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On the description of shape and spatial configuration inside buildings: convex partitions and their local properties

J Peponis, J Wineman, M Rashid, S Hong Kim, S Bafna
College of Architecture, Georgia Institute of Technology, Atlanta, GA 30332-0155, USA;
e-mail: john.peponis@arch.gatech.edu; jean.wineman@arch.gatech.edu;
sonit.bafna@arch.gatech.edu; mrashid@arch.gatech.edu; CJC632@chollian.dacon.co.kr
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Abstract. As we move through buildings, we experience not only continuous changes of perspective but also discrete transitions from one space to another. To describe movement as a pattern of such transitions we need methods for partitioning space into relevant elementary units. Here we explore several convex partitions including one based on the thresholds at which edges, corners, and surfaces appear into the field of vision of a moving subject, or disappear outside it. Our purpose is to contribute to the development of quantitative descriptions of building shape and spatial configuration.

1 Movement as a pattern of transitions

Buildings make space available to our experience, useful for human activities, and intelligible to our understanding, through the disposition and arrangement of boundaries. Boundaries are used to create patterns of enclosure, contiguity, containment, subdivision, accessibility, and visibility. Because interior space is configured according to the shape of the perimeter and subdivided according to the disposition of partitions, we cannot experience buildings in their entirety from any one of their points, except in the most simple cases. Consequently, movement is fundamental to our intuition of spatial patterns inside buildings.

The intimate relationship between movement and our understanding and experience of space is by no means a new idea. Frankl (1914) recognized movement as a precondition for retrieving a single mental description of form, the synthesis of the partial images that are collected from different observation points within a building. Cassirer (1955) has argued that our intuitions of form and movement are linked because movement can be interpreted as potential form and form can be interpreted as a structure of potential movement. In different ways Poincaré (1913) and Piaget and Inhelder (1967) have suggested that movement is the operational foundation that allows us to relate different views of complex spatial patterns to one another and to retrieve descriptions of spatial relationships by linking direct experience to abstract reason. The relationship between movement and visual perception is central to the work of Gibson (1979). He has argued that the observation of changes regarding the visible and the hidden, and more specifically the observation of the relationship between visibly extended, receding, and occluding edges, allows us to retrieve information on the three-dimensional (3-D) structure of environment. More recently, the relationship between the configuration of space and movement has been at the core of 'space syntax', a method for describing the relational structure of built space in conjunction with the development of theories regarding the generic social function and cultural meanings associated with buildings (Hillier and Hanson, 1984; Hillier, 1996). In this paper we introduce a description of the shape and spatial configuration of building plans to address the changes of spatial relationships that arise as we move and position ourselves inside buildings. The analysis is limited to two-dimensional (2-D) planar relationships even though the ideas presented

all possible points. One implication of this is that isovist analysis, although easily applicable if we have reason to select some particular viewing points, cannot readily be automated or proceduralized to deal with an entire plan. It is always necessary to devise ways for sampling the set of points from which we will draw the isovist, very much as we sample points when we develop contour site maps. Benedikt has noted that in general a small number of isovists are sufficient to collectively cover all surfaces, edges, and corners of a building. This follows from a mathematical theorem, known as the 'art gallery theorem', which demonstrates that the sufficient number of points needed to cover the entire surface of a plan is a function of the number of reflex angles (O'Rourke, 1994, pages 3 – 10). The identification of such strategic sets of points, and of potential paths that connect them, can be of great value, as suggested by Benedikt. It would still cover only one aspect of the experience of moving around buildings. Benedikt has also proposed that we can draw contours corresponding to points on a plan from which a certain property of isovists, say area, remains constant. Drawing such contour maps, however, can be a very laborious procedure depending on the level of detail that we wish to achieve and the sampling procedure that we follow.

It will be seen that isovists drawn from within the same e-space will encompass the same discontinuities and the same surfaces. The difference between one isovist and another, in this case, will be the progressive emergence or recession of certain surfaces behind certain edges, which is of course one of the 'constants' that describe the invariant structure of environment according to Gibson (1979). As we cross the boundary between e-spaces, the isovist will change more radically. In other words, from the point of view of information about an environment, we may consider that isovists can usefully be grouped according to sets which correspond to the points within e-spaces.

By providing us with the informationally stable convex constituents of a plan, the e-partition helps us to define movement as a finite pattern of discrete transitions rather than as an infinitely variable pattern of perspective views. The end-point partition is fundamental in several respects. It is mathematically well defined, finite, and specified uniquely for a given shape. It seems to capture the objective structure of environment that gives rise to the awareness of receding and occluding edges that is fundamental to Gibson's ecological theory of perception. Once we have identified elementary units we can seek to develop theories about how such units, and the information that attaches to them, can be coordinated into the more complex patterns of order and networks of accessibility that concern Piaget. And we can apply the measures associated with space syntax not only to these elementary spatial units but to more complicated entities that can be derived from them. This last possibility will be discussed in greater detail in the next section, bringing the idea of visual fields more firmly into the purview of the analysis.

3 The analysis of relationships of accessibility and visibility in convex partitions

Two convex spaces are adjacent if they have a face-to-face joint and thereby share at least part of an edge. They are connected to one another if their shared edge is not entirely covered by a wall, so that we can move from one to the other. Given a convex partition, we can easily establish the matrix of connections between spaces. The connectivity of each space is defined as a simple measure of the number of neighboring spaces to which it is connected. Connectivity, however, is a 'local' measure that does not describe how each space is related to the rest of the system.

Space syntax (Hillier and Hanson, 1984; Hillier, 1996) suggests that, from the point of view of the social use and cultural meanings of layouts, the relation of each space to the rest of the system is of far greater significance than its connectivity. Hillier and Hanson have proposed that the property of 'integration' describes the way in which the

parts of a system are linked into a whole. According to them, a space is said to be more integrated when all the other spaces can be reached after traversing a small number of intervening spaces; it is less integrated when the necessary number of intermediate spaces increases. From a computational point of view, the basis for computing the integration of a space is the formula $(k-2)/(2d^{\text{mean}}-1)$, where k is the number of elements in a system, and d^{mean} (mean depth) is the average minimum number of transitions, from one space to another, that must be made to reach every other part of the system. The value obtained by this formula is multiplied by the expression

$$\frac{6.644k\lg(k+2)-5.17k+2}{k^2-3k+2},$$

in order to facilitate the comparison of systems of different size, in light of the research findings regarding the behavior of the measure when applied to large samples of data.

For the greater part, research using space syntax has emphasized the analysis of plans in terms of their linear components. However there is a noteworthy body of analysis applying the convex partition of plans proposed by Hillier and Hanson (1984). This suggests that integration is related to the way in which layouts are used by their occupant organizations and also to the way they structure, instrumentally or symbolically, the relationships between their inhabitants (Hanson, 1994; Hillier and Penn, 1991; Hillier et al, 1987; Orhun et al, 1995; Peponis and Hedin, 1982; Peponis, 1985; 1993).

However, here we wish to treat integration as a purely formal measure and to discuss how the partitions proposed in this paper may allow us to describe subtler properties of shape and spatial configuration inside buildings. From an intuitive point of view, it would seem that our methods could be applied to studies of how environments become intelligible.

We will proceed through the discussion of an example. Figure 12 (see over) shows a simplified plan of Farnsworth House, designed by Mies Van der Rohe, and its convex partitions. Here we treat the plan as an elementary shape. We are not seeking to analyze the main architectural qualities of the building. For example, we will not be discussing the visual relationships to the external environment, or the placement of the building on its site, both of which are essential to its architectural quality.

The minimum and surface partitions of the interior of this built shape coincide. The structure of connections between spaces is essentially a continuous single ring and this is precisely why we have chosen the example. As a result, the integration values of all s-spaces are the same, so long as we do not take into account the effects of the entrance. As a purely interior space, therefore, this appears syntactically undifferentiated from the point of view of the minimum and surface partitions. We proceed to examine the e-partition and observe that the integration values of e-spaces fall into two groups. All main spaces along the perimeter have one value. The four spaces associated with internal convex recesses have a lower value. This elementary analytical discrimination still does not seem to reveal much about the properties of the shape.

However, the connectivity of e-spaces can be usefully expanded to take into account connections beyond the set of adjacent spaces. The way to do this is to consider relations of visibility that go beyond direct connections to adjacent spaces. This brings us back to Benedikt's isovists. Linking the analysis of isovists to the analysis of convex partitions is not a new idea. For example, Hillier (1993) and Hanson (1994) both adapt Benedikt's isovists so that they correspond to convex spaces rather than points. Intuitively, these isovists are intended to cover the areas visible from any of the points, either of the entire convex space under consideration or from one of its parts. However, unless the isovist that corresponds to a convex space is reduced to the isovist drawn from some specific point, like the center of gravity, or to the union of the isovists

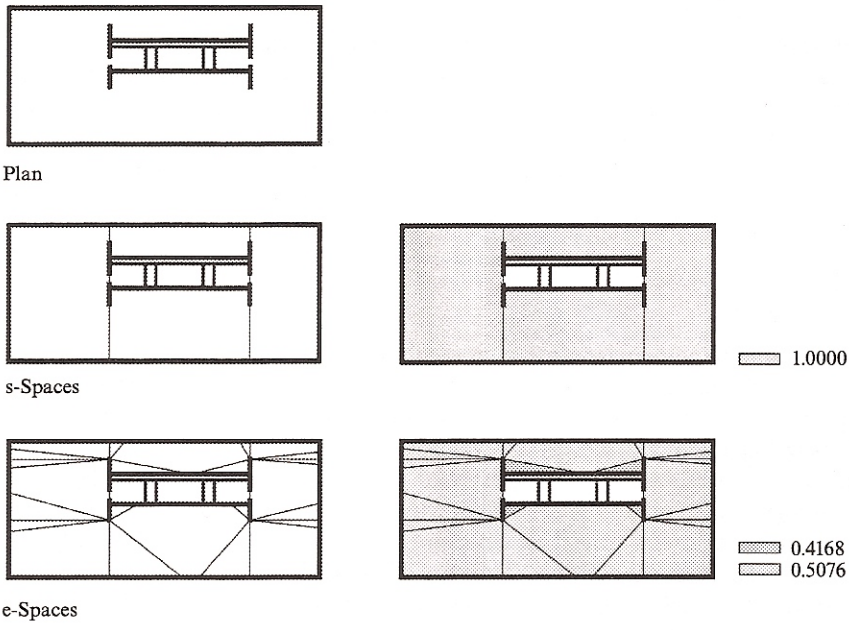


Figure 12. Plan and convex partitions of Farnsworth House, with integration values indicated.

drawn from a small set of points (for example, the middle of thresholds to adjoining spaces), the derivation of the isovist cannot easily be automated.

We believe that there is a way to analyze visual relationships between different areas systematically and automatically, across an entire plan. The key idea is to take pairs of convex spaces and to ask whether they can be 'rubber-banded' together, in order to create a bigger convex space not containing and not crossing a wall surface. In mathematical terms, rubber-banding is equivalent to determining the convex hull with the minimum perimeter that contains both spaces. If the convex spaces we start with have been produced by a minimum partition, the answer to this question will always be negative. If we deal with s-spaces or e-spaces, the answer can be interestingly positive.We can then take each space as a starting point and identify all other spaces that are convex to it, thus defining what we propose to call its 'convex span'. The convex span of a space contains all other spaces every point of which is fully visible from every point of the original space. Thus, convex spans provide us with an alternative way of describing visual fields.

We propose to use the term 'expanded connectivity' to describe the relationship of a space not only to its immediate accessible neighbors but to all members of its convex span. To illustrate these ideas, we present in figure 13 the convex span of a chosen e-space in Farnsworth House.

Given the expanded connectivity matrix that is based on the convex span of e-spaces, we can compute new integration values. The integration of an e-space now reflects how many convex spans other than its own must be traversed before all space becomes

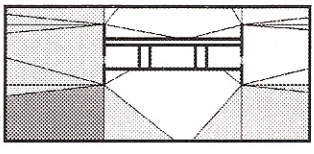


Figure 13. The convex span of an e-space in Farnsworth House.

visible. Figure 14 shows the results of this computation. It will be noticed that the spatial structure of Farnsworth House now appears highly differentiated. The pattern of differentiation matches our intuitive understanding of the plan. Spaces at the corners of the house appear more integrated than spaces in the middle because they enjoy expanded connections in two directions. Spaces associated with the convex alcoves are less integrated than other spaces contained between the inner and the outer surfaces. This example illustrates the potential advantages of using the more refined partitions, such as the end-point partition, to enrich the analysis of plans.

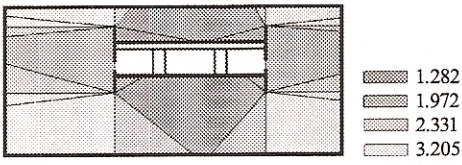


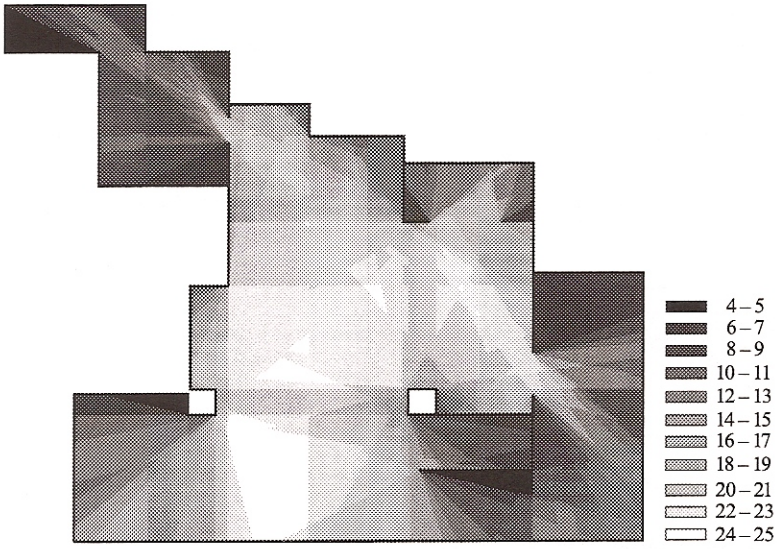
Figure 14. The integration pattern of Farnsworth House based on expanded connectivity.

4 Local properties of the spatial exposure of shape

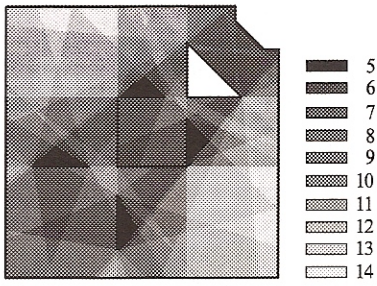
The partitions introduced here can provide us with a foundation for the analysis of our exposure to shape as we move inside buildings. We will proceed to illustrate a small number of properties among the range that are currently being investigated. Our aim is to show that the analysis of information available locally can clarify interesting aspects of shape and spatial configuration. A future paper may address the way in which we can develop more global descriptors and build up ways to recognize more encompassing shapes and subshapes inside buildings. To present our ideas we will use simple examples. It must be noted that we deal with interior spaces only, excluding the relation to the exterior, however important it might be architecturally; furthermore we treat the simplified plans as mere examples of built shape, not taking into account information about space use, architectural intentions or other matters.

The convex spaces produced by the end-point partition of a plan are linked visually to different sets of boundary discontinuities. The number of such discontinuities is an index of the information about the shape that is available locally and can easily be expressed as a ratio to the total number of discontinuities that are needed to define the shape. In figure 15 (see over) we present the simplified plans of Fallingwater by Lloyd Wright and the house at Riva San Vitale by Botta. These plans are then shaded according to the proportion of the shape discontinuities that are visible from their e-spaces. The basic numerical information concerning the visibility of discontinuities from e-spaces is summarized in table 1 (see over).

A visual inspection of the plans suggests that, in the case of the house at Riva San Vitale, more information about the overall shape becomes available from some of the e-spaces which are attached to the periphery of the plan. In the case of Fallingwater, information is more often maximized in e-spaces that are positioned centrally between the boundaries. The periphery, taken as a whole, provides less exposure to shape information. The contrast between the centrifugal and centripetal exposure of shape to view can easily be interpreted. In one case, the convex articulation of space arises from positioning one subshape inside the other. The center of the plan is occupied by walls. In the other case, articulation arises from the deformations and transformations of an external boundary. We can, however, understand the phenomenon better by taking a closer look at the behavior of the end-point partition of simpler theoretical plans.



Fallingwater



House at Riva San Vitale

Figure 15. Simplified house plans shaded according to the visiblity of discontinuities from e-spaces.

Table 1. The visibility of discontinuities from e-spaces in two house plans.

	Number of discontinuities	Visible discontinuities		
		mean	maximum	minimum
Fallingwater	45	0.332	0.55	0.09
House at Riva San Vitale	22	0.397	0.64	0.18

Figure 16 shows three pairs of plans. Each pair is generated by manipulating the same underlying shape. Therefore the members of each pair have the same number of discontinuities and generate the same number of e-lines. Their difference resides in the number of intersections that are produced by the e-lines and the corresponding number of e-spaces produced by the end-point partition. These seem to depend on the proportions of the plans and more specifically on the proportions of solid walls to their extensions. The numerical information concerning the visibility of discontinuities from e-spaces is provided in table 2. It will be noted that, in the case of a plan which is subdivided by virtue of placing a square inside a square, the e-spaces that get added at the periphery provide more information about the shape than any other space. By contrast, in the case of the plans produced by distorting an outer boundary, the e-spaces

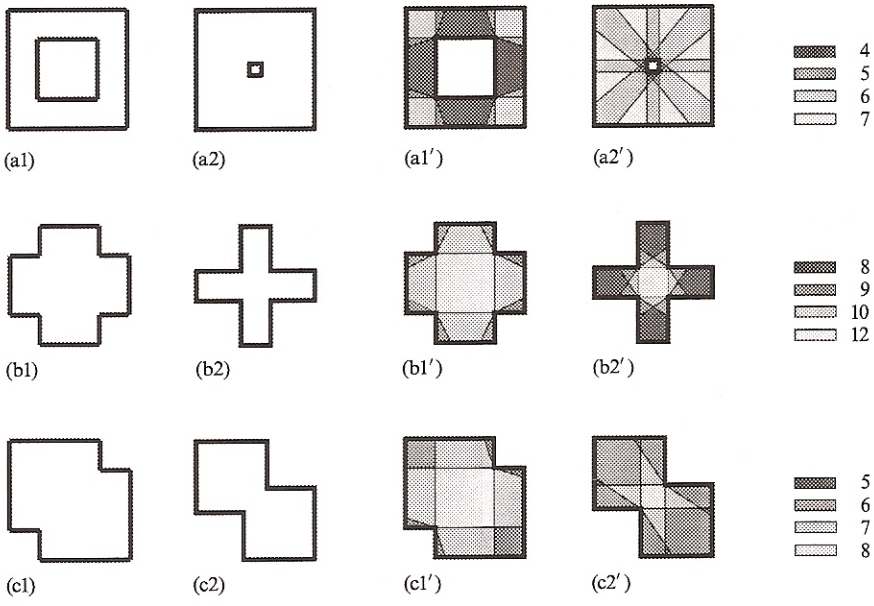


Figure 16. Three pairs of theoretical plans and their e-spaces shaded according to the number of visible discontinuities.

that get added at the periphery provide less information than any other space. It would seem that by studying elementary properties of the end-point partition we can potentially understand not only some critical differences between real buildings, but also the principles that govern the distribution of shape information as a function of elementary generators of shape.

As discussed thus far, the significance of the end-point partition resides in the creation of discrete informationally stable units of space, which punctuate our experience of movement. Because the end-point partition is so sensitive to all changes of our exposure to shape, it can coincide with the minimum and surface partitions only in the trivial case of a single closed convex polygon. Where we have at least two convex cells linked by a door, or at least one reflex angle in a polygon, the end-point partition differs from both the minimum and the surface partitions.

In comparison with the end-point partition, which provides us with informationally stable units, the surface partition defines spatial units as a function of the alignment and orientation of walls. In cellular plans, the surface partition coincides with the minimum partition; in open plans, it diverges from the minimum partition and captures the ambiguities and latent demarcations that govern the potential subdivision of space.

Table 2. The visibility of discontinuities from e-spaces in six theoretical house plans.

Plan	Number of		Visible discontinuities		
	discontinuities	e-spaces	mean	maximum	minimum
a1′	8	16	0.625	0.750	0.500
a2′	8	28	0.714	0.875	0.500
b1′	12	13	0.795	1.000	0.750
b2′	12	17	0.765	1.000	0.667
c1′	8	11	0.818	1.000	0.500
c2′	8	15	0.767	1.000	0.417

We propose to pursue this idea further through a comparison of Fallingwater, the house at Riva San Vitale, the design for a brick house by Mies Van der Rohe, and the design of the Villa Valmarana by Palladio. The four plans and their surface partitions are shown in figure 17.

The Villa Valmarana exemplifies a regular cellular plan, where the surface partition coincides with the minimum partition. The clear definition of rooms, however, is also based on another property, so obvious that it may not be noticed at first. All the corners that define individual s-spaces are physically manifest as wall intersections. By contrast, in all the other plans, the average proportion of physically manifest corners ranges from

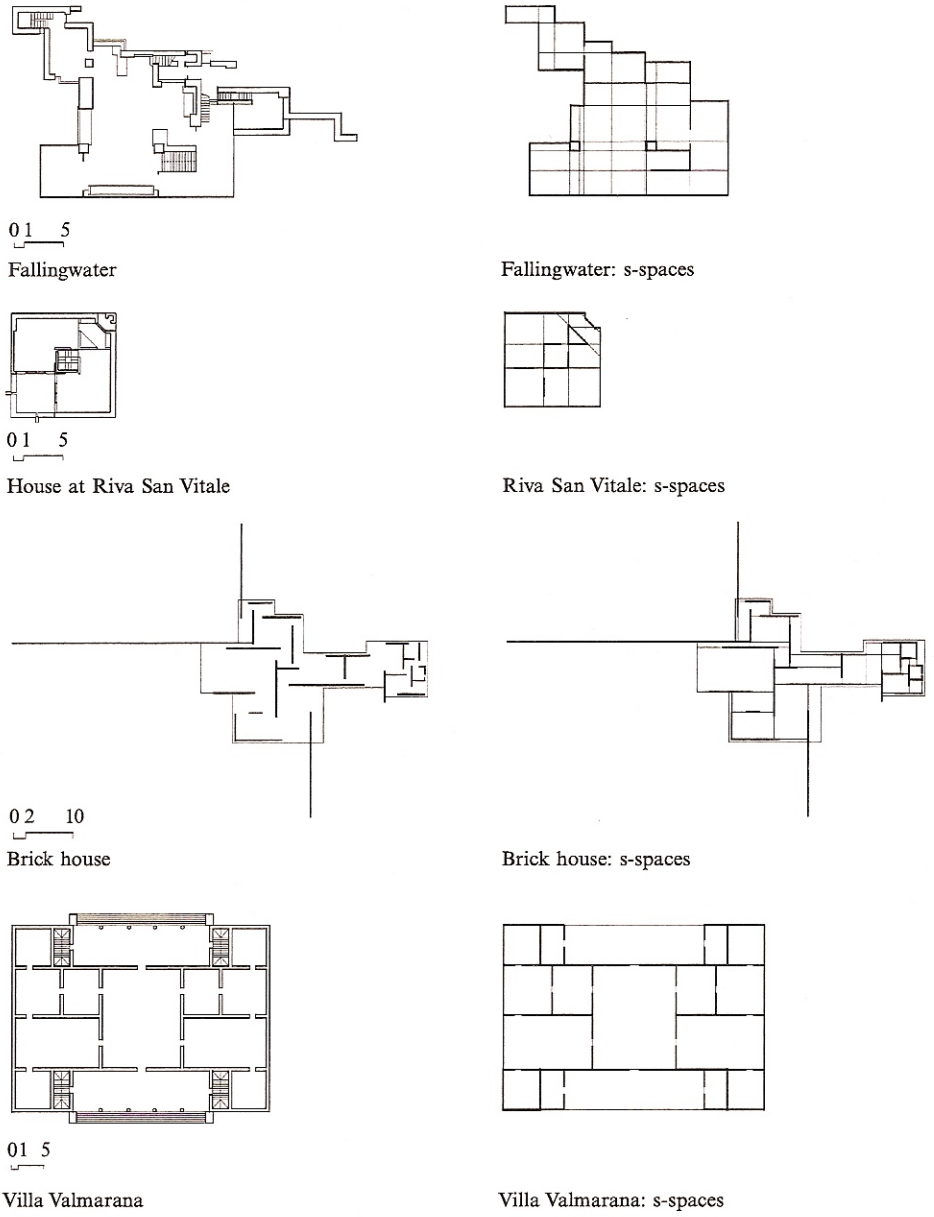


Figure 17. Four house plans and their s-spaces.

about 1/3 to about 1/2. The lack of physical definition of the corners of s-spaces may thus be treated as a distinguishing characteristic of various types of open plan.

There is, however, another property of the surface partition which, although intuitively obvious, is not as easy to quantify. In the brick house, for example, the s-spaces seem to be unambiguously delineated, whereas in the case of Fallingwater the surface partition creates spaces that may not otherwise have been recognized as discrete spatial units. We propose that this property can be formulated more rigorously if we ask a simple question, namely, whether it is possible to expand an s-space, without crossing a physical boundary, while preserving geometrical similarity. Furthermore we choose to deal with parallel expansions which occur while one of the vertices of the s-spaces is kept in its place, as shown in figure 18. We then measure the number of such expansions that are possible (holding one vertex stationary at a time) and express this as a proportion of the total number of vertices. We call this measure the 'expansion coefficient' of s-spaces. When the expansion coefficient score is low, the s-space has few possibilities for expansion. When the coefficient is high, the s-space can be expanded in many alternative ways. The more expandable an s-space is, the more ambiguous it will appear. The less expandable it is, the more it will seem well defined.

This measure reveals interesting differences between the open plans that are being compared, as shown in figure 19 (see over). In the brick house, practically all the s-spaces are defined perfectly. The average expansion coefficient is as low as 0.021, even though fewer than half the corners that define s-spaces are physically manifest. It would appear that the mode of spatial subdivision explored by Mies Van der Rohe creates a tension between two different and independent properties. On the one hand, walls extend beyond the limits of s-spaces, and physical corners are eliminated, so as to suggest continuity and flow across the shape. Although we are analyzing only the interior, it is clear that the same device produces the sense that the interior 'bleeds' to the outside. The vessels of such 'bleeding' are the extended boundaries themselves. On the other hand, interior space is compartmentalized in clearly defined constituent parts. Although they do not share the appearance of conventional rooms, these parts share with rooms

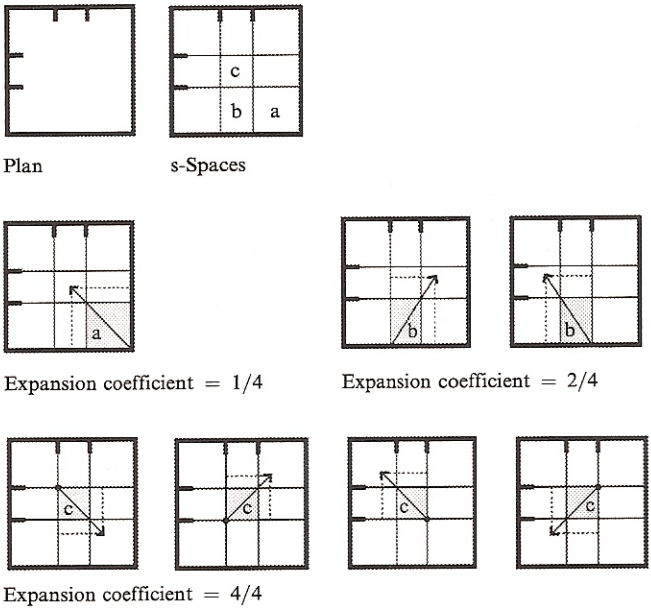


Figure 18. 'Fixed-vertex expansion' preserving similarity.

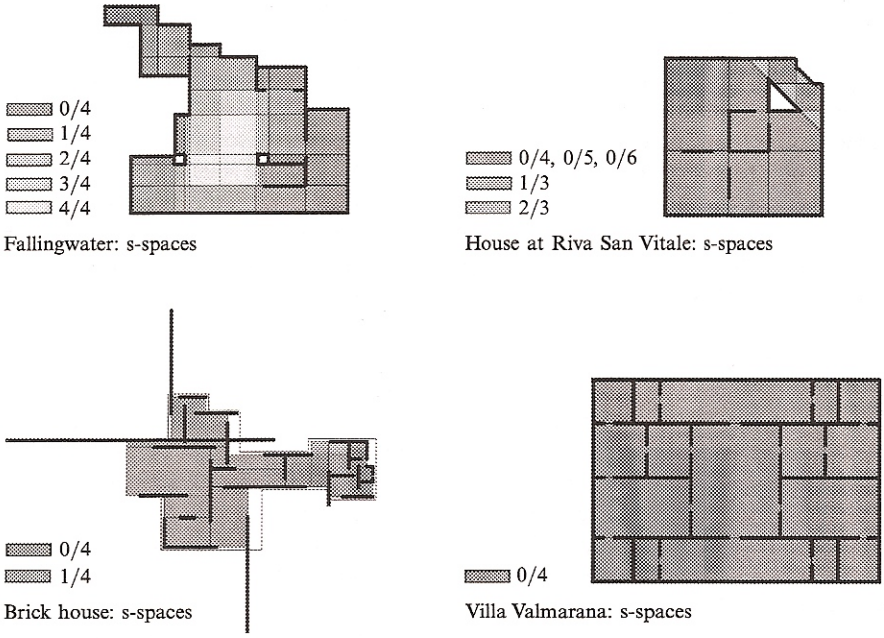


Figure 19. Expansion coefficients of s-spaces in four house plans.

the property of nonexpandability. By contrast, Fallingwater not only manifests fewer of the s-space corners physically, but also produces s-spaces with relatively high expansion coefficients. It would appear that in this case the free plan challenges the idea of discrete spatial units more fundamentally than in the case of the brick house. The house at Riva San Vitale produces relatively well-defined spaces in the greater part of its area, despite the uninterrupted potential movement around the core. Poorly defined s-spaces arise only near the fireplace, at the point where one of the corners of the outer square is truncated and articulated. These results are summarized in table 3.

The local properties of the surface partition are therefore as interesting as those of the end-point partition and seem to help us clarify different aspects of the interaction between the shape of the physical body of the building and the creation of a spatial structure for potential movement.

The last local property of convex partition that we want to introduce can best be understood by reconsidering the end-point partition. Given the number of discontinuities that are visible from an e-space, we may ask what is the minimum number of convex areas that can be defined so as to encompass all these points. Only in the case of a closed polygon would we get all the visible discontinuities to belong to a single convex area. In most cases the number of convex areas needed to cover all the discontinuities visible from an e-space will be greater, as shown in figure 20. We propose to define this number as the 'divergence' of an e-space.

Table 3. Properties of the surface partition of four houses.

	Number of		Proportion of manifest corners	Expansion coefficient
	discontinuities	s-spaces		
Fallingwater	45	46	0.333	0.380
House at San Vitale	22	16	0.486	0.210
Brick house	38	24	0.427	0.021
Valmarana	104	17	1.000	0.000

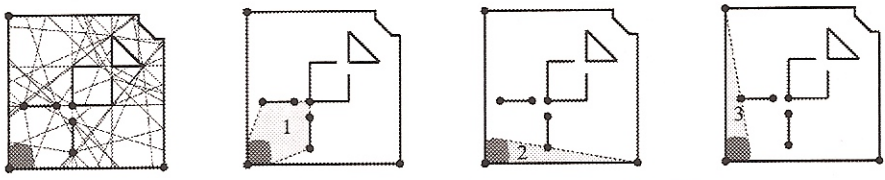


Figure 20. The divergence of an e-space in the house at Riva San Vitale.

In figure 21 (see over) we compare the divergence values of e-spaces in Fallingwater and the brick house. Numerical information is provided in table 4. In the case of Fallingwater, most e-spaces have a high divergence value. More particularly, the spaces in the middle of the living area have a divergence value of 6. This suggests that in Fallingwater the embracing quality of the wall boundaries, which are 'fine-tuned' to create alcoves for placing furniture, objects or seats, is combined with a property of openness rather than enclosure. The number of different convex areas that are implied by the visible discontinuities provides a sense of multiplicity rather than simple unity. On the contrary, in the case of the brick house, the e-spaces with higher divergence values are located mostly near thresholds and to some extent around the outer edges of spaces. The central space is characterized by e-spaces which have lower divergence values. Thus the brick house suggests a greater emphasis on finiteness, if not enclosure, despite the fact that boundaries are often seen to extend one's field of vision. In fact, it may be appropriate to say that, in the case of the brick house, boundaries, not spaces, enjoy continuity and flow.

This comparison concludes our presentation of some local properties of the convex partitions introduced in this paper. In each case we have used our comparisons to try to capture familiar properties that cannot always be defined clearly and rigorously: centrifugal and centripetal arrangements, well-defined or ambiguously defined sub-spaces, multiplicity versus discrete enclosure. At times we might have clarified a substantive aspect of the chosen examples, but mostly we have used the examples to introduce the theoretical and methodological ideas.

Table 4. Divergence values of two houses.

	Number of		Divergence		
	discontinuities	s-spaces	mean	maximum	minimum
Fallingwater	45	45	4.05	7	1
Brick house	38	24	3.84	5	1

5 Concluding comments

In this paper we have documented an approach to the refinement of the description of shape and spatial configuration inside buildings. We anticipate that these methodologies, as they are developed further, will contribute in several areas of inquiry, including the study of how buildings become intelligible; the characterization of different design styles; and the assessment of the effects of space upon aspects of building function such as display, interaction, and movement. These methods can assist in design education as a tool for the interactive analysis of plans and in the creation of libraries of comparative formal analysis. Admittedly we have not touched upon the more difficult problem: how to retrieve descriptions of more complicated subshapes that are implied by the overall building layout. This means that the relationship between the compositional principles used by architects and the potential spatial

here can potentially be extended to deal with three dimensions or be applied to an analysis of sections. Building plans that involve curves will not be discussed.

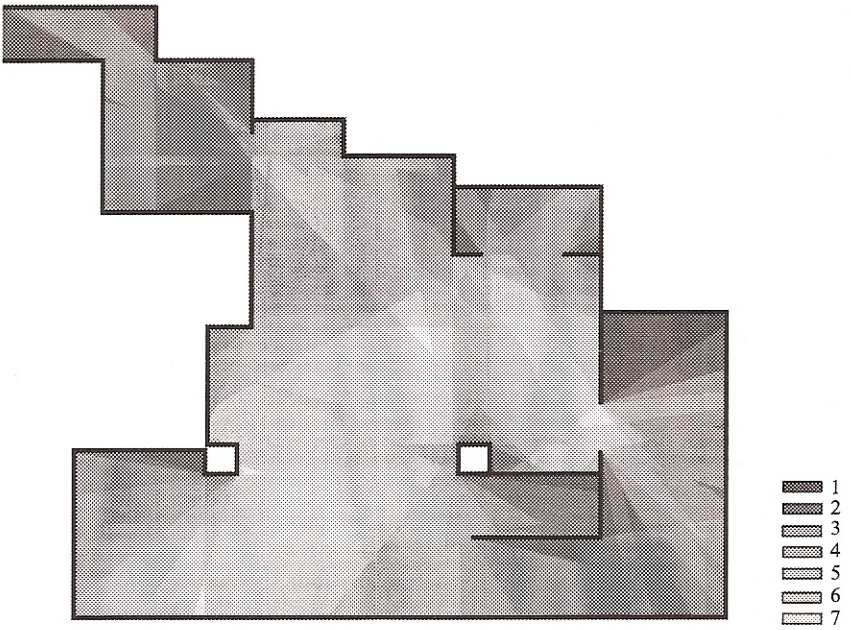
Movement is about changes of position, and positions can themselves be differentiated according to the views of the building that they offer. We can distinguish two kinds of differentiation. The first relates to our changing perspective upon a given set of elements of the environment and is continuous while the set of elements remains constant. The second relates to our exposure to different parts of the environment; it entails discrete transitions from one space to another or from a visual field comprising some set of environmental elements, such as surfaces, corners, and edges, to another visual field comprising a different set. Therefore a study of shape and spatial configuration from the point of view of movement must show how we can partition space so as to identify transitions from one spatial position to another. In this way we can account for the experience of potential movement in terms of the objective structure of environment. In this paper, transitions are ultimately defined according to the appearance or disappearance of corners, edges, or surfaces, as we move inside buildings. We identify units of space within which the visual information regarding corners, edges, and surfaces remains stable. This allows us to describe a given plan as a pattern of potential transitions from one informationally stable area to another.

We define the *shape* of a building plan as a set of wall surfaces and a set of *discontinuities*. We define discontinuities to include the edges of freestanding walls and the corners formed at the intersection of two wall surfaces. As wall surfaces extend between discontinuities, the crucial determinants of shape are the discontinuities themselves. In this paper we allow walls to be represented as lines without thickness, not only for simplicity but also to allow for the architectural intuition that there can be elements of building plans which are conceptually linear. At various stages as we move through a building, discontinuities and surfaces either appear into our field of vision or disappear outside it. If we assume a theoretical observer occupying a single point and possessing 360° of vision, we may say that at any moment in time the observer sees those discontinuities that can be linked to his or her position through uninterrupted straight lines, or lines that do not cross a physical boundary. Accordingly, a transition with respect to shape is a change of the set of such visible and accessible discontinuities. This is illustrated in figure 1. Our positing of a theoretical observer endowed with 360° of vision is a convenient way to take into account the fact that we experience space over time, looking in different directions, so that we are ultimately aware of our complete surroundings. Gibson (1979), for example, has argued extensively that vision should not be considered in terms of single visual frames, taken at a given moment in time, but rather in terms of this broader awareness which involves movements of the eyes, the head, and the body. As our aim is to describe the structure of environment, not the processes of perception and cognition, the postulate of a theoretical observer allows us to summarize the relevant properties of environment leaving open the question of how such properties are gradually recognized by actual subjects.

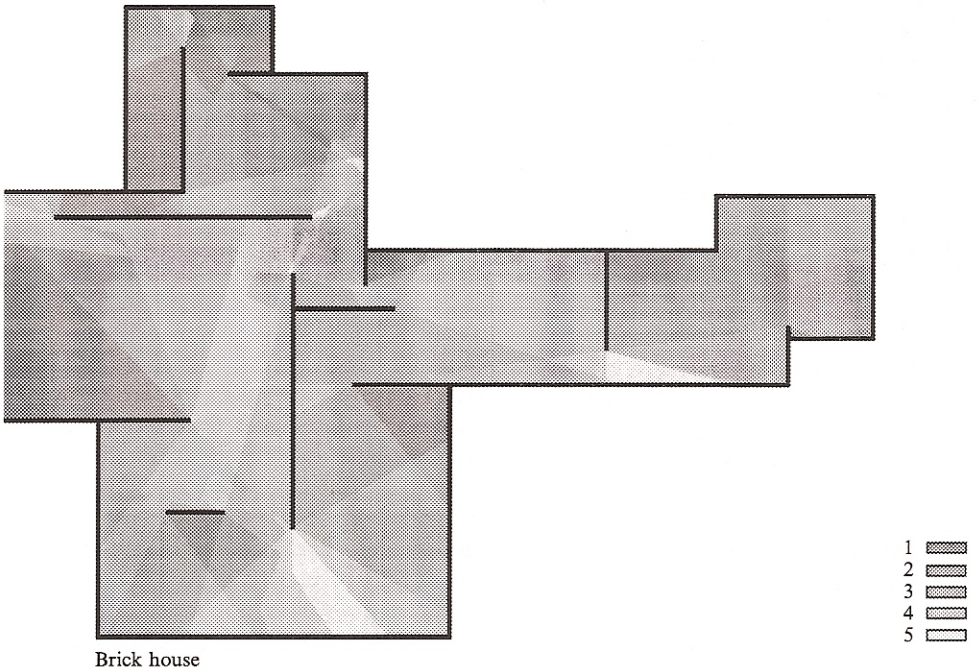
The differentiation of interior space, however, should not be considered only as a function of our exposure to different elements of built shape. We can look at space, and

Figure 1. Changing relationships of a moving subject to the discontinuities that define a shape.

experiences that are engendered by buildings has not been properly addressed. It is possible, however, that the partitions and properties introduced here may assist a future response to this larger task.



Fallingwater



Brick house

Figure 21. Distribution of divergence values in two houses.

The methods of analysis proposed here are very laborious because even simple plans produce complicated partitions and even simple properties require that many relationships be identified and counted. Any analysis of substantial bodies of data is therefore dependent upon the availability of computer programs for the automated analysis of layouts. Such programs are currently under development on a microstation platform, through our collaboration with IdeaGraphix, Atlanta, GA. As a consequence, in this paper we have discussed a very limited number of examples and a small set of variables of potential interest. The implementation of automated analysis will allow us to test whether the ideas introduced above reveal interesting spatial properties in larger samples of data, and whether these properties can be consistently linked to aspects of design style, building function, and building use[3].

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[3] The work reported here, and the development of related software, is made possible by an Academic Initiative Grant from the Georgia Tech Foundation, 1996 – 98.

transitions across spaces, from the point of view of the relationships between different potential occupants. We may initially conceive of undivided space as an infinite collection of points on the plane, representing possible positions that can be occupied by a subject. The continuity of empty space implies that any three subjects A, B, and C are directly visible and accessible from each other, so that we can link them by a straight line. With the addition of walls this is no longer the case for all positions. Belonging to the same space is now a special condition. We can appreciate this by imagining three theoretical subjects and by observing that their relationship can vary. In some situations, if subjects A and B see each other and subjects A and C also see each other, then we can infer that subjects B and C see each other as well. In such situations, all relationships of visibility are not only reciprocated but also commutative. This is what we normally mean when we say that 'we are together' in a space. In other situations, the opposite condition prevails and none of the subjects sees another. In still other cases, the relations which hold are not commutative: A and B, as well as B and C, see each other, but A cannot see C. These possibilities are shown in figure 2.

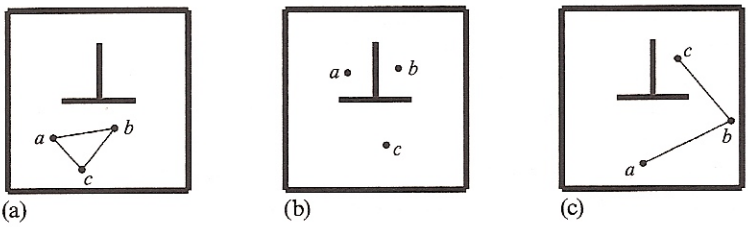


Figure 2. Convex and nonconvex relationships between positions on a plan.

From a mathematical point of view, we are discussing properties of convexity. According to the standard definition, a space is convex when any two of its points can be joined by a line that lies entirely within the space or, to state the same thing differently, a line that does not cross the boundary of the shape. Convexity is the underlying property that we recognize when we identify an area as an integral and discrete spatial unit. A set of positions on a plan are in convex relationship to each other if there is a convex polygon that contains all three in its interior. Two areas are in a nonconvex relationship to each other when there is no convex polygon that includes both in its interior. In figure 2(a) the three subjects are in a convex relationship, whereas in figure 2(b) they are in a nonconvex relationship. In figure 2(c), only two pairs of subjects are in a convex relationship. The consequence of boundaries is the creation of areas that are not convex to each other as well as the creation of relationships of adjacency and/or connection between convex areas. Two convex areas will be said to be connected when they are adjacent and when their shared edge or partly shared edge is not fully occupied by a wall. Thus, a spatial configuration can be described according to the pattern of convex spaces that it engenders and the connections between them. From this point of view, a transition can be defined as a movement from one convex area to another.

The description of building plans as patterns of convex spaces and their connections of permeability is one of the major methodological propositions of space syntax. Hillier and Hanson (1984) have argued that convex spaces correspond to our intuition of 2-D spatial units which are completely available to our direct experience from any of their points. To them, the convex representation of plans is a counterpoint to their representation according to the pattern of intersection of the longest lines that can be drawn without meeting a wall. Such lines, called 'axial lines', cut across several convex spaces and correspond more closely to our intuition of space as a field of movement.

This is because movement presupposes a sense of potential destinations, either final or intermediate, that may be only partly visible, at a distance. The major thrust of space syntax has been to describe space and movement as a dimension of social copresence. The way in which the structure of space and movement affects our exposure to the elements of shape has been a secondary consideration. For the purposes of the argument developed in this paper, the emphasis is reversed. An attempt is made, however, to show that, from the point of view of formal analysis, these two ways of looking at spatial configuration are complementary and can be developed from the same foundation.

We will propose that the description of shape and spatial configuration from the point of view of the moving subject can be discussed by linking projective and convex relationships. We use the word projective in a most elementary sense, to refer to relationships of incidence between lines. As mentioned earlier, given a point in space, and given the lines that project from it to the discontinuities that define the built shape, the question is which of these lines intersect walls (and therefore do not represent a relation of visibility) and which do not (thus corresponding to a relation of visibility). On the other hand, the idea of convexity is fundamental because it is linked to the structure of space as a field of potential copresence of the occupants of the building. The two ideas, of elementary projective relationships and of elementary convex areas come together as we seek to identify informationally stable units of space. From a logical point of view, a spatial unit can only be informationally stable if every point within it is linked not only to the same discontinuities of shape around and beyond, but also to all other points inside the unit. Informational stability has both internal and external components. We will proceed to introduce some concepts and representations through the use of examples. Where appropriate, we will relate and contrast our approach to other descriptive methods that share an intellectual foundation similar to our own and that have served as starting points for the development of our work, most notably space syntax as well as the 'isovists' proposed by Benedikt (1979).

2 Convex partitions

In this section we discuss different ways of partitioning a plan into convex spaces, culminating with a partition which provides us with spatial elements which are informationally stable with respect to their exposure to shape.

As space syntax, more than other methodologies for the analysis of buildings, has proposed that plans can be represented as sets of interrelated convex spaces, we may take it as a starting point. Hillier and Hanson (1984, page 92) originally proposed that the convex representation of a plan should comprise "the least set of the fattest spaces that covers the system". Subsequently, they proposed that if visual distinctions are difficult the map can be derived by locating the largest possible circles that can be drawn without intersecting a wall and then by expanding each circle to the largest space possible without reducing the fatness of any other space (page 98). A number of questions are raised by the degree of completion and rigor of the definition. Not only are we not always sure how to balance rigorously the search for large spaces against the requirement that we preserve fatness, but it is also unclear how the requirements of size and fatness should interact with the requirement that we break the system into as small a number of spaces as possible given our other requirements.

We illustrate some of these difficulties in figure 3. In discussing the examples, we interpret fatness to mean the area/perimeter ratio. However, we relativize this ratio to allow comparisons between spaces of different areas. We do this by comparing a given space to a circle of the same area; we divide the perimeter of a circle whose area is equal to that of the space under consideration by the perimeter of that space. In figure 3(a) we start with two equal circles and end up with two convex spaces;

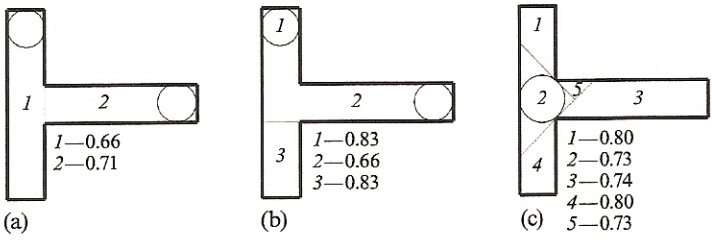


Figure 3. Alternative attempts to partition a shape into the least set of fattest convex spaces (fatness values of individual convex spaces are indicated alongside).

in figure 3(b) we start with two circles and end up with three spaces. Even choosing between those two solutions is not as easy as it seems because the solution with three spaces gives us higher fatness values. In figure 3(c) we start with the largest single circle and end up with five spaces. It seems that, if we want to minimize the number of spaces, we cannot always start by drawing the largest possible circle. These examples are not intended to challenge the earlier syntactic methods regarding convex partitions in general. In most real cases the methods can be applied by the student without generating puzzles. A substantial body of research suggests that the method recaptures properties of spatial arrangements that are essential to our understanding and use of buildings. Some of the relevant studies will be mentioned later in this paper. Our comments are intended to highlight some of the technical problems that remain unresolved with the earlier attempts to divide building plans into convex spaces.

The development of space syntax has led to new ways of handling convex partitions. The most notable method allows for the identification of overlapping convex spaces (Hillier, 1996, page 125). According to this partition, convex elements are defined with reference to the surfaces of built form; the edges of convex spaces are collinear with the lines produced by extending wall surfaces, where this is possible, until the extensions reach another wall surface. The process has been automated through 'Space Box', a computer program developed by Penn and Dalton, working at Hillier's Space Syntax Laboratory, University College, London[1]. When we extend the lines defined by wall surfaces, we produce a potentially very large number of overlapping convex spaces. As we understand it, the convex partition mentioned by Hillier (1996) comprises only those convex spaces each side of which contains a wall surface of the system. Essentially, this means that only the 'largest' convex spaces defined by the various combinations of extended wall surfaces are considered. Figure 4 presents a simple example of overlapping convex spaces according to our understanding of the procedure developed by Penn, Dalton, and Hillier.

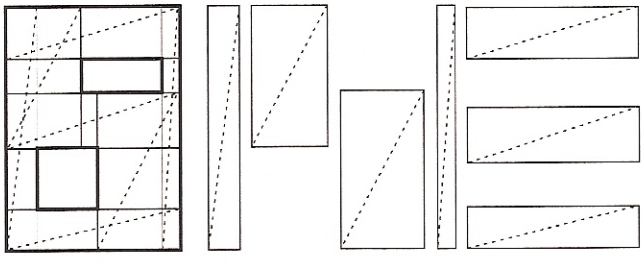


Figure 4. Partition of shape into overlapping convex spaces.

[1] To the best of our knowledge, details about the program and the theoretical arguments supporting its development, or applied studies using it, have not been published yet. We are grateful that direct communication has allowed us to remain in touch with some ongoing work at the Space Syntax Laboratory.

The comparison between the two methodologies of convex partition mentioned above reveals a rather interesting theoretical issue. In the first case, the analysis starts from space and treats built shape as a constraint that limits the extent to which space can retain its convex integrity. Convex integrity can, in this case, be seen as a foundation for the functioning of space as a field of reciprocal and commutative copresence regardless of the exposure to particular elements of shape. In the second case, the partition of space proceeds according to the components of the shape. The built shape drives the analysis, and space itself is shaped into convex elements as a consequence of the presence of built shape. One practical consequence of this rather subtle theoretical difference is the different number of convex spaces and the different kinds of relationships (adjacency versus overlap) produced in each case. As a consequence, space as a field of copresence appears more diversified in the second analysis. But the theoretical possibility, that we may look at the interaction of space and shape from space up towards shape as well as from shape down towards space, remains intriguing to us. Though shape and spatial structure cannot, quite self-evidently, be discussed independently of one another, the issue of whether they may be conceptually and analytically distinguishable remains a concern for any theory of architectural form.

We will now proceed to propose or discuss other convex partitions. Defining the minimum number of convex spaces into which a plan can be partitioned provides us with the simplest representation of its spatial structure. At the same time, the plan can be explored from the point of view of the underlying relations of copresence and separation between occupants that it engenders. We propose to call the partition of a plan into the minimum number of convex spaces that are needed to cover all its area a 'minimum partition'. We will refer to the convex spaces created by a minimum partition as 'm-spaces', and the lines drawn to demarcate them as 'min-lines'. From a mathematical point of view, the minimum partition describes how far space can retain its convex integrity subject to the presence of built shape. The minimum number of convex spaces will on average also be the largest in area (because the total area is divided by a smaller number) and thus the 'minimum partition' would come intuitively close to the one proposed by Hillier and Hanson (1984), if we were prepared to ignore the question of fatness and relax the requirement that the largest individual spaces are drawn first.

We have to treat two questions. Can a minimum partition be drawn? And can a minimum partition be specified uniquely? We think that we have developed a process that gives a minimum partition but we have not found a way to ensure that this partition is specified uniquely in all cases. To develop a minimum partition for shapes without curves, we start with the observation that each reflex angle must be divided into two convex ones by drawing a partition line. A reflex angle is one that is greater than 180°. Here, reflex angles include the freestanding edges of walls which are taken to be angles of 360°. It follows logically that we should first draw any lines that connect two reflex angles in such a way that reflex angles are eliminated at both ends and then deal with the remaining reflex angles. The following logical procedure seems to work well in the cases we have tested.

(1) We draw each line that connects two reflex corners in a way which eliminates concavity at both ends.

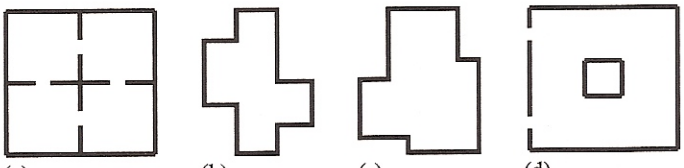
(2) When these lines intersect or share end points we start a process of elimination. The aim is to retain the largest number of such lines which do not intersect with each other or share an end point.

(3) For all remaining reflex angles we extend one of their sides until it meets a wall or a previously drawn partition line. We proceed until all necessary extensions are drawn.

(4) We check whether any two adjacent convex spaces thus produced, whose common edge does not include any part of a wall, can be treated as parts of a single larger convex space. If so, we eliminate the demarcation line that separates them.

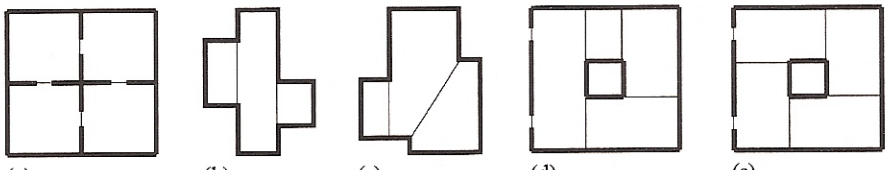
It should be noted that the minimum partition thus created is not always specified uniquely. There are alternative ways of drawing the partition, for example, by choosing to extend different sides of reflex angles or to deal with the reflex angles in a different order. It is important to realize, however, that the problem of whether a minimum partition can be drawn is theoretically independent from the problem of whether it can be specified uniquely. Specificity can be handled as a matter of imposing additional constraints. We will briefly illustrate this. We can limit the minimum partition alternatives by choosing to draw the shortest possible extension lines first. This takes care of the order in which we will deal with reflex angles, as well as of the choice of which side to extend. Figure 5 shows four elementary hypothetical plans and figure 6 shows their minimum partitions. Following all the above rules, we can derive only one minimum partition for cases 5(a), 5(b), and 5(c); a number of alternatives, however, would equally well be derived for 5(d), as shown in figure 6. For example, the two external doors can appear connected either to the same space or to different spaces, as shown in figures 6(d) and 6(e), which may have serious implications for further analysis. To deal with such problems, we may stipulate that we draw the bisector of reflex angles rather than extend one of their sides. In addition, the shortest possible bisectors should be drawn first. This allows us to specify uniquely the minimum partitions of all shapes in figure 5, as shown in figure 7. Our conventions, however, cannot help us to specify uniquely the minimum partition for the shape shown in figure 8 (see over). Thus the development of a procedure that leads to a minimum partition specified uniquely for any shape remains an unresolved problem.

As we discussed earlier, seeking to preserve the maximum convex integrity is only one way to approach the interaction between built shape and space. We now come to propose convex partitions that take into account the way in which built shape appears to a moving subject. We first consider the partition which is obtained by extending



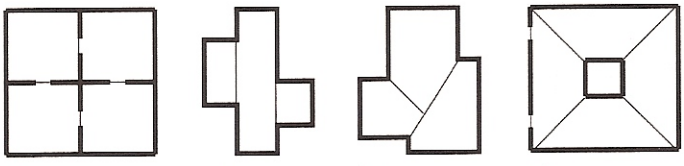
(a) (b) (c) (d)

Figure 5. Four hypothetical plans.



(a) (b) (c) (d) (e)

Figure 6. Minimum partitions of hypothetical plans.



(a) (b) (c) (d)

Figure 7. Modified minimum partitions of hypothetical plans.

both sides of all reflex angles as well as all walls terminating at a freestanding end point. As the partition is obtained by extending surfaces, we propose to call it a 'surface partition'. We refer to the corresponding lines and spaces as 's-lines' and 's-spaces', respectively. The s-partitions of the shapes presented in figure 5 are presented in figure 9. It is noticeable that in the case of plan 9(a) the surface partition coincides with the minimum partition; in the case of plans 9(b), 9(c), and 9(d), this is not so. In these three cases, the surface partition produces spaces whose corners do not correspond to the intersection of two walls. In fact, at least one solid wall extends continuously across most s-lines. The continuity of boundaries across the demarcations of the surface partition may be treated as a defining characteristic of a broad family of open plans. This property is absent from the family of more cellular arrangements, illustrated in plan 9(a).

It will be noted that the overlapping convex spaces mentioned by Hillier (1996) can be derived by creating the union of sets comprising some of the discrete spaces produced by the s-partition. The connection between the two partitions should not, however, detract from the distinct focus of the argument that is being developed here. To us, the s-partition is a first step towards capturing the experience of shape that is available to a moving observer. Each time such an observer crosses an s-line, an entire surface either appears into the visual field or disappears outside it. For any two different s-spaces, there is at least one wall surface which is entirely visible from one but not from the other. Thus transitions from one s-space to another are associated with changes in the available information about shape.

The reverse, however, is not true. Surfaces and parts of surfaces may appear or disappear without crossing an s-line. The information about shape changes while a moving observer remains within the same s-space. Quite clearly, different points within the same s-space may differ by being linked to a different set of discontinuities of the built shape as illustrated in figure 10. To obtain informationally stable spaces, and to

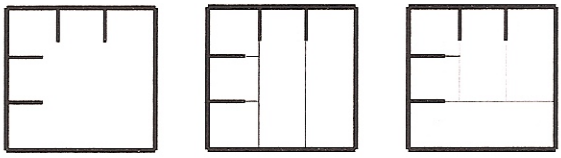
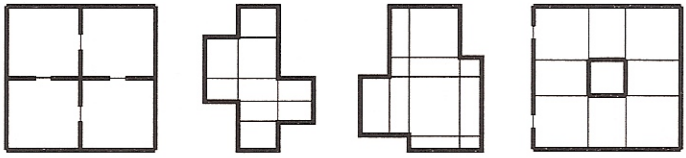


Figure 8. Alternative minimum partitions of a hypothetical plan.



(a) (b) (c) (d)

Figure 9. s-Partitions of four hypothetical plans.

(a) (b)

Figure 10. Changing relationships of a moving subject to the discontinuities available to an s-space.

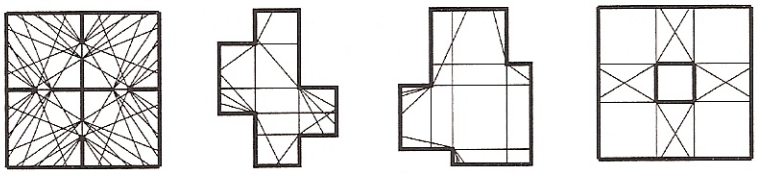
identify explicitly all thresholds at which information regarding shape changes, we propose another partition.

We begin by considering the diagonals that can be drawn in a shape and their extensions. A diagonal is simply defined as a line that joins two discontinuities without crossing a wall. Some diagonals cannot be extended without going outside the shape. We call these 'nonextendible' diagonals. Other diagonals can be extended inside the shape at one, or both, of their ends until they meet a wall. We call these diagonals 'extendible'. We now propose to obtain a new convex partition which includes the extensions of the extendible diagonals in addition to all the lines used to generate the surface partition. The diagonals themselves are now drawn.

The resulting partition has two interesting properties. First, every time we cross a demarcation line, a discontinuity either appears into or disappears from our field of vision. Second, the convex subshapes defined by this partition are informationally stable. We propose to call this partition an 'end-point partition'. The corresponding demarcation lines and convex spaces will be referred to as 'e-lines' and 'e-spaces'. Figure 11 presents the end-point partition of the four theoretical shapes discussed earlier[2].

The significance of the e-partition can be highlighted if we relate it to the idea of 'isovist'. In a key paper Benedikt (1979) has defined the 'isovist' as the set of points visible from a vantage point in space, with respect to an environment. He then proceeded to propose a number of properties of isovists which have a mathematical and intuitive significance as descriptors of the structure of an environment seen from a point. These include the area of the isovist, the length of real surfaces exposed to the isovist, the length of occlusive radial boundaries of the isovist (these are the boundaries that do not correspond to real surfaces but are generated from the manner in which surfaces are placed in front of each other with respect to the observer), and the shape and compactness of the isovist.

Isovists can be drawn from a great number of positions in any plan (in mathematical terms, they can be drawn from an infinite number of positions). Benedikt's (1979, page 50) statement that "describing an environment in terms of the position of its real surfaces ... is entirely equivalent to describing it by the set of all possible isovists corresponding to all points" could inadvertently conceal a real methodological dilemma. Though surfaces can be described completely according to the positions of edges and corners, which are always a finite set, isovists can never be drawn from



(a) (b) (c) (d)

Figure 11. e-Partitions of four hypothetical plans.

[2] Hillier (1996) discusses the representation of spatial relationships according to all the linear elements that can be drawn by linking any two vertices of a plan that are visible from each other and by extending the lines in both directions until they meet a wall. The generation of such 'all lines maps' resembles the first steps to the generation of the e-partition. There are two differences. The e-partition does not include the diagonals but their extensions; it also includes the extensions of surfaces. The main difference though is theoretical and conceptual. Hillier studies lines of potential movement and visibility. He analyzes lines and their intersections as representing spatial configuration. We treat 2-D spaces and seek to determine convex spaces that are informationally stable.