Cutting edge imaging

Theory of image reconstruction

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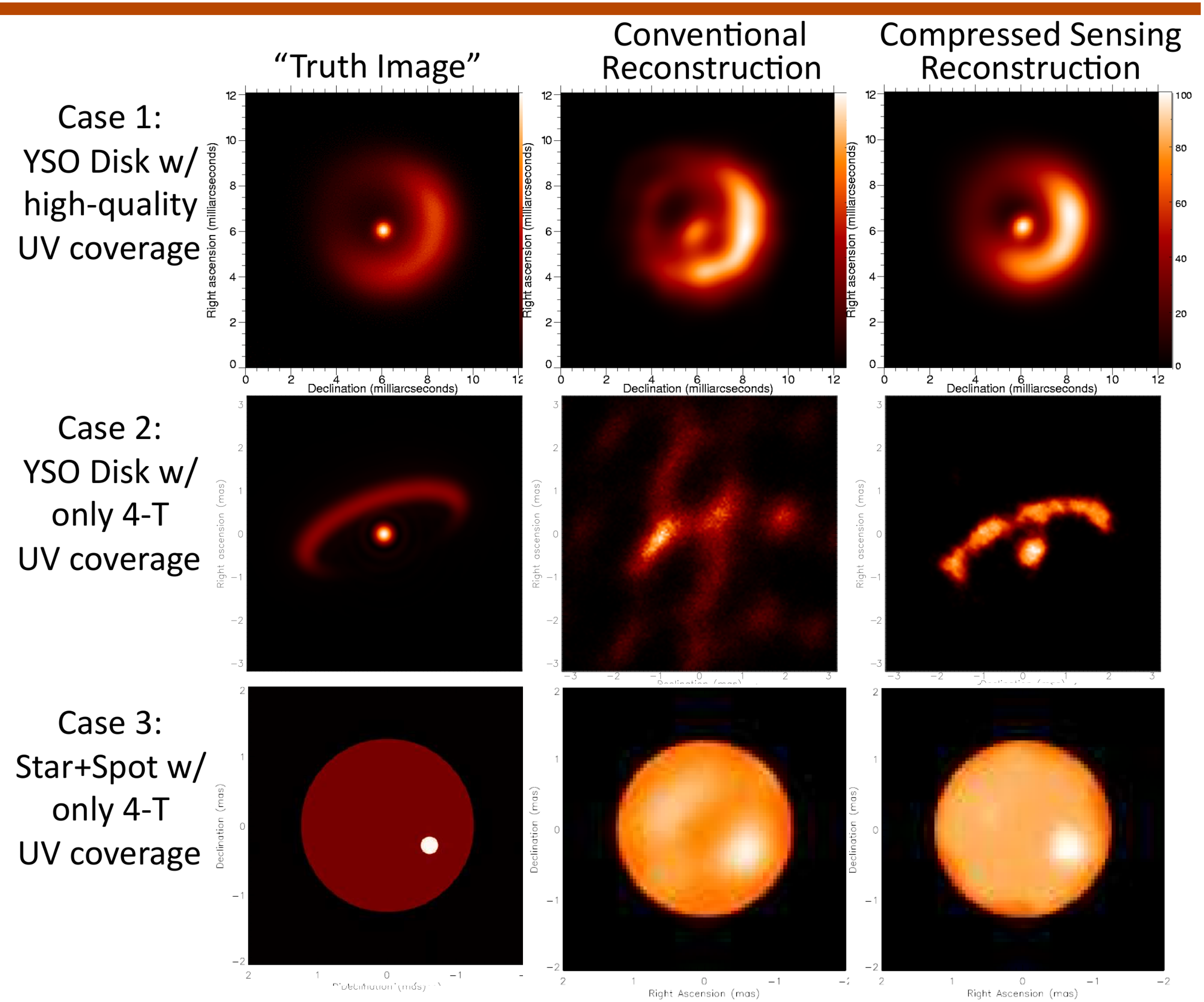
Interferometric imaging
Main research axes

- New regularizers (e.g. Compressed Sensing)
- Simultaneous model fitting
- Reconstruction from multi-frequency datasets
- Reconstruction on non-planar surface (spheroids)

COMPRESSED SENSING

Compressed sensing Performance



Baron et al., 2012 in prep

New regularizers sparsity and compressed sensing

- Sparsity: *an image is said sparse in a basis* $\boldsymbol{W}$ *if it can be expressed as a small number of non-zero coefficients* $\boldsymbol{c}$ *in this basis.* Example: point sources on pixel grid
- **For a sparse image, CS supersedes Shannon sampling**
- CS theory assumes linear measurement equation

$$\boldsymbol{v} = \boldsymbol{H}\boldsymbol{i} + \boldsymbol{n} = \boldsymbol{H}\boldsymbol{W}\boldsymbol{c} + \boldsymbol{n}$$

- Optimal reconstruction (Candes 2007) of this small number of coefficients achieved by $\boldsymbol{W}\widetilde{\boldsymbol{c}}$ with $\widetilde{\boldsymbol{c}}$ solution:

$$\underset{\boldsymbol{c}\in\mathbb{R}^P}{\operatorname{argmin}} ||\boldsymbol{c}||_0 \quad \mathrm{s.t.} \quad ||\boldsymbol{v} - \boldsymbol{H}\boldsymbol{W}\boldsymbol{c}||_2 \leq \epsilon$$

- ℓ_0 regularization on the coefficients, hard to minimize...
- ℓ_1 regularization on the coefficients, give similar solutions (Candes 2006) and is convex...

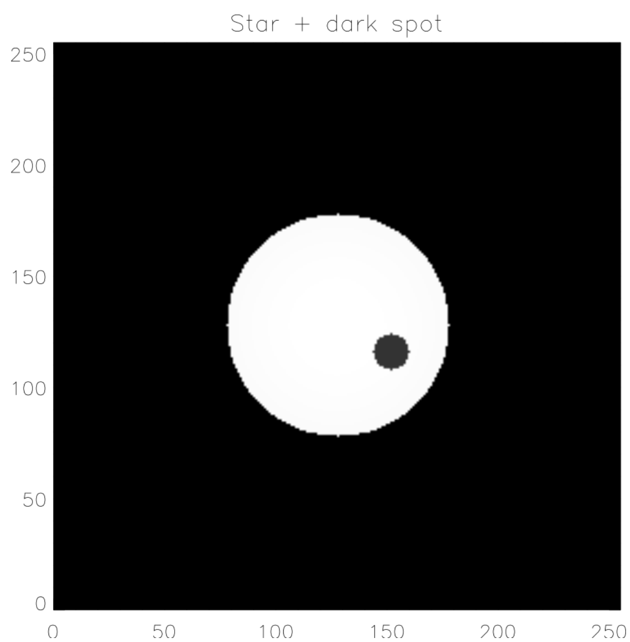
Building a spot regularizer

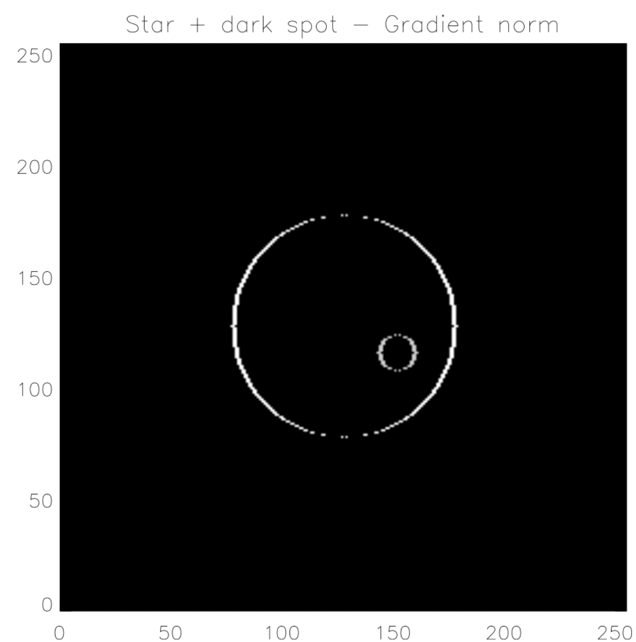
- Total variation, successful regularizer (Renard et al. , 2011)

$$R_{\mathrm{TV}}(\boldsymbol{i}) = \ell_1(\boldsymbol{\nabla i}) = \sum_{x=1}^{N_x}\sum_{y=1}^{N_y} |\nabla i|_{xy}$$

where ∇i is the local spatial gradient

- Better with $\ell_p, p < 1$ (Chartrand 2007) ? $\ell_p(\boldsymbol{\nabla i}) = \left(\sum |\nabla i_n|^p\right)^{1/p}$





Building a spot regularizer

- For given spot flux F, regularizer should prefer 1 spot to 2 spots (Occam's)
- For given spot flux F, the regularizer should not prefer a specific spot size
- 1 spot of diameter 2D: flux density = $F/(\pi D^2)$
- 2 spots of diameter D: flux density = $2F/(\pi D^2)$
- For a gradient regularizer, regularization = $2\pi D$ x reg(flux density)

$$\mathrm{TV} = \ell_1(\nabla i) = \sum_n |\nabla i|_n$$

$$\mathrm{TV}(\text{one spot}) = F/D$$

$$\mathrm{TV}(2\text{ spots}) = 2F/D$$

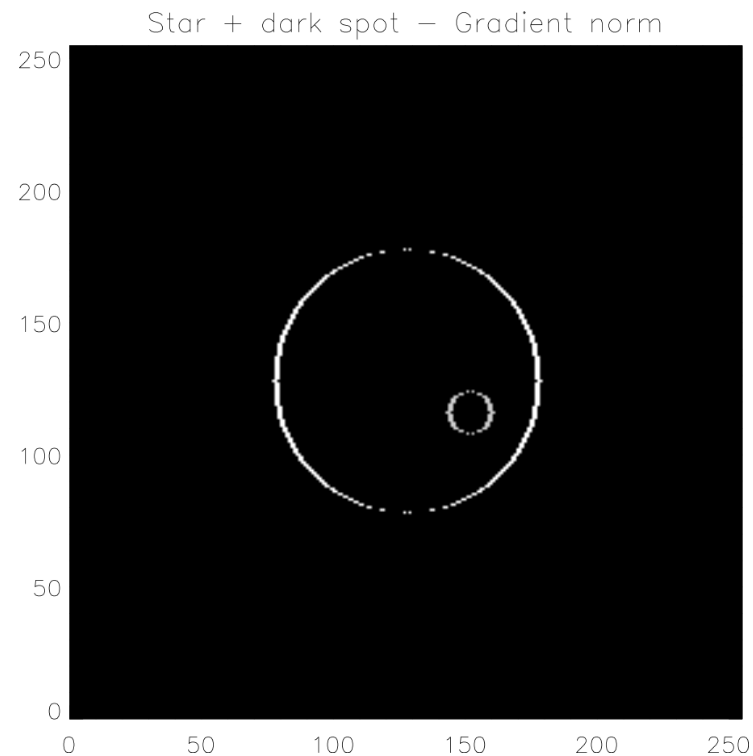
→ Bad
Scale dependent

$$\ell_{\frac{1}{2}}(\boldsymbol{\nabla i}) = \left(\sum \sqrt{|\nabla i|}\right)^2$$

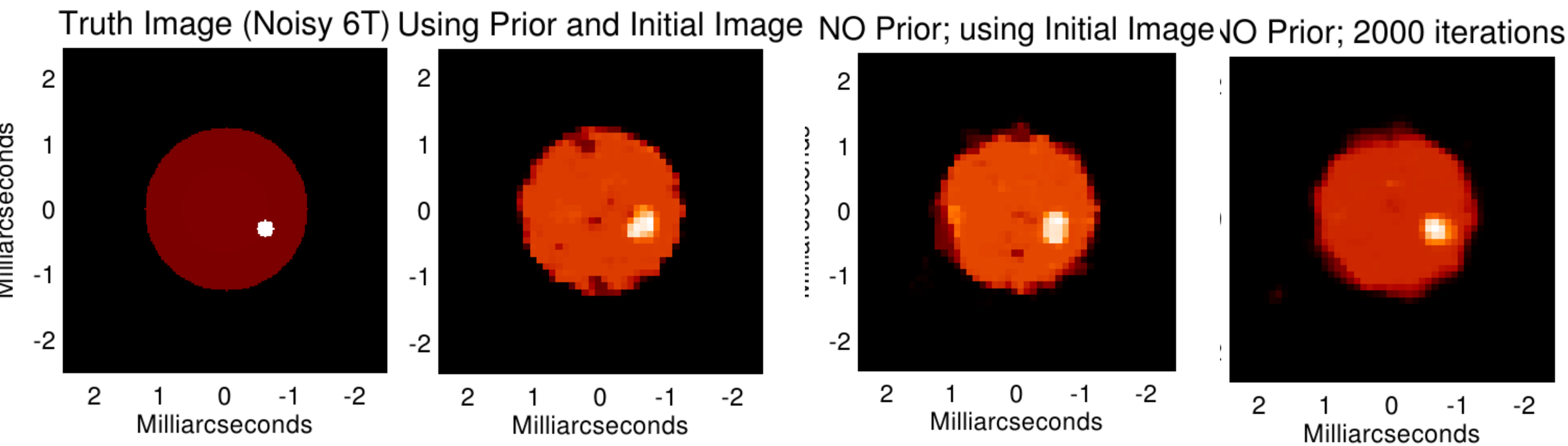
$$\ell_{\frac{1}{2}}(\text{one spot}) = 4\pi F$$

$$\ell_{\frac{1}{2}}(\text{two spots}) = 8\pi F$$

→ Good
Scale independent

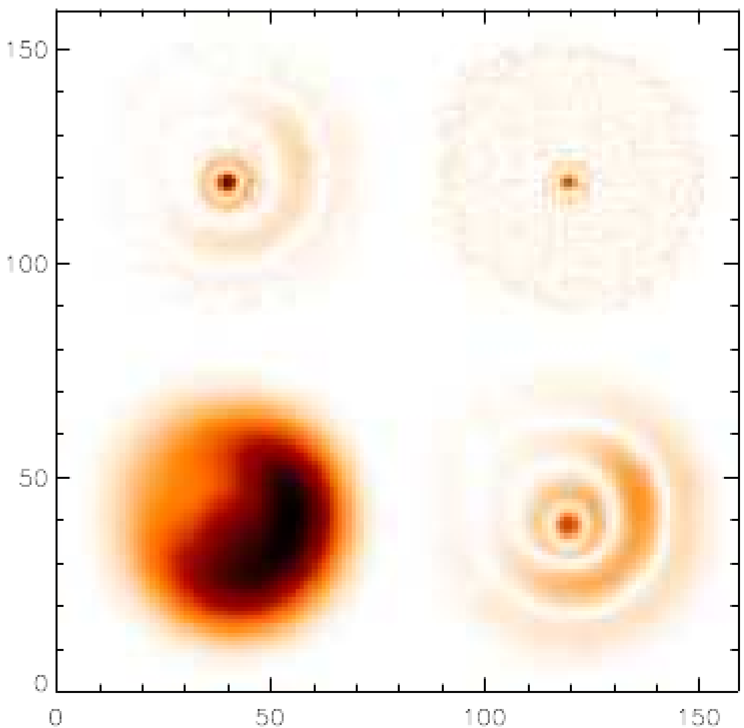


Characteristics of the UD regularizer

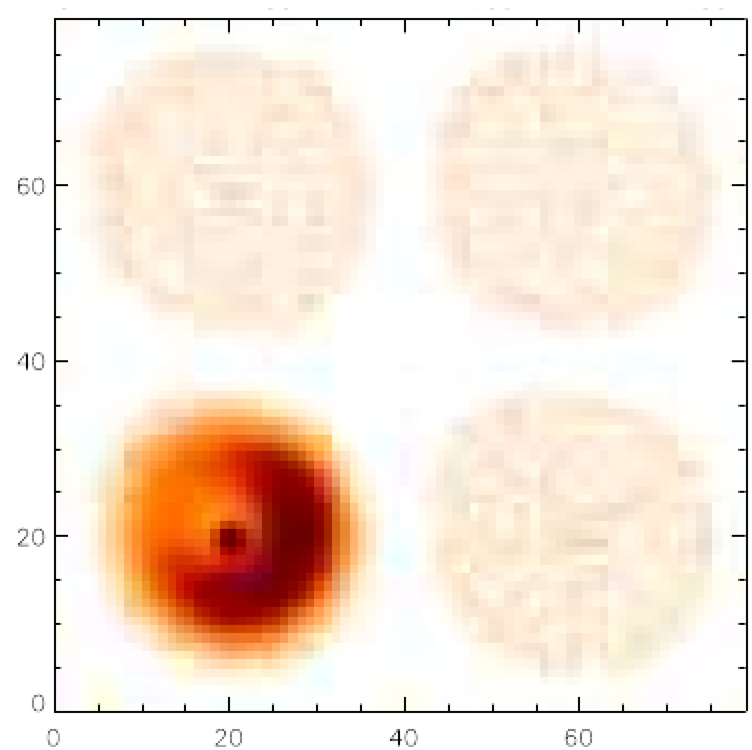


- Local correlations taken into account
  - constraining disc prior not required any more
  - pixels group together, initial image not as important
    - but this can prevent image exploration
- Works well on simulated spots
- In MACIM, use the -bm flag to use the UD regularizer

Compressed Sensing
Wavelets



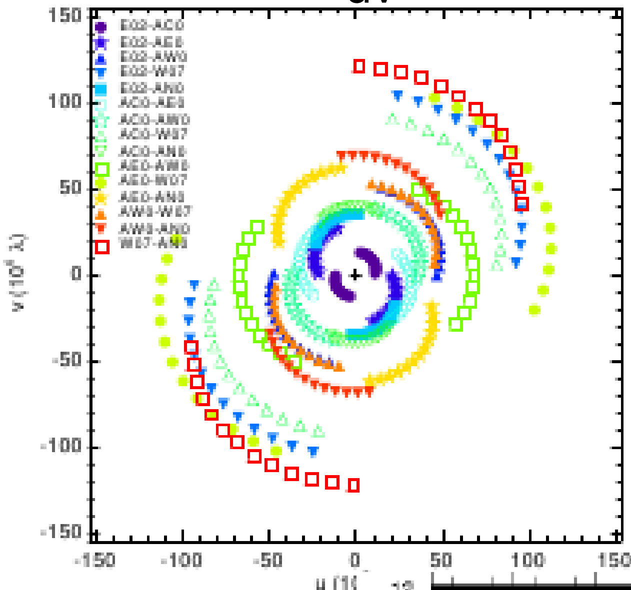
Undecimated isotropic wavelets
4 scales



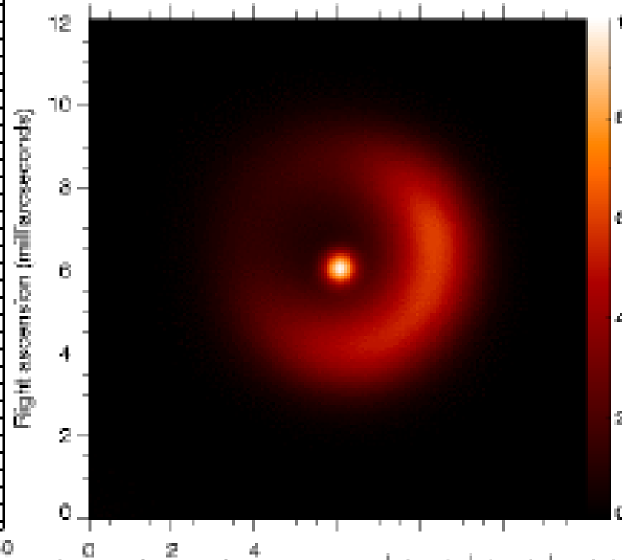
CDF 5/3 wavelet decomposition
(JPEG 2000)
4 scales

Compressed sensing Wavelet super-resolution

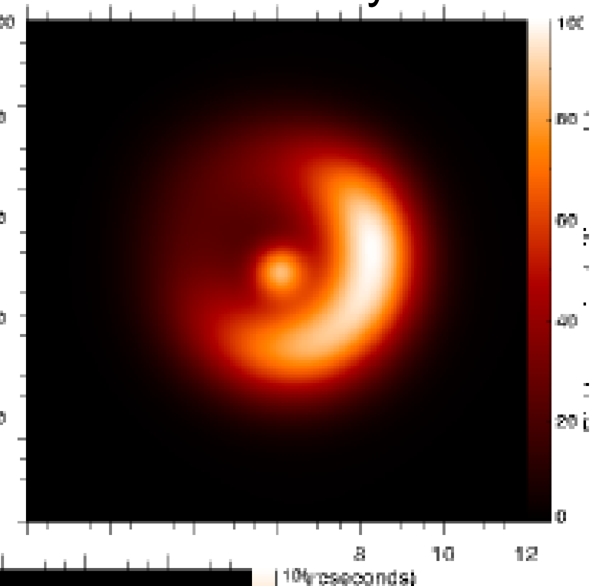
uv



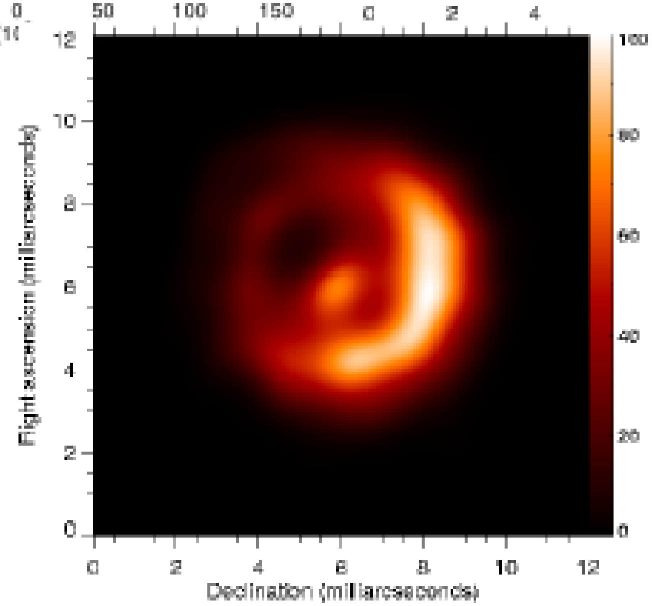
Truth

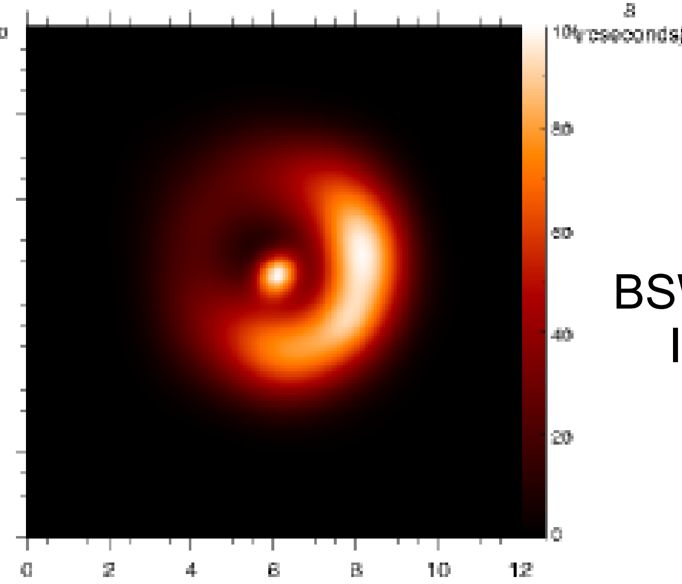


Convolved to array resolution



BSMEM
pixels

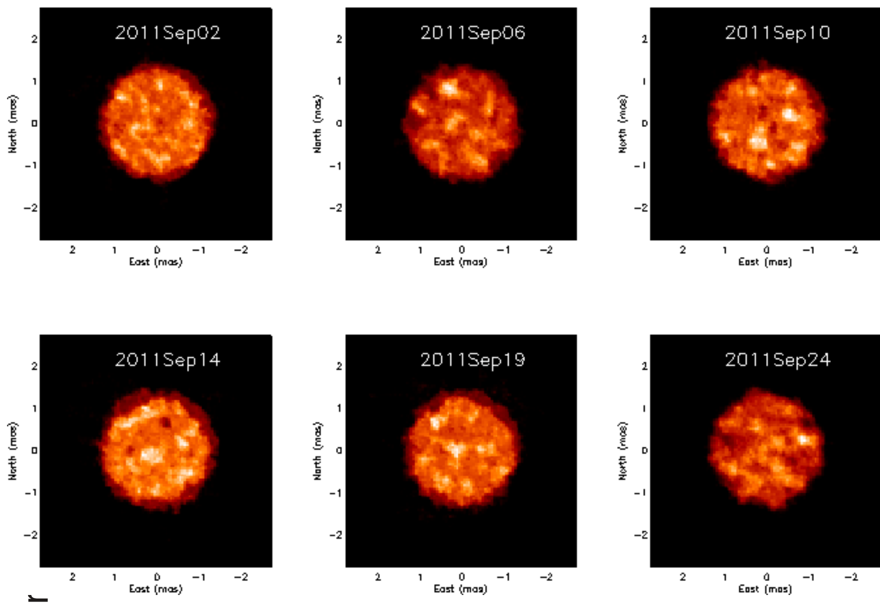




BSWAVE
IUW

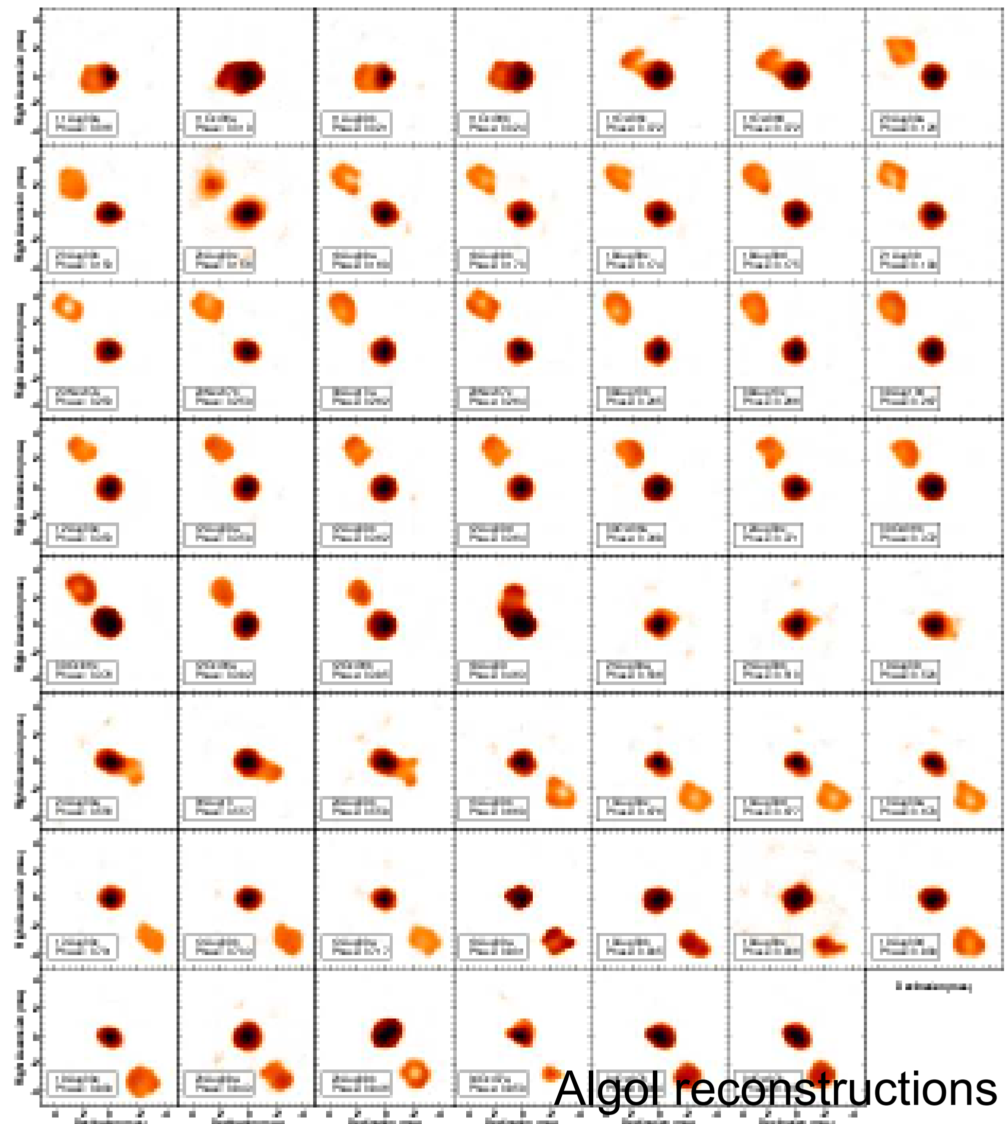
IMAGING ON SPHEROIDS

Issues when imaging stellar surfaces



Lam And reconstructions, Parks, GSU

- Fast movement: data has to be split into time chunks
- Reconstruction of the next chunk does not use knowledge of previous ones
- Constraining the star geometry improves the images



Algol reconstructions
Baron et al., 2012

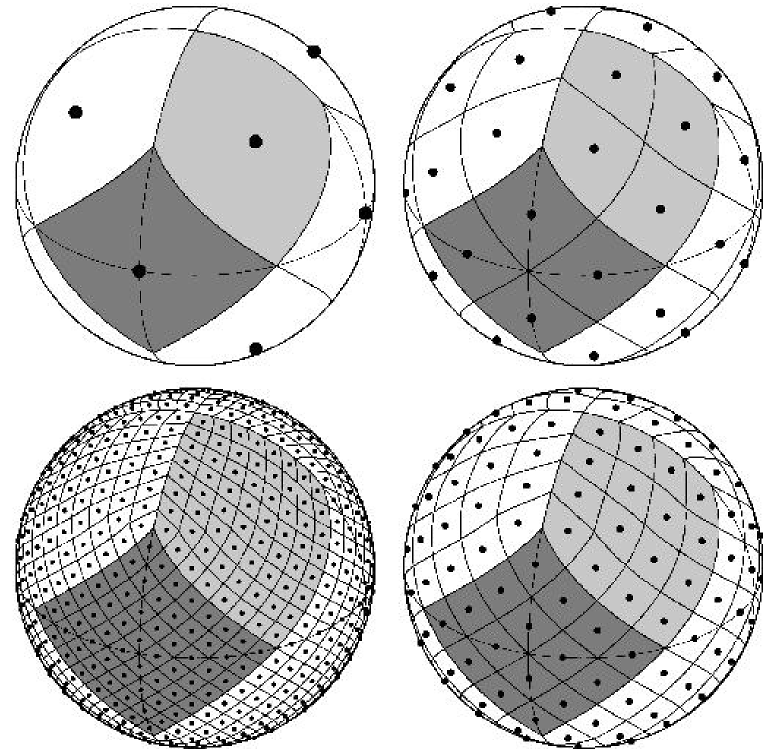
“4D” image reconstruction

- 3D = imaging on spheroids
- + 1D, time dependency
- (+ 1D, wavelength dependency ?)
- Mixes model-fitting and imaging
  - Temporal effects have to be modelled
  - Reconstruction engine has to be flexible and allow simultaneous imaging/model-fitting, e.g. MACIM
- Uses all the data available + derive geometric/dynamic parameters
- $\chi^2(t, \lambda)$ easy to compute if we can generate the image
  - but maybe slow process...
- Regularizers have to be adapted to spheroids
  - easy for UD regularizer

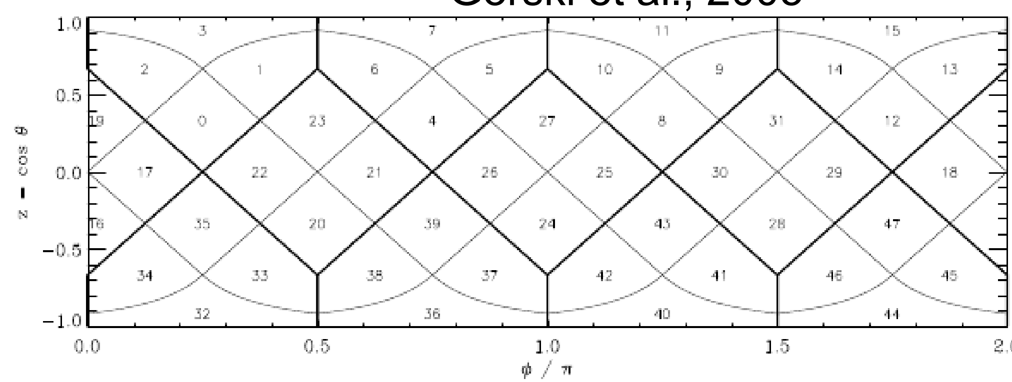
Spheroid parameters

- Pixellation $n \leftrightarrow (\theta, \phi)$
- Healpix equisurface subdivision of the sphere
- $r(\theta, \phi)$: equipotential surface equation (rapid rotators and Roche lobes)

$$V_1 = -G\frac{M_1}{|\boldsymbol{r}_1|} - G\frac{M_2}{|\boldsymbol{r}_2|} - \frac{1}{2}|\boldsymbol{\Omega}_1|^2(X^2 + Y^2) + G\frac{M_2 X}{D^2}$$

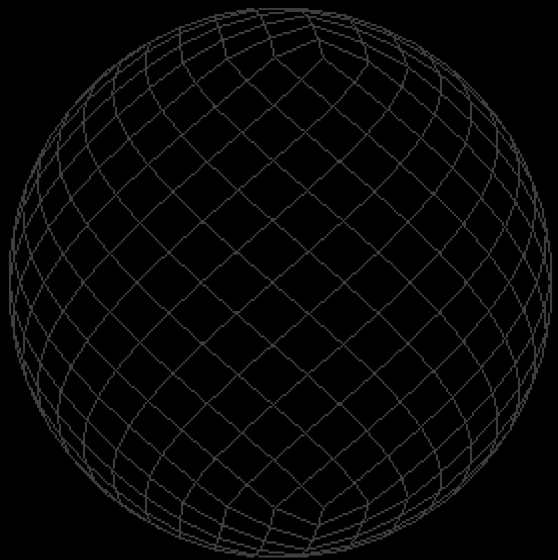


Gorski et al., 2005

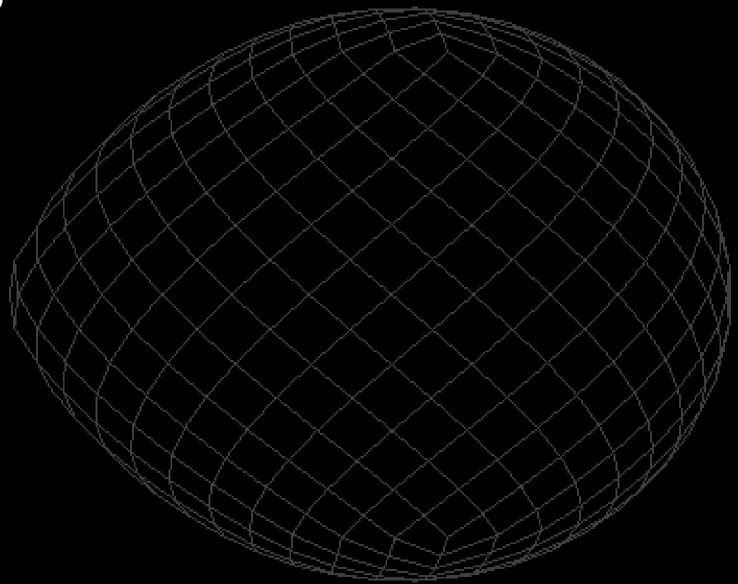


- Temporal parameters
  - speed + axis orientation
  - differential rot. (hard ! TBD)
  - orbital elements

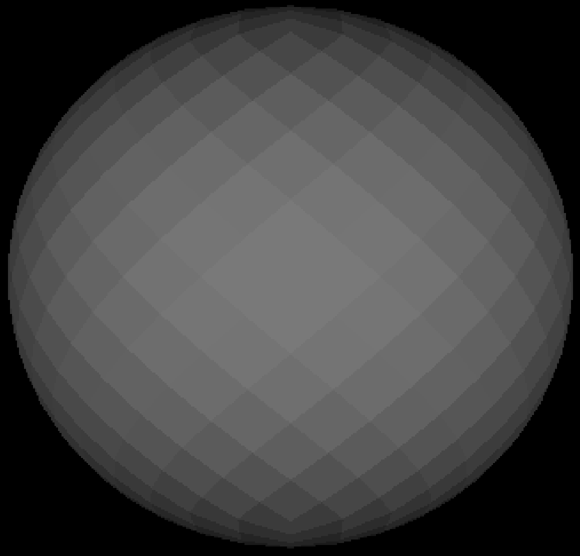
Examples



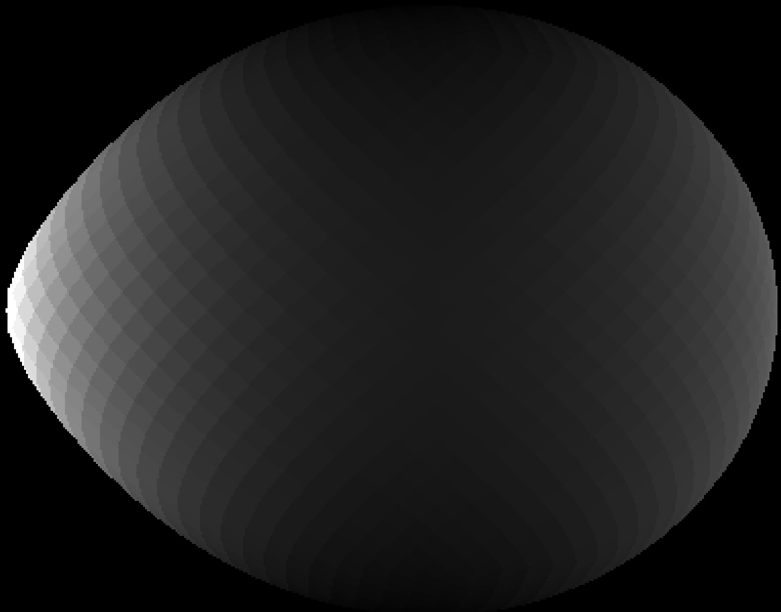
Basic Healpix



Healpix + Roche surface



Texturing, Roche back



Texturing, Roche side, Gravity Darkenin

Fast 3D rendering: OpenGL

- GPU accerated 3D rendering onto the 2D screen
- 3D rendering + FFT not as fast as analytic
  - but very flexible for discs (Eps Aur), YSOs,...
  - easy path for occlusion (binaries)
- OpenGL manages the star geometry, i.e. $r(\theta, \phi)$
  - surface defined by the Healpix/Roche vertices
  - fast translation/rotation operations based on orbits or rotation info
- To each Healpix pixel is associated a temperature (not flux)
  - applied as a texture on top of surface mesh
- Post-processing:
  - blending (opacity)
  - shaders: LDD (see B. Kloppenborg's SIMTOI)

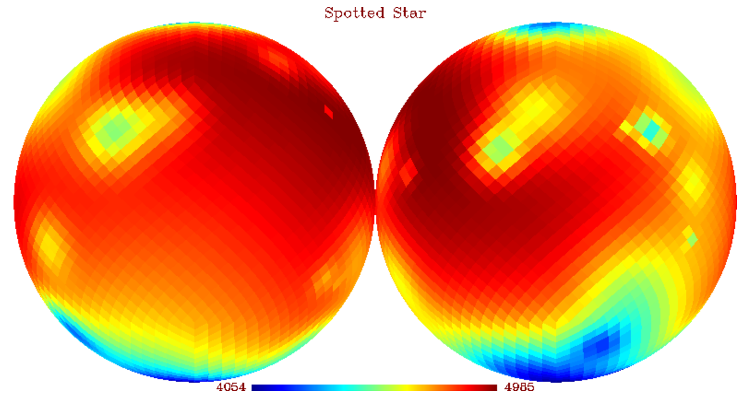
Fast parallel computation: OpenCL

OpenCL

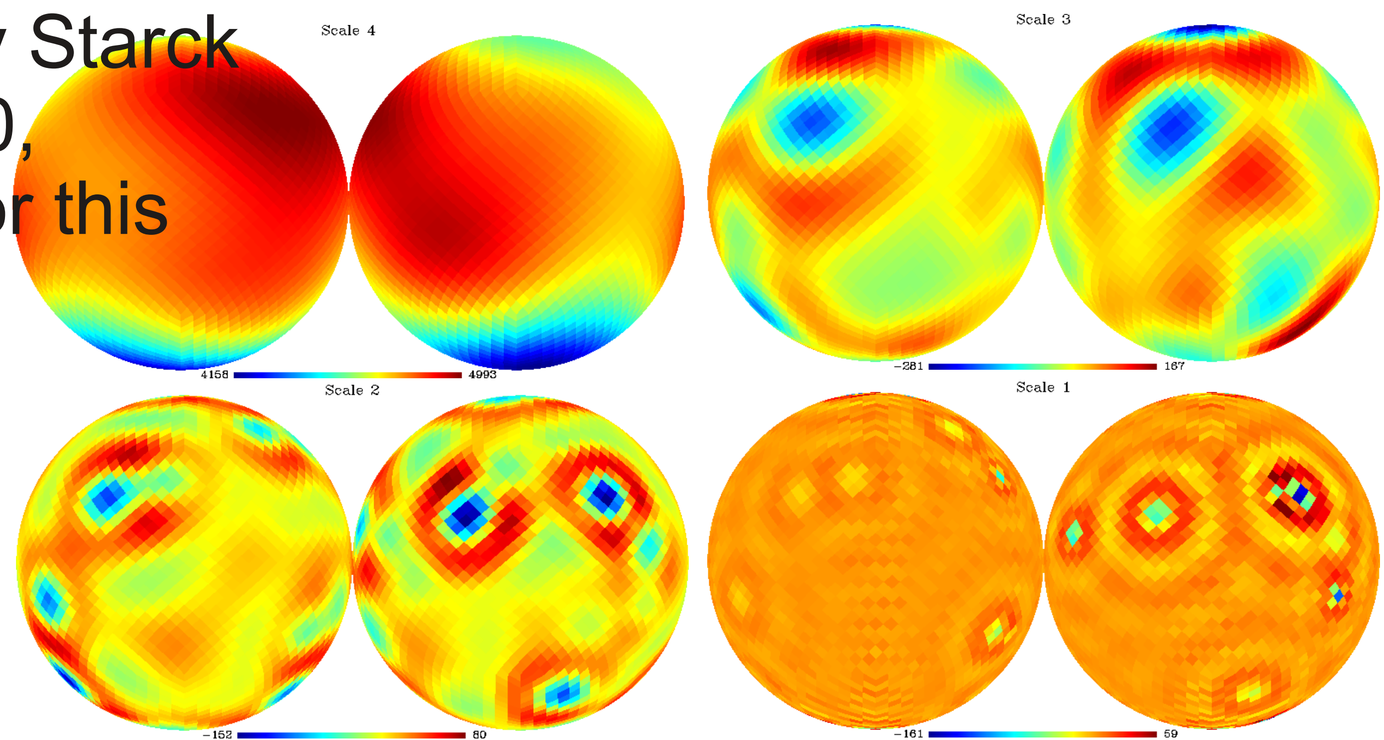
- GPUs have hundreds to thousand cores
- Imaging = massively parallel problem
  - vector/array operations
  - demonstrated with GPAIR (Baron & Kloppenborg, 2010)
  - high performance gain vs CPU code (speed-up > 100)
- OpenGL/OpenCL interoperation mode
  - fast computation of Roche Lobe surface
  - visibility computation via DFT, NFFT algorithm TBD
  - $\chi^2(\lambda, t)$ computations
  - physical computations (e.g. for model-fitting)
    - surface gravity, Doppler map, etc.
  - synergy with Kloppenborg's SIMTOI model-fitting tool
  - light curves as a bonus for 3D+time (total flux)

Compressed Sensing on spheroids Wavelets

- Isotropic undecimated wavelets, CDF 5/3, CDF 9/7 on sphere
- MRS3D by Starck et al., 2010, tweaked for this code



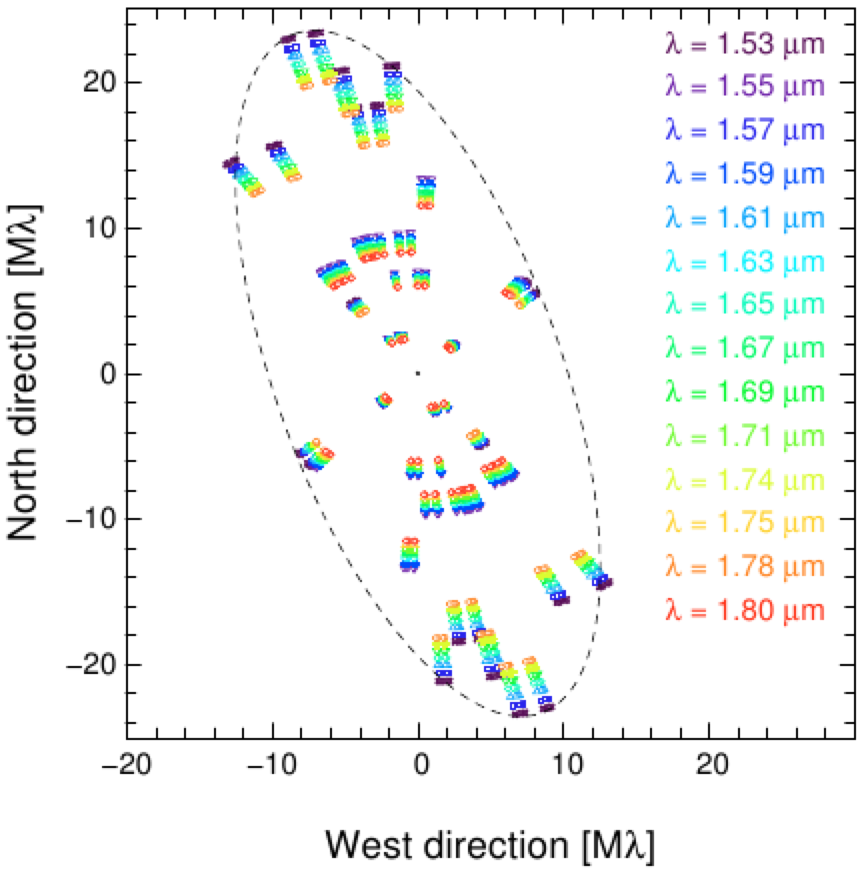
synthetic spotted star



4-scale decomposition on spheric undecimated isotropic wavelets

MULTI-WAVELENGTH RECONSTRUCTIONS

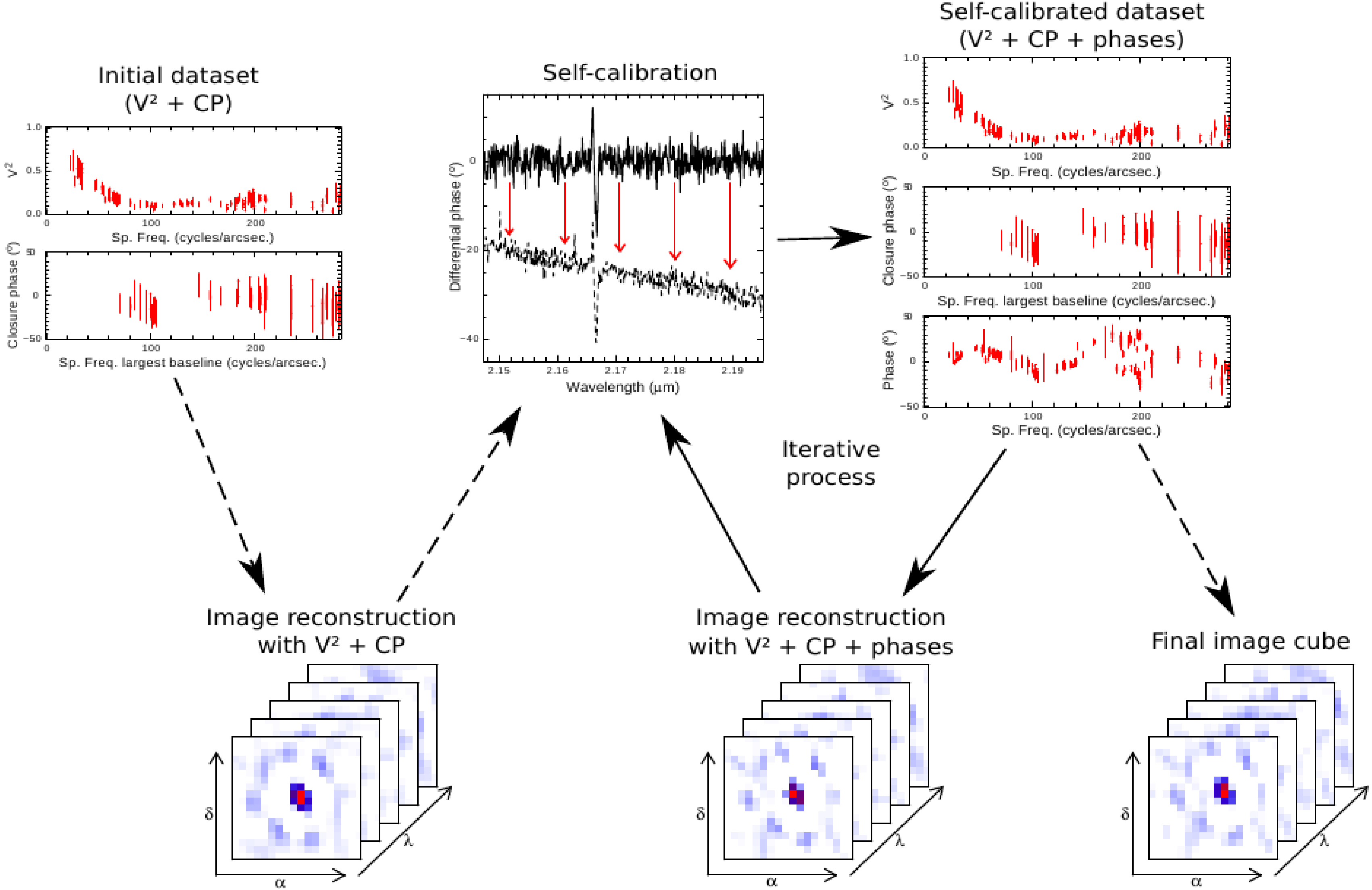
General polychromatic image reconstruction



Arcturus with IOTA, Lacour et al. 2008

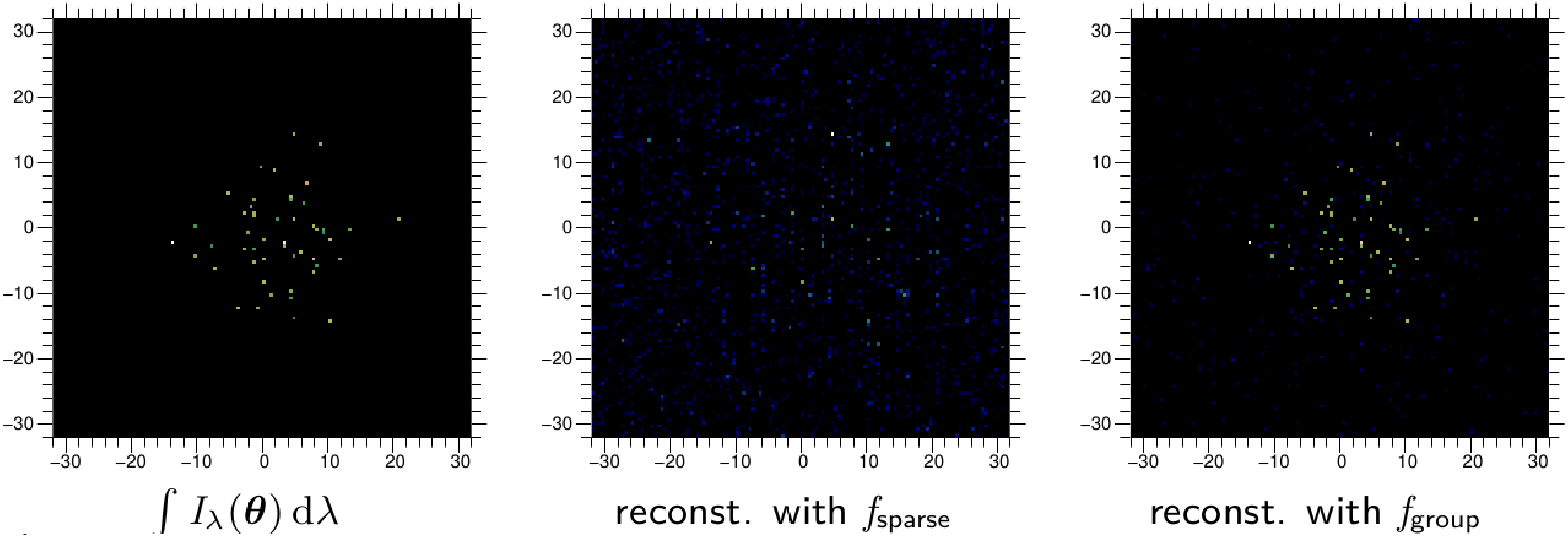
VLTI/AMBER and CHARA/VEGA
Differential phases/visibilities

Self-calibration



Millour et al. 2011, unstable process...

General polychromatic image reconstruction



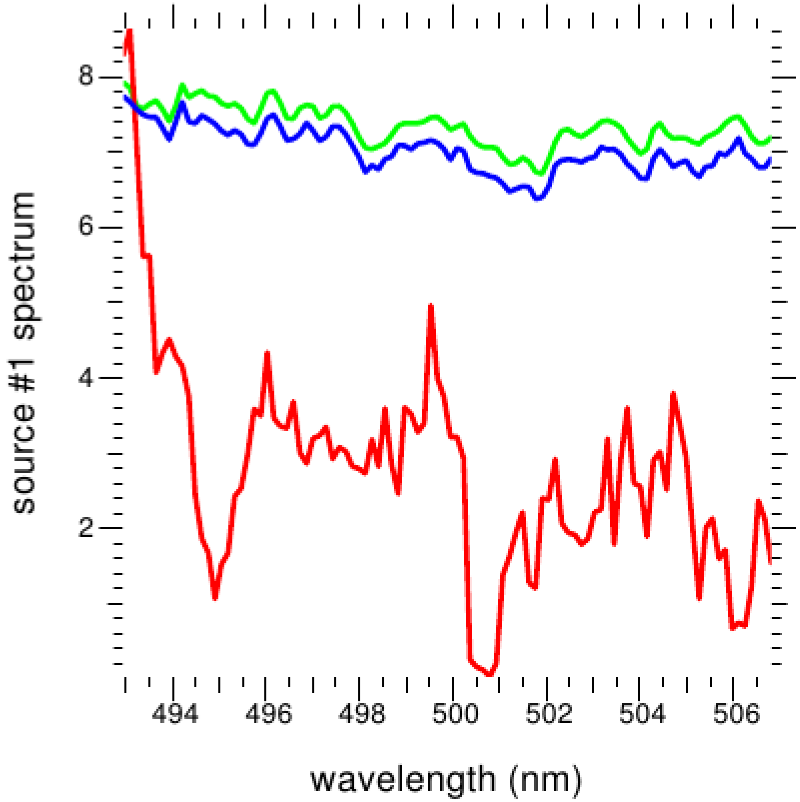
Thiebaut 2012

Fabien B

$$f_{\mathsf{sparse}}(\boldsymbol{x}) = \|\boldsymbol{x}\|_1 \qquad f_{\mathsf{group}}(\boldsymbol{x}) = \sum_n \Big[\sum_\ell x_{n,\ell}^2\Big]^{\frac{1}{2}}$$

Spectral reconstruction

Thiebaut 2012



- in green: true spectrum
- in red: spectrum in 3-D reconstruction with f_{sparse}
- in blue: spectrum in 3-D reconstruction with f_{group}

MODEL SELECTION

Model selection

- How do we select regularizers, regularization weights, compressed sensing basis ?
- Bayesian model selection:

$$\frac{p(\boldsymbol{M_1}|\boldsymbol{D})}{p(\boldsymbol{M}_2|\boldsymbol{D})} = \frac{p(\boldsymbol{M}_1)}{p(\boldsymbol{M}_2)}\frac{p(\boldsymbol{D}|\boldsymbol{M_1})}{p(\boldsymbol{D}|\boldsymbol{M_2})} = \frac{p(\boldsymbol{D}|\boldsymbol{M_1})}{p(\boldsymbol{D}|\boldsymbol{M_2})}$$

- The “evidence” for a model measures its probability

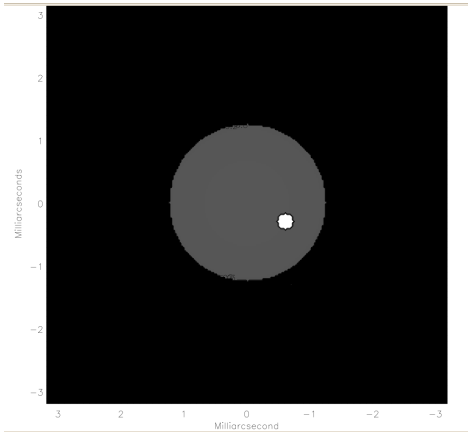
$$\Pr(\boldsymbol{i}|\boldsymbol{D},\boldsymbol{M}) = \frac{\Pr(\boldsymbol{i}|\boldsymbol{M})\Pr(\boldsymbol{D}|\boldsymbol{i},\boldsymbol{M})}{\Pr(\boldsymbol{D}|\boldsymbol{M})}$$

$$p(\boldsymbol{D}|\boldsymbol{M}) = \int p(\boldsymbol{D}|\boldsymbol{i},\boldsymbol{M})p(\boldsymbol{i}|\boldsymbol{M})d\boldsymbol{i}$$

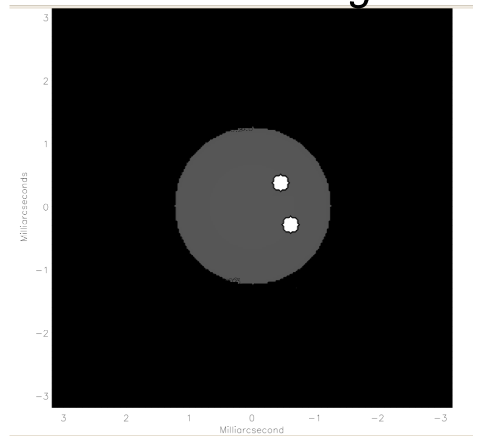
- Integration of posterior probability = difficult numerical problem
  - BSMEM engine
  - Nested Sampling (Skilling 2006)

Application of the evidence framework

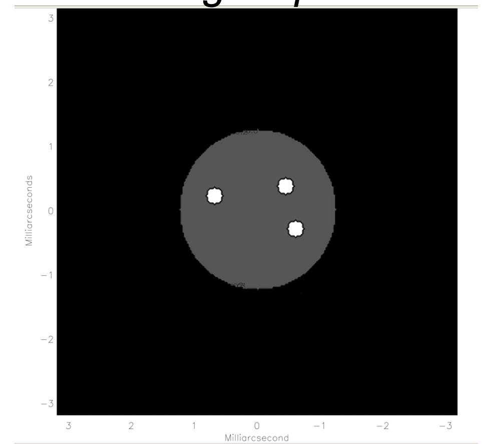
Evidence evaluation for model-fitting of a star with a single spot



$\log P(D|M) = 0.00 \pm 0.12$

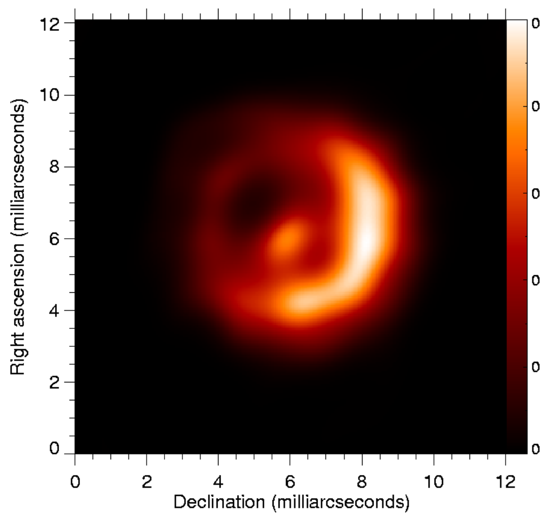


$\log P(D|M) = -2.34 \pm 0.45$



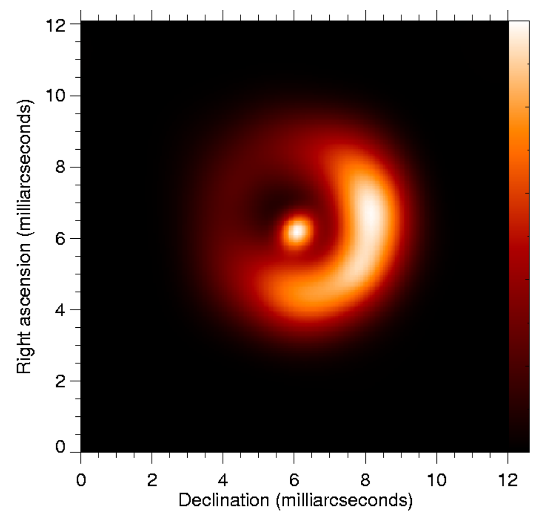
$\log P(D|M) = -15.83 \pm 1.36$

BSMEM



$\log P(D|M) = 8.13 \pm 0.34$

BSWAVE



$\log P(D|M) = 9.63 \pm 0.12$