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# On the generation of linear representations of spatial configuration

**J Peponis, J Wineman, S Bafna, M Rashid, S H Kim**
College of Architecture, Georgia Institute of Technology, Atlanta, GA 30332-0155, USA;
e-mail: john.peponis@arch.gatech.edu; jean.wineman@arch.gatech.edu;
sonit.bafna@arch.gatech.edu; mahbub.rashid@arch.gatech.edu; shkim@scucc.scu.ac.kr
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**Abstract. It has been shown that, with space syntax, spatial configuration can be described as a set of lines covering all the areas of a layout and all the ways of moving around the one-dimensional and two-dimensional boundaries that it comprises. In this paper we propose alternative formal definitions of linear representations of spatial configuration and the ways in which they can be generated. The significance of these representations is discussed briefly.**

## Space syntax and the linear representation of spatial configuration

Our aim in this paper is largely theoretical. We are dealing with the analysis of built shape and space in two dimensions, as documented in plans. Research associated with space syntax establishes the significance of linear representations of spatial configuration. Given this, we propose to explore the way in which such representations can be defined formally with greater rigor. We examine how they can be generated automatically, at least in principle, because we take the ability to specify the relevant procedures as a test of the clarity of our conceptualization of such representations. Finally, we discuss briefly the significance of linear representations in an attempt to bring to light the relationship between geometrical and architectural intuitions. As space syntax has been our point of departure, and as the aim is to contribute to the clarification of its mathematical nature, we will begin with a brief review of relevant literature.

With space syntax, spatial configuration has been described largely as a set of lines that represent directions of uninterrupted movement and visibility and cover all the areas of a plan and all the ways of moving around one-dimensional (1-D) and two-dimensional (2-D) boundaries situated within it. These linear representations have been originally referred to as 'axial maps'. It was proposed that axial maps can be derived by drawing the longest straight line possible, and then the next longest until "all convex spaces are crossed and all axial lines that can be linked to other axial lines without repetition are so linked" (Hillier and Hanson, 1984, page 99). This method of deriving linear representations of spatial configuration is dependent upon the prior definition of a convex partition of space. The stipulated 'convex map' of a configuration was taken to comprise the fewest and largest possible convex spaces needed to cover the entire area (Hillier and Hanson, 1984).

The original linear representations of configuration have been powerful tools in the analysis of the social and cultural functions of space. They have been associated with the discovery that spatial configuration is correlated with the distribution of movement patterns and the probabilistic generation of encounter in urban areas (Hillier et al, 1987; Hillier et al, 1993; Peponis et al, 1989); they have been used in the empirical study of the organizational use of space in buildings (Hillier and Penn, 1991; Peatross and Peponis, 1995) and the empirical study of the intelligibility of layouts (Peponis et al, 1990). They have been applied to the analysis and interpretation of historical evidence (Hanson, 1989; Markus, 1993). They have even been found to be useful in the analysis of satisfy the conditions just described, they also take us around some physical object or, more precisely, around some of the surfaces that make up the plan shape.

The realization that the maps described here do not necessarily take us around all nontrivial circulation rings is rather fundamental. It attests to the fact that we can get everywhere and still not capture the essential topological properties of a plan. To obtain a complete linear representation of a spatial configuration, the issue of nontrivial circulation rings has to be considered more closely. In order to avoid confusion, and recognize the aforementioned facts, we will refer to the maps described in this section as 'partial linear accessibility maps'.

**An economic set of lines that get everywhere and recognize plan topology**

Following upon the remarks made in the preceding section, our last linear representation is aimed at recognizing plan topology. The presence of nontrivial circulation rings and their number can best be determined as a function of the relationships between wall surfaces. It is to these that we turn our attention now. A wall surface is physically connected to another if it shares an edge with it. On plan, each wall surface can be connected to two others at most. If we represent the pattern of these connections as graphs, using nodes to denote wall surfaces and lines to denote connections, the overall pattern will take the form of one or more rings. This is easily intuited if we recognize that, for the sake of example, a rectangular room with a single freestanding wall inside it has six surfaces and generates two rings, one with two nodes and one with four.

The basic number of nontrivial circulation loops is equal to the number of rings of surfaces that are surrounded by interior space. The ring of surfaces that includes the perimeter of a plan shape does not generate a circulation loop. For the sake of convenience, we will refer to a set of continuously connected wall surfaces surrounded by a circulation loop as an 'island of surfaces'. We are now looking for a test that will allow us to achieve two things. First, it must be determined whether the partial accessibility map fully encircles every island of surfaces or whether it must be expanded by including additional m-lines. Second, we must determine where to add the appropriate m-lines in cases where islands of surfaces are not already encircled. The algorithm that we present below is not computationally efficient but it is logically elegant.

The logic of the test is as follows: for each surface of an island, we seek to determine a polygon attached to it and extending outward as much as m-lines and the disposition of other surfaces will allow. If the island is surrounded fully by m-lines, the polygon will not have edges comprising surfaces not lying on the same island. If such edges are discovered, we know that additional m-lines must be drawn.

The algorithm is as follows. We list all the islands, each with its set of surfaces. All the s-lines that are incident upon each surface of an island are considered; s-lines extending island surfaces as well as s-lines extending surfaces of other islands and/or the perimeter are included. We mark the points of incidence and allow them to divide the surfaces of the island into a larger set of subsurfaces.

We then consider the partition of space into discrete convex elements which is created by the s-lines and the m-lines already drawn—this is a hybrid partition that does not correspond to either the e-partition or the s-partition as defined above. Given this convex partition, we determine the strong visibility polygon of each surface or subsurface. The m-lines intersecting this polygon are identified. We determine on which side of the m-line the surface lies and we cut away that part of the strong visibility polygon which lies on the opposite side. Where an m-line is incident upon the surface or subsurface, so that the surface lies on both its sides, we ignore that m-line, so far as this particular part of the procedure is concerned. If no m-line intersects the visibility polygon, we retain the entire polygon. When this is done for all m-lines, we obtain a polygon which is equal to, or smaller than, the original visibility polygon.

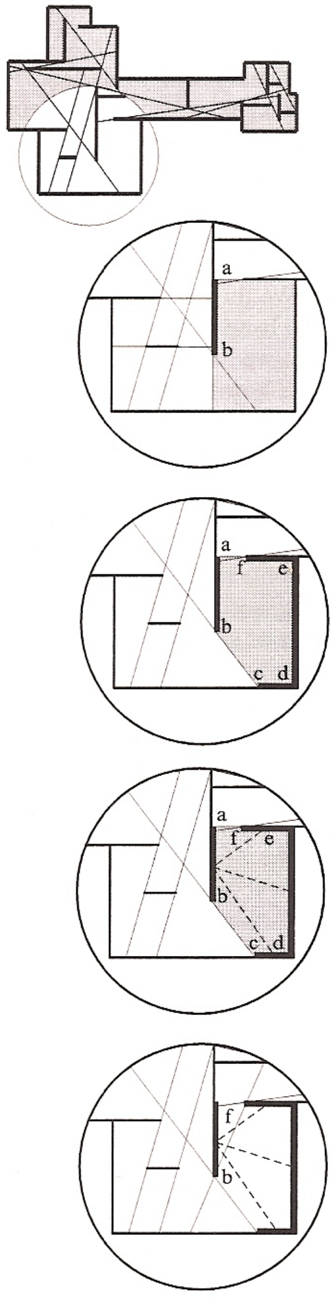
We now consider the edges of the polygon previously determined. If the island is surrounded fully by m-lines, no edge of the polygon should include a wall surface, or part of a wall surface, that does not belong to the island. If such edges are found, we have identified a 'problem condition'. To address this condition, we link the midpoint of the original island surface to the midpoint of each of the unacceptable surfaces in all cases where this is possible without crossing a surface. We treat this as a set of 'problem lines' and list it.

This procedure is repeated for all surfaces of all islands. We thus find the set of all the problem lines associated with all the 'problem conditions'. It must be noted that we cannot be sure we have identified all problem conditions until we have examined every surface or subsurface as described above. We will now discuss how to remove problem conditions.

Each problem line identified is intersected by a number of diagonals of the plan shape. At this stage, our purpose is to find the smallest set of such diagonals that intersects all problem lines and to add these diagonals to the m-lines that represent the shape. We choose the diagonal that intersects the largest number of problem lines. Where several diagonals intersect the same number of problem lines but not the same lines we choose them all. Where several diagonals intersect the same problem lines we choose the one that also intersects the largest number of s-lines. Where several diagonals intersect the same problem lines and the same number of s-lines we choose the one that intersects the largest number of m-lines. Where even this is the same for two or more diagonals we chose the largest one. Where even length is the same we simply eliminate one at random. We eliminate from our consideration the problem lines thus intersected and repeat the procedure until all problem lines have been removed. The selected diagonals are added to the set of m-lines. This completes the derivation of the full linear representation of spatial configuration. Figure 5 (see over) illustrates some of the aspects of the algorithm described above. Figure 6 (see over) presents the complete accessibility maps for the plans taken as examples. In table 1, rows 7, 11, 12, and 13 give the number of m-lines needed to get everywhere and to recognize plan topology as a proportion of the number of surfaces in the plan and as a proportion of the number of diagonals that can be drawn without crossing a wall surface. In all cases we need fewer m-lines than 20% of the number of vertices, fewer than 10% of the number of extendible diagonals, and fewer than 9% of the total number of diagonals. These results demonstrate the economic efficiency of our completed linear representation as compared with Hillier's (1996) all-lines map, which includes the extendible diagonals only.

It must be noted that in some cases our elimination procedures will meet with pairs of diagonals that are the same in every respect. Keeping all such diagonals will lead to 'weighting' the connectivity pattern towards that part of the system, however slightly. Elimination of one at random will lead to another kind of bias in the local connectivity pattern. Thus the final linear representation is not always uniquely specified, even though the number of lines and the number of nontrivial circulation loops that it comprises are. Unique specification is harder in systems endowed with symmetry, especially where symmetry applies to metric relations as well as to relations of incidence. A discussion of properties of symmetry as a subject in its own right is outside the scope of this paper.

At this stage we can compare the linear representation developed in the two preceding sections and the traditional axial maps drawn manually according to the definitions offered by Hillier and Hanson (1984). We have selected two examples that are well used in the earlier literature on space syntax, namely Gassin (Hillier and Hanson, 1984) and Apt (Hillier et al, 1983; Hillier, 1989). In addition to the fact that



The plan shown in figure 4(d), highlighting two islands of surfaces that are not fully surrounded by m-lines. One of these will be examined as an example.

We consider subsurface ab, where a is the point of incidence of an s-line upon a surface on an island, and b is an edge of the same surface. The visibility polygon of the subsurface comprises the convex spaces all points of which see the subsurface in its entirety. At this stage, the underlying convex partition used is created by s-lines and m-lines taken together.

We consider the m-lines that intersect the visibility polygon. One of these (bc) leaves the surface on one side. We eliminate that part of the visibility polygon which lies on its opposite side. The remaining polygon is then examined. Three of its edges, cd, de, ef, comprise surfaces that do not lie on the same island as ab. We identify this as a problem condition.

We draw the lines that connect the midpoint of surface ab to the midpoints of the problem edges of the polygon. These are listed as problem lines.

All problem lines identified previously are intersected by the diagonal bf, which is thereby added to the m-lines, previously identified. This helps to complete the circulation loop around the island. The same procedure is used to complete all remaining circulation loops and to generate the complete linear representation of the plan, shown in figure 6(b).

**Figure 5.** Illustration of the procedure for completing the accessibility map.

these examples are familiar, they are convenient because reasonably clear urban plans have also been published along with their axial representations. The axial maps developed according to our procedure were compared with those published originally and analyzed with 'Axman', the software for axial analysis developed at the Space Syntax Laboratory at University College London. Figure 7 presents the two pairs of linear maps. Table 2 (see over) summarizes the syntactic profile of the two cases. For these two examples, the linear representations drawn according to our algorithms

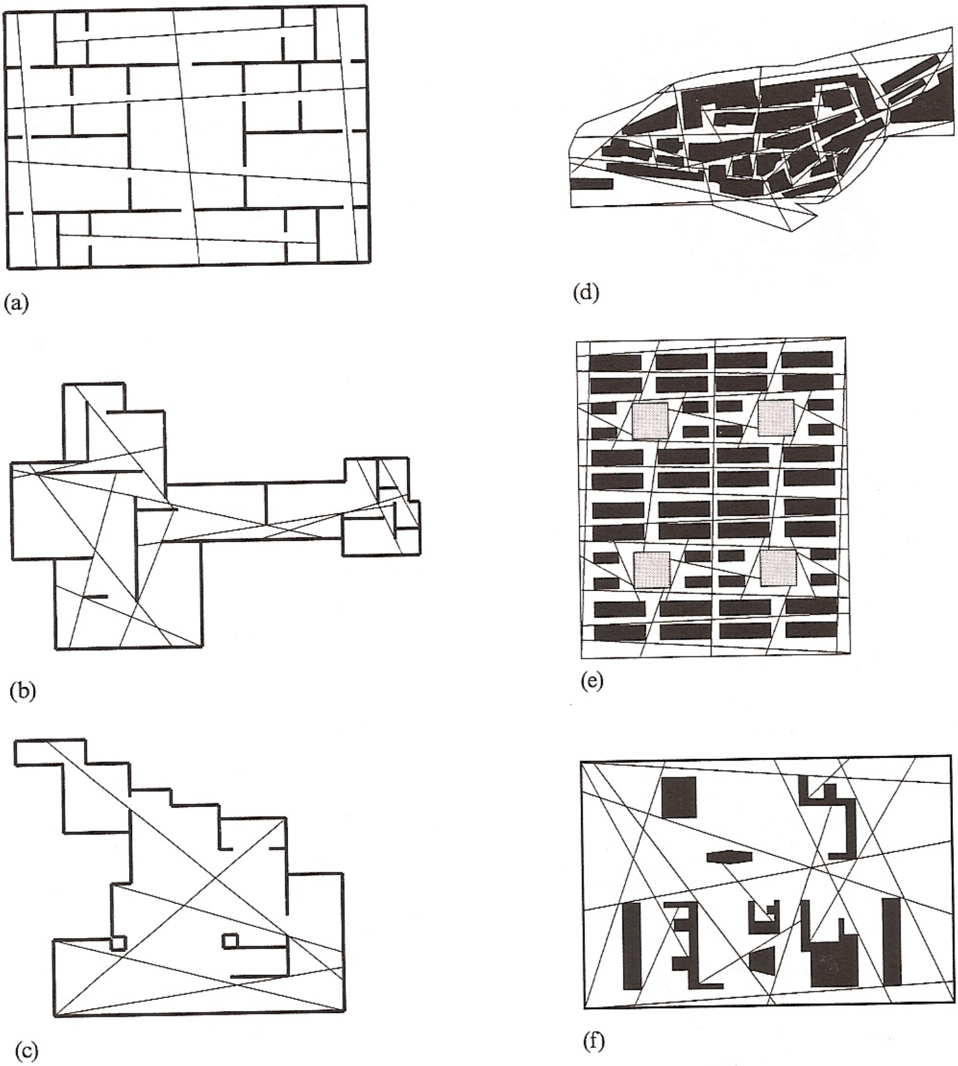
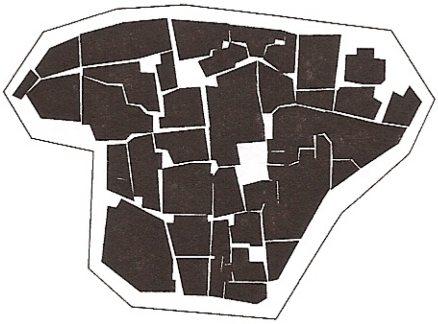
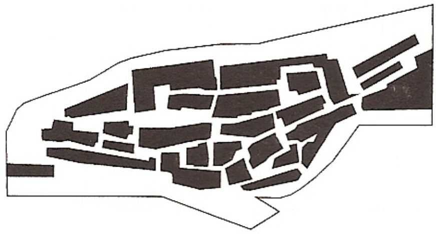



**Figure 6.** Complete linear accessibility maps.

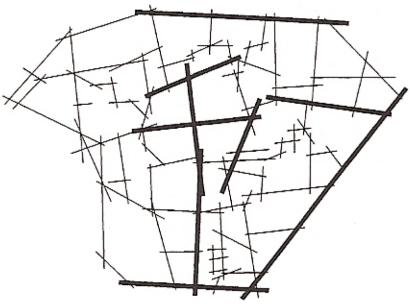
approximate quite well the original, manually drawn, axial maps. In both cases our method comes up with fewer lines. It is possible that this is due to small discrepancies between the electronically redrawn maps we have used and the original maps. Alternatively, our algorithms may have recognized possibilities that were not noticed when the original maps were manually drawn. Mean connectivity and integration values are quite close and the 10% most integrated lines are practically the same. Only the correlation between connectivity and integration for Gassin is appreciably better according to our map. The examples therefore suggest that dense settlement patterns arranged so as to generate relatively linear spaces can be analyzed by our algorithms in ways that approximate the original, manually drawn, axial maps.



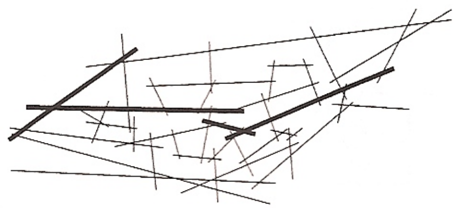
Plan of Apt (source: Hillier et al, 1983)



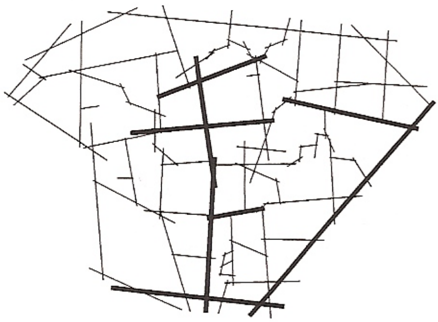
Plan of Gassin (source: Hillier and Hanson, 1984)



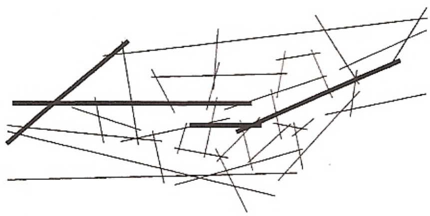
Axial map of Apt as published by Hillier et al (1983); 10% integration core shown as thicker lines



Axial map of Gassin as published by Hillier and Hanson (1984); 10% integration core shown as thicker lines



Linear representation of Apt generated by our algorithms; 10% integration core shown as thicker lines



Linear representation of Gassin generated by our algorithms; 10% integration core shown as thicker lines

**Figure 7.** A comparison between published axial maps and complete linear maps derived according to our procedures.

**Table 2.** A syntactic comparison between two settlement patterns analyzed according to manually drawn axial-maps and according to our algorithms.

| | Lines | Integration | Connectivity | Correlation between integration and connectivity |
|---|---|---|---|---|
| Apt (Hillier et al, 1983) | 89 | 1.29 | 3.71 | 0.79 |
| Apt according to our algorithms | 82 | 1.30 | 3.75 | 0.79 |
| Gassin (Hillier and Hanson, 1984) | 41 | 1.49 | 4.00 | 0.77 |
| Gassin according to our algorithms | 39 | 1.51 | 4.00 | 0.84 |

## Do linear maps describe 'link' or 'directional' distances?

By definition, an integrated axial line is linked into the pattern of intersections of an axial map such that few other lines need to be used in order to reach any particular destination line in the system (Hillier and Hanson, 1984; Hillier, 1996). Another way to express this is to say that an integrated line is positioned such that we need few changes of direction in order to reach any particular line in the system. In other words, implicit in the analysis of the axial map and its interpretation is the notion of axial distance between a line taken as origin and all the other lines in a pattern. Unlike metric distance, axial distance is about changes in direction. This is why it corresponds to our sense of intelligibility of spatial patterns and our sense of orientation within them. The question then arises as to whether a linear map also offers us a shorthand for dealing with directional distances between any two points of a spatial configuration, not just points on the axial lines. This has theoretical as well as practical consequences. The theoretical consequence is that the linear map would then be interpreted not merely as an economical set of lines that get around the system, but also as the structure underlying directional distances between pairs of positions.

Interestingly, there are ways to deal with directional distance without evoking a linear representation of spatial configuration. In the literature on computational geometry the expression 'link distance' has been used to describe the number of directional changes that are necessary in order to move between two positions (Suri, 1997). Given a point in a simple polygon, link distances from any other point can be described according to a partition into subpolygons known as a 'window partition' (Suri, 1990). This is obtained by drawing the visibility polygon from the starting point first. Those parts of the perimeter of the visibility polygon which are not edges are treated as 'windows' to areas beyond. Further windows are determined by taking each of the first-order windows as the starting point. By a successive process of window determination, the entire original polygon is eventually covered by subpolygons. The minimum link distance between the starting point and any other point is then a function of the number of windows that have to be crossed.

Having established the window partition from a given point, it is possible to compute the minimum link distance between any two points in a polygon within a level of accuracy of $\pm 2$ (Suri, 1990). If we are willing to compute a window partition from each of the vertices, the minimum link distance between any two points can be computed with perfect accuracy (Arkin et al, 1992). Unfortunately, these procedures have only been applied to polygons without holes. Dealing with polygons that include holes, such as most architectural plans, is not an easy problem and there is no relevant solution of the problem in the literature on computational geometry.

Given a connected linear representation of a spatial configuration, however, we could propose the following procedure for exploring link or directional distances. Given two points in the configuration, we first compute their visibility polygons. If those polygons intersect, and if the points lie within the intersection, then their link distance is 1. If the polygons intersect, but the two points do not lie on the intersection, then their link distance is 2. If the two polygons do not intersect with each other, then we determine which m-lines are intersected by each. We then search for the shortest connection between one of the m-lines intersected by the first polygon and one of the m-lines intersected by the second. The link distance we are looking for would be equal to the number of m-lines needed to link the visibility polygons, plus 2 for the lines representing the transition from the original points to the relevant m-lines. According to this procedure, the linear representation of spatial configurations becomes an efficient basis for exploring link distances between any two positions. The added advantage, from an intuitive point of view, is that the linear representation of spatial configuration offers a sense of the directions and intersections of movement patterns that is not as readily accessible through the window partition.

The question is whether a procedure such as the above, when applied according to the map of lines that get everywhere and recognize circulation loops, would provide good approximations to minimum link distances and what the range of approximation would be. We have not resolved this question yet. However, it would seem that it is, in principle, possible to interpret the linear representations of spatial configuration as an efficient scaffolding that allows us to deal with link distances between any positions inside it. From the point of view of architectural analysis, that is theoretically significant because changes in direction are involved with the intelligibility of plans from the point of view of a moving subject, while also affecting orientation. From a practical point of view, the complete linear representation of spatial configuration provides us with a way of describing the structure of all potential movements, an issue we will take up again in the next section.

**Concluding comments**

The lines involved in the representations of spatial configuration discussed in this paper, just as the lines of the axial map proposed originally by Hillier and Hanson (1984), are first and foremost lines of potential movement. From a logical point of view, a straight line constitutes an elementary and well-defined relationship between any two points that it links, with order and succession defined along its length. From a perceptual point of view, a straight line is the only path of movement that we are sure to be able to see 'all at once' from any of its points. By contrast, broken or curvilinear paths can partly disappear behind visual obstacles. Thus the straight line implies both an ordering of the successive positions through which we can potentially move and a visual synchronization of these positions. In addition, a pattern of intersecting straight lines offers a clear and simple reference for describing changes of direction and for developing an understanding of directional distances. These properties imply that the representation of potential movement in terms of straight lines contributes to our understanding of spatial configurations as intelligible patterns.

The significance of linear representations arises from the conceptual framework within which they are defined. Their relevance depends upon the establishment of correlations between their properties and the aspects of space occupancy, space use, and cultural meaning in the built environment. Their objectivity, however, arises from the rigor and repeatability of the procedures used to generate them. Our claim is that our linear maps are economical and can be derived objectively for all relevant cases. Thus they represent a useful contribution within the general framework of space syntax. The procedures described in this paper, however, should not be interpreted solely as an attempt to approximate the manually drawn and intuitively appealing axial map. We have shown that in some cases such approximation does indeed arise. In other cases it may not, either because the axial map appears underdetermined and difficult to draw with certainty or because intuition would suggest the addition or removal of lines for reasons that cannot be spelled out in a rigorous and generalized manner. It is possible that future work may suggest new principles for generating linear representations, not merely in the interests of algorithmic efficiency but rather in order to capture some property of configuration other than the ones we have dealt with: visibility, getting everywhere, and recognizing topological structure. If our paper in any way facilitates or precipitates this, it will have served its methodological and theoretical purpose well.

In this paper we also point to the idea that the same properties of configuration are at the foundation of the structure of potential movement as well as of the structure of visibility. We are dealing essentially with the pattern of incidence of wall surfaces, the occurrence and intersection of diagonals which do not intersect wall surfaces, and the patterns of extension of diagonals and surfaces. These are the relational building blocks of configurational patterns. Movement and visibility are also linked from the point of view of ordinary perception, even though movement and orientation can be achieved in the absence of sight. Given these observations, it would seem that the ability to distinguish systematically between linear representations which only guarantee complete visibility, representations which also relate to universal accessibility, and representations which recognize topological structure in addition to everything else, may serve to clarify aspects of configurational structure which otherwise remain compounded and can appear confusing. Such successive linear representations allow us to explore layers of configurational structure while manipulating the same elementary relational ideas. They also provide us with a clearer understanding of how visibility and movement are balanced as aspects of configurational descriptions.

Reference to movement and the patterns of potential movement brings us to our last comment. Actually observed paths of movement may be linear, they may merely be approximated by straight lines or they may be entirely unrelated to such lines. However, we can project any path of movement which does not intersect itself except after encircling a set of wall surfaces, onto a system of straight lines, so that every point along the paths is linked visibly to its projection conjugate, and also so that relationships of order and succession are preserved. We may think of this in terms of a rubber band whose ends are pinned to the coordinates corresponding to the origin and the destination of the path. The aim is to transform the rubber band so that it gets superimposed on the smallest number of lines that connect origin and destination, while satisfying the previously mentioned conditions. Such an exercise is always possible if our linear representation has been developed to recognize the topological structure of the plan. In many cases, the transformation will result in a more economical description of the path from the point of view of directional changes. In some cases, the transformation may lead from a direct path to a broken one, if the linear trajectory of the path does not correspond to an m-line that has been drawn. All paths, however, can be reduced to some part of the completed linear map. By implication, the complete linear maps, though generated according to the properties of shape, can also be interpreted as representations of certain structural features of paths of movement, whether actually observed or simply potentially present in the configuration. This is an additional reason why linear representations of spatial configuration have proved so fundamental to the theoretical insights of space syntax.

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small building plans (Hanson, 1994). Most importantly, they have been used as expert tools applied to the formulation, evaluation, and reformulation of designs on the ground (Hillier, 1993; Stonor, 1997). At the first international space syntax symposium, held in London in 1997, 23 out of 35 contributions referred to axial maps in some way.

The above definition of axial maps is, however, dependent upon the prior establishment of a unique and economic partition of a plan into 2-D convex elements. This cannot be treated as an easy task. In the literature on computational geometry, we can identify algorithms that provide us with partitions of polygons into the minimum number of convex subpolygons, drawing diagonals only (Keil, 1985), or also drawing lines which meet at internal intersections known as 'Steiner points' (Chazelle and Dobkin, 1979). These partitions are not always uniquely specified. The main problem, however, is the fact that dealing with polygons which involve 'holes' has remained 'intractable' (Suri, 1997). In architectural plans, a hole arises when we have a circulation loop around a set of walls in a building, or a set of streets around an urban block in an urban layout. In addition, the original convex partition (Hillier and Hanson, 1984), although appealing initially, cannot be formalized with mathematical rigor (Peponis et al, 1997). In other words, the original convex partition cannot be treated as a well-defined minimum partition and cannot be drawn consistently as a merely economic partition. Thus it is desirable that we explore linear representations of spatial configurations which do not presuppose an economic convex partition, at least until such partition can be specified more conveniently.

More recently, a second linear representation of spatial configuration has been proposed. In this case, all lines that connect vertices without intersecting wall surfaces are drawn and these lines are extended until they meet a wall surface (Hillier, 1996). This linear representation of spatial configuration can be completed according to the relationships of linear visibility and accessibility between vertices, without explicit reference to the convex structure of a layout. There is published evidence that 'all-lines' maps will prove fruitful in various areas of empirical research on the use and organization of space (Penn, 1997; Stonor, 1997). However, axial maps involve a small number of lines that can intuitively be taken to correspond to major directions of movement, whereas the all-lines map includes a large number of lines that have no equally obvious single intuitive interpretation. On the one hand, all-lines maps represent clearly the relationships of visual connections between vertices; on the other hand, they represent potential paths of movement, but do so by allowing a considerable degree of redundancy.

Another difference between the axial map and the all-lines map is evident; although appearing technical at first, this may, upon further consideration, turn out to be of theoretical relevance as well. The generation of all-lines maps can be automated because we always have a finite number of vertices. All that is needed is to connect all these vertices by pairs, to test which pairs can be connected in this way without intersecting a wall, and then to extend the connecting lines that satisfy this test until they meet a wall. By contrast, the generation of the axial map has not yet been automated. This initially appears to result from two difficulties, in addition to the problem of establishing an economical convex partition as a basis for determining the linear map. First, there is the difficulty of inventing algorithms that can find 'the longest line that can be drawn without intersecting a wall', and continue to produce lines until 'all connections have been made'. Second, there is the intuitive problem which arises when we deal with sparsely built systems involving large open spaces, when many alternative lines can be drawn if we insist that 'all axial lines that can be linked to other axial lines without repetition are linked in this way'. It is no coincidence that most published axial maps of urban systems represent dense systems where open spaces are clearly delineated streets. This intuitive problem disappears when we are prepared to deal with the all-lines map.

Thus, the easier implementation of the all-lines map as compared with the axial map points to some implicit conceptual ambiguities of the latter, in addition to the purely technical issues involved.

**An economic set of lines that 'see everything'**

The rest of this paper is devoted to defining three different linear representations of spatial configuration which can be generated following routine procedures. Only the third representation can be compared usefully with the axial map proposed originally by Hillier and Hanson (1984). However, the comparison and contrast between the three representations will serve to clarify issues that are relevant to the 'syntactic analysis' of configuration.

The first representation is to include a small set of lines such that, if we move along their length looking around 360° at every stage, we can see all the wall surfaces that form a built plan in their entirety. The issue of visibility is of obvious interest in the analysis of architectural space. The set of lines that we seek to identify can be taken to represent the *linear visual core* of a built plan. The arrangement of lines provides a useful basis for characterizing built plans.

We acknowledge that the problem of determining how many lines are needed so as to see all surfaces in a plan is equivalent to the problem in computational geometry, that of determining the number of necessary and sufficient 'mobile guards' that can see a 'gallery' of polygonal shape (O'Rourke, 1987). However, the theorem that $\lfloor n/4 \rfloor$ guards moving along diagonals will be sufficient for all polygons with n vertices has only been proved for polygons without holes. Our literature search has not led us to identify already available solutions to our problem with regard either to the sufficient number of lines, or to their actual specification.

For the rest of our argument we will assume that all diagonals connecting vertices without intersecting walls have been drawn and that they have subsequently been extended beyond their defining vertices until they meet a wall surface; lines that connect vertices and graze wall surfaces are included in the set of diagonals. Our task is to find ways that allow us to eliminate most of these diagonals and retain a small set according to criteria that can be justified logically and experientially. For the sake of reference we will refer to the diagonals that are retained after the application of the various tests that we propose as 'movement lines' or m-lines.

A plan shape not involving curves will be treated as a set of wall surfaces extending between edges and/or corners and intersecting other surfaces only at their endpoints. But first we must acknowledge the problem of determining the perimeter of a plan shape. When we analyze buildings, the perimeter can be treated as closed. There will always be an uninterrupted ring of connected surfaces that surrounds all internal areas. When we deal with settlement configurations, however, this is not necessarily the case. We propose to deal with the perimeter of settlements in one of three ways. First, we can draw the perimeter as a definite closed polygon when our understanding of the object justifies it. Second, we can surround the system of building blocks by its convex hull, thus producing an artificial perimeter. Third, we can imagine that the system of building blocks is set in a homogeneous carrier space, such that a continuous circle of movement is possible at some conventionally fixed radius. Any radial lines of movement are simply stopped at their intersection with that circle or are held to be connected to each other through it, depending on whether we want to treat the theoretical carrier space as a barrier or as a connector. These possibilities are illustrated in figure 1 (see over).

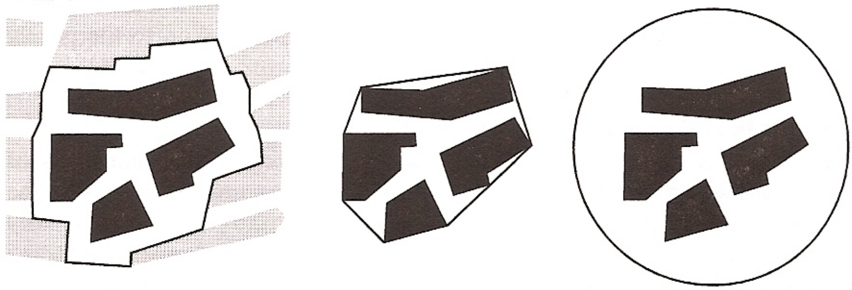


**Figure 1.** Alternative definitions of perimeter for the analysis of a settlement layout.

Having determined that any shape can be treated as having an external boundary, we are ready to proceed with the analysis. At this stage we need to introduce four definitions.

First, we will treat two points as visible from each other when the line that connects them lies entirely within the plan shape and does not intersect a wall surface. Second, we will define the visibility polygon (O'Rourke, 1987), or 'isovist' (Benedikt, 1979), of a point as including all other points in a plan that are visible from it. Algorithms for determining such visibility polygons are provided in the literature (El Gindy and Avis, 1981; Joe and Simpson, 1987; Lee, 1983). Third, we will define the 'strong visibility polygon' associated with a wall surface as the one which includes all points which are visible from each of the points on the surface in question. This is in contrast to a 'weak visibility polygon' which would include all points which are visible from at least one point of the wall surface. Fourth, we define the 'e-partition' of a plan as the partition into convex spaces which arises by extending the sides of all reflex angles until they meet a wall surface, and also by drawing the extensions of all extendible diagonals which connect corners which are visible from each other (Peponis et al, 1997). For the purposes of this partition, the freestanding edges of walls are treated as reflex angles of 360°; consequently, walls terminating at a freestanding edge also get extended.

The convex spaces defined by the e-partition are stable with respect to visual information concerning the vertices of a plan, whether corners or freestanding edges. While we remain inside such a convex space, referred to as an 'e-space', the same vertices are visible to us. It follows that the strong visibility polygon of a wall surface can be treated as the union of some number of e-spaces. Thus we can proceed assuming that we have determined the visibility polygons of all wall surfaces in a plan. One way to do this, logically, is to find the visibility polygon from one point within each e-space, say the centroid, and then to determine which visibility polygons include a given surface, in its entirety, as part of one of their edges. The union of all e-spaces whose visibility polygon includes a given edge is the strong visibility polygon of that edge.

For the rest of our argument in this section we will suppose that any areas which, for the purpose of the analysis, cannot be reached have been marked specifically, so that the surfaces associated with them are not treated as parts of the plan shape. The process of recognizing automatically the presence of isolated areas contained within a larger plan shape can itself be automated; however, because it is of no intellectual relevance to our present argument we will be content with removing isolated areas from consideration. Isolated areas include service cores in building plans, or private premises in the analysis of settlement layouts.

Each of the diagonals of a plan shape, as defined above, intersects a number of strong visibility polygons of surfaces, as shown in figure 2. We rank the diagonals according to the number of visibility polygons that they intersect and select the

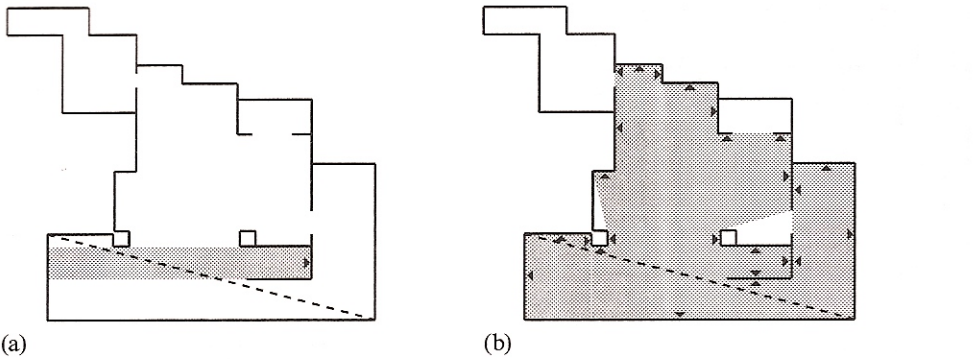



**Figure 2.** (a) The visibility polygon of a wall surface intersected by a line of movement, and (b) all wall surfaces whose spatial convex span is intersected by the same line of movement.

diagonal which intersects the greater number. The polygons that have thus been intersected are eliminated. We then rank the diagonals once more, according to the number of remaining visibility polygons that they intersect. The process is repeated until all visibility polygons have been intersected by at least one selected diagonal. Where two diagonals intersect the same number of visibility polygons, but not the same polygons, we include them both. Where two diagonals intersect the same set of wall-surface visibility polygons, we choose the longest diagonal. This choice is aimed at maximizing the chance that the selected diagonals may intersect each other. We thus select a small number of m-lines.

The set of m-lines that we have thus selected at the end of the process is ranked according to the number of visibility polygons intersected. We then proceed to test whether an m-line of higher rank can be eliminated without leaving any of the polygons unintersected. This test is necessary because some combination of m-lines of equal or lower rank may make any given m-line redundant. If several m-lines of the same rank can equally be eliminated, one at a time, we eliminate the shortest one first and repeat the test. If several m-lines of the same rank and the same length can equally be eliminated, one at a time, we randomly eliminate one and repeat the test. We proceed until no further m-line can be eliminated. The end result provides us with a small and economic set of linear paths that must be covered by a moving subject so that all the surfaces which form the built shape have become visible from at least one point. We will refer to this set of m-lines as the 'linear visibility map'. Linear visibility maps for a number of examples are shown in figure 3 (see over). The examples include three buildings and three settlement configurations. In the case of settlements, we have chosen to terminate the shape with a closed, deliberately drawn, perimeter. In table 1 (see over), rows 5, 8, 9, and 10 present some basic information regarding those plans, so that we can express the number of m-lines needed to see everything as a proportion of the number of vertices in the plan and also as a proportion of the number of diagonals that can be drawn without intersecting a wall. In all cases we need fewer m-lines than 10% of the number of vertices, fewer than 5% of the number of extendible diagonals [which is the set of diagonals included in the all-lines map (Hillier, 1996)], and finally fewer than 4% of the total number of diagonals. In other words, table 1 establishes the level of economy that results from the application of our procedure, with respect to all the indexes used.

The lines included in the linear visibility map, as described above, are a small and economic set but we cannot prove that they will be the minimum set for all cases. This is associated with a rather more general problem of computational mathematics known as 'set cover'. Given a set of points and a collection of subsets, the problem of set cover is

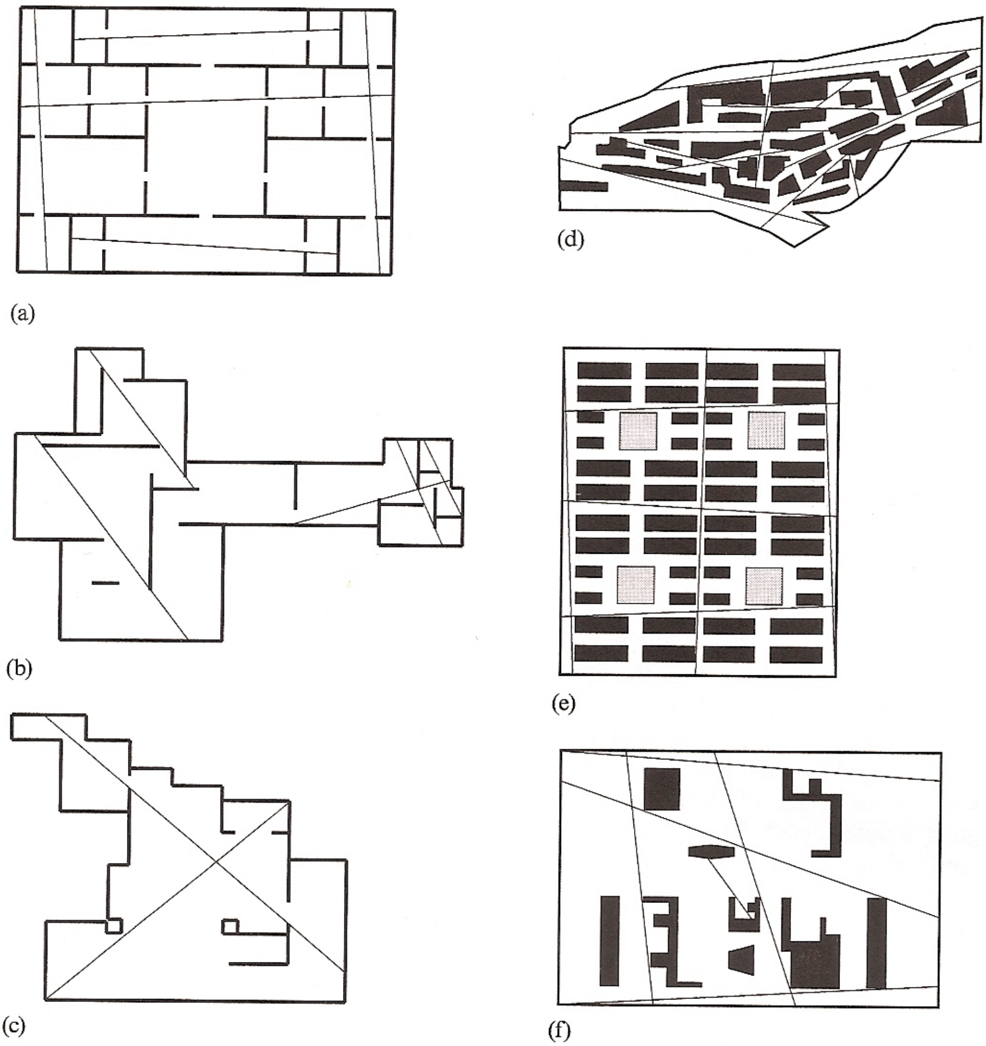



**Figure 3.** The minimum set of lines that 'see' every surface in six plan shapes.

to select as few subsets as possible so that every point of the original set is contained in at least one subset. In our case, the set in question comprises all the edge strong visibility polygons of a plan, and the subsets comprise those polygons which are intersected from each one of the diagonals. It is generally understood that the set cover problem is difficult to solve algorithmically and that we have to contend ourselves with approximations, the best of which are based on 'greedy algorithms' which operate on principles similar to those that we have proposed (Fiege, 1996; Slavik, 1996).

The small and economic set of lines that 'see everything', as determined above, are not necessarily intersecting each other so as to create a continuous pattern of connections. In fact, for the examples presented, continuity of connections is available only in the case of the plan in figure 3(c). If we wish to obtain the minimum set of connected paths that see everything, some additional lines may have to be added. We can select these from among the m-lines of the full accessibility map that will be presented below, following a similar procedure to determine what is the smallest set of additional lines that are needed to connect the set of m-lines that see all surfaces in their entirety.

**Table 1.** The numbers of lines of the various linear representations of spatial configuration compared with the numbers of vertices and the numbers of diagonals in six examples.

| | Building | | | Area | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 1 | 2 | 3 |
| (1) Vertices | 104 | 55 | 45 | 223 | 212 | 80 |
| (2) Nonextendible diagonals | 34 | 13 | 34 | 240 | 298 | 78 |
| (3) Extendible diagonals | 268 | 121 | 140 | 634 | 1104 | 285 |
| (4) All diagonals | 302 | 134 | 174 | 876 | 1402 | 363 |
| (5) Visibility lines | 5 | 5 | 2 | 13 | 6 | 6 |
| (6) Access lines | 7 | 9 | 4 | 37 | 34 | 13 |
| (7) Complete linear maps | 7 | 11 | 5 | 39 | 34 | 15 |
| (8) Visibility lines/vertices | 0.048 | 0.091 | 0.044 | 0.058 | 0.028 | 0.075 |
| (9) Visibility lines/extendible diagonals | 0.019 | 0.041 | 0.014 | 0.021 | 0.005 | 0.021 |
| (10) Visibility lines/all diagonals | 0.017 | 0.037 | 0.011 | 0.015 | 0.004 | 0.017 |
| (11) Complete map lines/vertices | 0.068 | 0.200 | 0.111 | 0.175 | 0.160 | 0.188 |
| (12) Complete map lines/extendible diagonals | 0.026 | 0.091 | 0.036 | 0.062 | 0.031 | 0.053 |
| (13) Complete map lines/all diagonals | 0.023 | 0.082 | 0.029 | 0.045 | 0.024 | 0.041 |

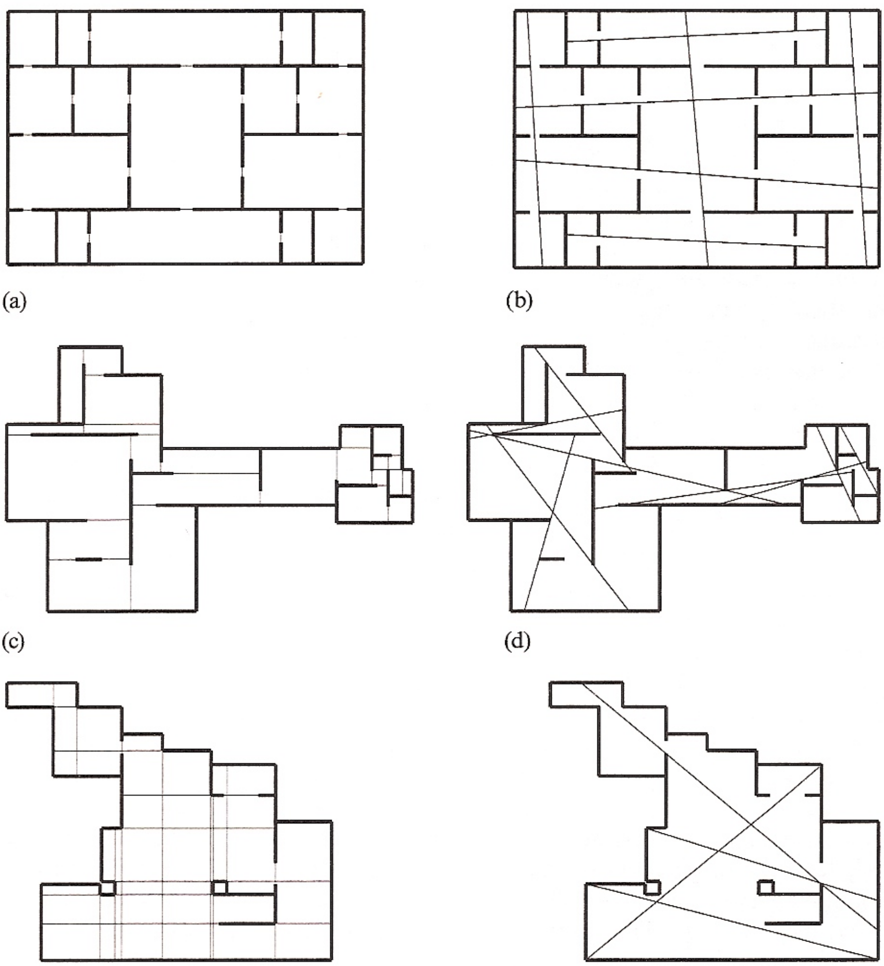
**An economic set of lines that 'get everywhere'**

The second representation is to include a small set of lines that 'get everywhere'. Thus we must provide a plausible interpretation to the idea of getting everywhere. In the procedure for deriving the traditional axial map, this particular question is addressed by requiring that all convex spaces should be covered. We will propose an alternative interpretation that does not require us to presuppose an economic convex partition. We take as our starting point the fact that the elementary act in the arrangement of space is the construction of a single wall with two surfaces. This has the implication of dividing the plane into two halves. As boundaries are added, more complicated patterns of division are produced and, in turn, these can coalesce into more complex relationships such as containment, juxtaposition or alignment. The division of space through boundaries is the basic logical operation involved in the construction of built space; the idea of getting everywhere can plausibly be interpreted as getting to both sides of each boundary.

To formalize this idea we will introduce a definition. The 'surface convex partition' (Peponis et al, 1997) is derived by extending all the freestanding edges of walls and all the sides of reflex angles until they meet a wall surface. We refer to these extension lines as 'surface lines' or 's-lines' and to the resulting discrete convex polygons as 's-spaces'. As an s-line is crossed, at least one entire wall surface either appears into our field of vision or disappears outside it. We now propose that to 'get everywhere' means to cross every s-line. If one has been on both sides of every extendible surface, one has, in an intuitive sense, covered a continuously connected plan shape in its entirety; one has crossed all critical thresholds. It must of course be noted that areas that have been marked off as isolates are ignored in our present derivation of the s-partition.

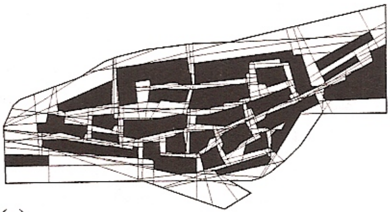
The procedure for deriving a linear path map that gets everywhere becomes rather obvious. We take the original set of diagonal lines and determine the intersections of each diagonal with the s-lines. The diagonals are then ranked according to the number of s-lines that they intersect. The diagonal that intersects the greatest number of s-lines is selected. We then remove the s-lines already intersected from the set of s-lines and rank the diagonals again. We proceed in this manner until every s-line has been intersected by at least one diagonal. Where two or more diagonals intersect the same number of s-lines, but not the same s-lines, all of them are selected. Where two or more diagonals intersect the same s-lines, only the longest is selected. This choice is aimed at maximizing the probabilities that the diagonals selected will also intersect each other. Where two diagonals are indistinguishable according to the set of s-lines that they intersect and to their length, we randomly select one. The set of diagonals thus selected is ranked according to the number of s-lines that are intersected by each diagonal.

We then examine the ranked set of selected diagonals in order to determine if any can be eliminated, one at a time, without leaving any s-line uncrossed. This is a necessary test, because some combination of diagonals of equal or lower rank may make any given diagonal redundant. Where a diagonal can be eliminated without leaving an s-line not intersected by at least one diagonal, we eliminate it. If a number of diagonals of the same rank can equally be eliminated, one at a time, then we first eliminate the shortest one and repeat the test. If several diagonals of the same rank and length can equally be eliminated, one at a time, we randomly eliminate one and repeat the test. We check in this manner until no diagonal can be removed without leaving an s-line not intersected by at least one diagonal.

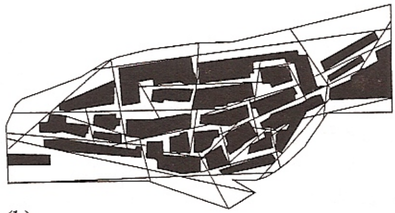


(a) (b) (c) (d) (e) (f)

**Figure 4.** The minimum set of lines that 'get' everywhere: (a), (c), (e), (g), (i), (k) show the s-partition lines, all of which are crossed by the selected lines of movement shown in (b), (d), (f), (h), (j), (l).



(g)



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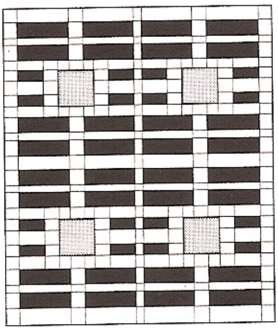
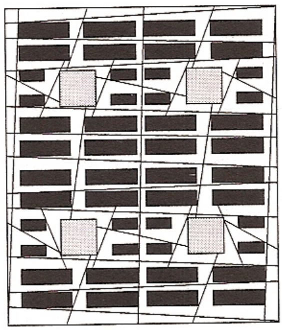
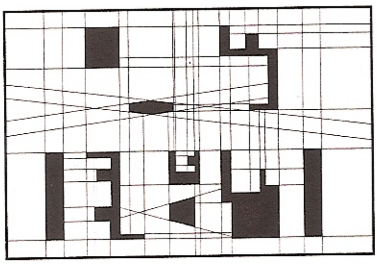


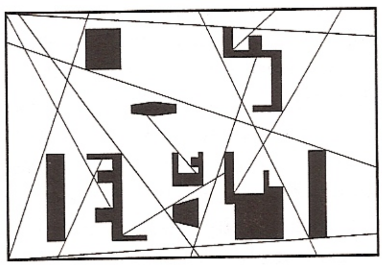
(i)



(j)



(k)



(l)

**Figure 4** (continued).

The set of m-lines thus obtained can be treated as an economic linear map that gets everywhere. The problem of set cover referred to earlier also applies here, so that we can have no proof that the minimum set of lines will be selected in all cases. Examples of such economic linear maps that get everywhere are provided in figure 4.

These maps can be analyzed according to the usual methods associated with space syntax. For the purposes of such analysis, two lines are said to intersect when they cross each other, or when they meet at the same vertex of the plan shape. By examining the maps provided in figure 4, however, we note that they differ from traditional axial maps in one critical respect. They do not always cover the nontrivial circulation rings that are available in the system. Here, a circulation ring is said to arise when linear paths intersect in such a manner that we can move from one line to another, and after a number of such steps back to the original line, without ever moving along any intervening line twice. Now, circulation rings will frequently arise when three or more lines intersect each other in an open space. These rings, however, are intuitively trivial because they do not take us around a physical object. Nontrivial rings not only