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Role of Students' Representations in the Mathematics Classroom

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Abstract: Learning science research has examined how external representations can be used to foster student reasoning and understanding. This paper describes additional roles that students' representations play in a reform mathematics classroom. The paper describes three ways in which students' representations were used in a classroom: (a) to provide information on how an individual student thinks about a mathematical issue, (b) to provide information on patterns and trends in knowledge across students, and (c) to serve as an classroom tool for the students and teacher.

Keywords: Curriculum, mathematics education, teacher learning

Purpose

Research in the area of external representations has focused on how representations benefit student learning (Kaput 1987; Resnick & Omanson, 1987). We draw on this literature and expand it by examining other roles students' representations could play in classroom learning. The following perspectives in current research guide our work. Firstly, constructivist teaching and learning emphasizes teachers and students working together to create understanding (Brown, Collins, & Duguid, 1989; Cobb, 1994). This places an additional burden on teachers in terms of their instruction. In addition, math reform emphasizes that teachers pay attention to student thinking (Ball, 1997; Fennema, Franke, Carpenter & Carey, 1993). Research on student learning has shown how representations can be thought of as externalizations of students' mental models that can be manipulated, and used as shared information. Given the above, this work explores how students' representations can be used to provide a bridge towards a student-centered classroom in two ways: (a) by helping teachers understand students' ideas and (b) by allowing the representations to be used as objects of discussion by the class.

Context and Methodology

This research was conducted within the context of a reform mathematics curriculum called the Childrens Math Worlds (CMW) (Fuson et al, in press). Three features of the curriculum make it a valuable context within which to examine the issues raised above. 1) Students' representations (anything that a student creates to externalize and show his or her work) are a major focus of the curriculum structure. 2) CMW emphasizes the use of the Solve-and-explain process, in which two to three students work at the board drawing and explaining their solution strategies. 3) Students show their work in daily journals. This research examines how curricular supports and activity structures support learning and teaching.

The data for this study comes from the first three months of school and is based on four teachers who are in their first year of implementing the curriculum. Two of the teachers teach grade 2 and two of the teachers teach grade 3. On average, each of the teachers was observed once a week, and most observations were videotaped. Field notes were also taken at each observation. Following the observations, the teachers were interviewed. These interviews were audiotaped and later transcribed. Sizeable portions of the classroom observations were transcribed as well.

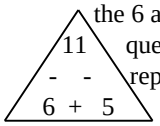
Results and Discussion

We found three ways in which student representations were used in this context. 1) *Representations were used to provide information to the teacher and the class on how an individual student thinks about a mathematical issue or context.* For example, in a class on subtraction, the teacher called 3 students to the board to show their work. Two of the students had a representation of the following type

11 – 7 = 4 but one of the students had the following representation 11 – 7 = 4

7 o o o o o o o o o o o o

In the first representation, the student started with the lower number 7, drew circles until he reached 11, and then counted the number of circles to get the answer. In the 2nd representation, the student drew 11 circles, crossed 7 out and then counted the remaining circles. In the post-observation interview the teacher explained how the third student's representation gave her an opportunity to 'know how he thought' as she put it. She also mentioned that she chose that student's work, as it gave her an opportunity to show the class how this could be done in a different way. 2) *Representations were used to provide information on patterns and trends across students*. For example in a lesson where students had to make shapes (circles, squares, triangles) using ten counters to recognize breaking numbers by place value, the teacher mentioned at the interview that as she walked around looking at the arrangements she was surprised at how few of the students knew some of the shapes. Thus she was able to see a pattern among the students in their lack of knowledge on some shapes. 3) *Representations were used by both the teacher and other students as a classroom tool.* For example in a lesson on seeing the relationship between addition and subtraction using a triangle representation, the students took over the discussion using a student's work at the board. The students used the representation to ask a lot of interesting questions where they explored the underlying concepts of addition and subtraction and how they fit together. Example questions "What if you made the 5 a 3 and the 6 an 8". "What if you switched the 5 and 6". The teacher explained that she was impressed at the questions that the students asked and at how she did not have to ask a single question. Thus the representation here was used by other students as an object around which discussion took place.



After we identified the categories, a couple of interesting results emerged. *A single instance of representation use could be used in multiple ways.* For example, a teacher could use a student's representation to learn about that student and also use that as an instructional tool to show a different solution method. *There is movement between the categories*. Thus, for example, a teacher may notice something about an individual student; then decide for example to see if the pattern persists across students. While identification of these categories provides input on how teachers and students can use representations in a classroom and is an important first step, the next step for this research will be to see how these categories would allow us to answer the broader research questions posed early on. Perhaps one way of answering these is to use the categories and see how movements among them allow learning and the establishment of classroom culture.

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