reserve University. A. K. was supported by the Director, Office of Science, of the DoE under contract DE-AC03-76SF00098.

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