# Stars in the Spotlight: Mapping Starspots via Light-curve Inversion 

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## THE

# ASTROPHYSICAL JOURNAL 

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

volume xxiv

## JULY I 906

NUMBER I

## ON THE LIGHT-VARIATIONS OF ASTEROIDS AND SATELLITES

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§ I. It has often been suggested that the light-changes of those variable stars which are regularly periodic may be due to the existence of spots on their surfaces, which are hidden or brought into view as the star rotates about its axis. The same explanation has been given more plausibly for the variability shown by certain satellites and asteroids, for in this case the only rival hypothesis is that which ascribes the light-changes to the departure of the body from a spherical form.

It may therefore be worth while to discuss some results of these hypotheses, and consider ( I ) what is the character of the lightcurve produced by the rotation of an arbitrarily spotted body, and (2) how far it is possible to reason backward from such a lightcurve to the spots which produce it.

How long a spot is visible during each rotation depends on its latitude and on $i$.


Comparing light curves for different limb darkening gives information about spot latitudes.




## Surface Partition

Divide the surface into $N$ latitude bands of equal angular widths. Subdivide each band into patches such that the number of patches $M_{n}$ in band $n$ is proportional to the cosine of the latitude, to within the constraint that the number of patches must be an integer. This yields a partition such that all the patches have nearly equal areas.


## Mathematical Formulation

- Assume each patch is uniformly bright across its face. Let $J_{n ; i j}$ be the specific intensity along the outward normal of the $i^{\text {th }}$ patch in the $j^{\text {th }}$ latitude band as seen though the $n^{\text {th }}$ photometric filter.
- Let $t_{n k}$ be the time of the $k^{\text {th }}$ observation through filter $n$. The observed intensity at the photometer at this time is

$$
I_{n k}=\sum_{i=1}^{N_{s}} \sum_{j=1}^{M_{i}} \Omega_{n k ; i j} L_{n k ; i j} J_{n ; i j}
$$

where
$\Omega_{n k ; i j}=$ solid angle subtended by patch $(i, j)$ at time $t_{n k}$
$L_{n k ; i j}=$ limb darkening of patch $(i, j)$ at time $t_{n k}$

- We want to invert the light curve to find the patch intensities, but the problem is ill-posed.
- Below we use a caret ${ }^{\wedge}$ over a symbol to denote a reconstructed quantity


## Input parameters:

- Inclination angle $i$ of rotation axis to line of sight
- Limb darkening model
- $s_{n}=\frac{J_{n ; \text { spot }}}{J_{n ; \text { phot }}}=$ spot-to-photosphere intensity ratio for each filter
- $r_{n}=\frac{J_{n ; \text { phot }}}{J_{1 ; \text { phot }}}=$ ratio of photosphere intensity for filter $n$ to that for filter 1


## Scaling Patch Intensities for Multi-filter Inversions

- In order to simultaneously invert light curves through multiple filters, we need to use a single set of patch intensities to generate all the light curves.
- To do so, we use the linear scaling
$\hat{J}_{n ; i j} \equiv \frac{r_{n}}{1-s_{1}}\left[\left(s_{n}-s_{1}\right) \hat{J}_{1 ; \mathrm{avg}}+\left(1-s_{n}\right) \hat{J}_{1 ; i j}\right]$
where we use $J_{1 ; \text { avg }}$ as a proxy for the photosphere intensity for filter 1 .
- Note that $J_{1 ; i j}=J_{1 ; \text { avg }} \rightarrow J_{n ; i j}=r_{n} J_{1 ; \text { avg }}$ and

$$
J_{1 ; i j}=s_{1} J_{1 ; \mathrm{avg}} \rightarrow J_{n ; i j}=r_{n} s_{n} J_{1 ; \mathrm{avg}} .
$$

## Regularization

- Objective function to be minimized is

$$
E(\hat{\mathbf{J}}, \mathbf{I}, \lambda, B)=G(\hat{\mathbf{J}}, \mathbf{I})+\lambda S(\hat{\mathbf{J}}, B)
$$

where the vector $\mathbf{I}$ is the set of observed intensities, $\mathbf{J}$ is the set of reconstructed patch intensities for filter $1, \lambda$ is the smoothing parameter, and $B$ is the "bias" parameter (described later).

- The goodness-of-fit measure is
$G(\hat{\mathbf{J}}, \mathbf{I})=\frac{\left(2.5 \log _{10} e\right)^{2}}{P} \sum_{n=1}^{Q} \frac{1}{\sigma_{n}^{2}} \sum_{k=1}^{P_{n}}\left(\frac{I_{n k}-\hat{I}_{n k}}{I_{n k}}\right)^{2}$
where $Q$ is the number of filters, $\sigma_{n}^{2}$ is the estimated variance in magnitudes of the noise for filter $n, P_{n}$ is the number of points in the light curve for filter $n$, $P$ is the total number of points in all the filter light curves, and the $\hat{I}_{n k}$ are the intensities of the reconstructed light curves implied by $\mathbf{J}$.


## Regularization

- The prior is that the surface contains dark spots on a uniformly bright photosphere.
- The penalty function used to enforce this is

$$
S(\hat{\mathbf{J}}, \boldsymbol{B})=\sum_{i=1}^{N} \sum_{j=1}^{M_{i}} c_{i j}\left(\hat{J}_{i j}-\hat{J}_{\mathrm{avg}}\right)^{2}
$$

where $B$ is the "bias parameter," and $c_{i j}=1$ if $\hat{J}_{i j} \leqslant \hat{J}_{\text {avg }}$, while $c_{i j}=B$ if $\hat{J}_{i j}>\hat{J}_{\text {avg }}$.

- If $B>1$, the penalty for a patch being brighter than average is $B$ times larger than for being dimmer than average by the same amount.
- In this way we bias the distribution of patch intensities so that most are just slightly brighter than average (the "photosphere") while a small number are dimmer than average so as to fit the light curve.


## Determination of $\boldsymbol{\lambda}$



Fig. 2.- $V$ light-curve inversions for a pair of circular spots of radius $20^{\circ}$ for a polar inclination to the line of sight of $45^{\circ}$. The left-hand image in each pair is the true surface; the right-hand image is the MLI reconstruction. The ratio of the estimated noise in the light curve expressed in rms magnitudes to the true noise decreases down each column. On the left-hand side, the values are $1,0.98$, and 0.96 ; on the right-hand side, the values are $0.94,0.92$, and 0.90 .


## $V$-filter inversions for simulated surfaces



## BVRI-filter inversions for simulated surfaces

# A STUDY OF DIFFERENTIAL ROTATION ON II PEGASI VIA PHOTOMETRIC STARSPOT IMAGING 

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#### Abstract

We present the results of a study of differential rotation on the K2 IV primary of the RS CVn binary II Pegasi (HD 224085) performed by inverting light curves to produce images of the dark starspots on its surface. The data were obtained in the standard Johnson $B$ and $V$ filter passbands via the Tennessee State University T3 0.4 m Automated Photometric Telescope from JD 2447115.8086-2455222.6238 (1987 November 16-2010 January 26). The observations were subdivided into 79 data sets consisting of pairs of $B$ and $V$ light curves, which were then inverted using a constrained nonlinear inversion algorithm that makes no a priori assumptions regarding the number of spots or their shapes. The resulting surface images were then assigned to 24 groups corresponding to time intervals over which we could observe the evolution of a given group of spots (except for three groups consisting of single data sets). Of these 24 groups, six showed convincing evidence of differential rotation over time intervals of several months. For the others, the spot configuration was such that differential rotation was neither exhibited nor contraindicated. The differential rotation we infer is in the same sense as that on the Sun: lower latitudes have shorter rotation periods. From plots of the range in longitude spanned by the spotted regions versus time, we obtain estimates of the differential rotation coefficient $k$ defined as in earlier work by Henry et al. and show that our results for its value are consistent with the value obtained therein.


Key words: binaries: close - stars: activity - stars: imaging - stars: individual (II Pegasi) - starspots - stars: variables: general
Online-only material: figure set

The T3 Automated Photometric Telescope


The B and V Light Curves of II Pegasi, 1987 Nov 16 - 2010 Jan 26

$B$ Light Curve for Data Set 3

$B$ Light Curve for Data Set 4

$B$ Light Curve for Data Set 5





Group 2: Data Sets 3 - 6


Fig. 3.- Inversions of the four data sets assigned to Group 2, acquired between heliocentric MJD 47417.7681-47556.5813 (1988 September 11 - 1989 January 30) The assumed spot temperature is $T_{\text {spot }}=3500 \mathrm{~K}$. The top row is for an assumed inclination angle between the rotation axis and the line of sight of $\alpha=45^{\circ}$, while the bottom row is for $\alpha=60^{\circ}$.


Fig. 4.- Plots of the span in longitude $\Delta \phi$ of the active regions shown in the inversions in Figure 3 versus the heliocentric MJD of the midpoint of the time spanned by each data set, along with the least-squares best-fit line. The method used to determine $\Delta \phi$ is detailed in the text. In this figure and in similar figures which follow it, the slopes of the best fit lines expressed in units of degrees per day are indicated on the plots.

$$
\begin{aligned}
& \Omega(\theta)=\Omega_{\mathrm{eq}}\left(1-k \sin ^{2} \theta\right) \\
& k=\frac{\Delta \Omega}{\Omega_{\mathrm{eq}}\left(\sin ^{2} \theta_{1}-\sin ^{2} \theta_{2}\right)}
\end{aligned}
$$

Table 2. Differential Rotation Coefficient, $k$

| Group | $\Delta \Omega\left(\operatorname{deg~d}^{-1}\right)$ |  | $\theta=\left(0^{\circ}, 90^{\circ}\right)$ |  | $\theta=\left(45^{\circ}, 80^{\circ}\right)$ |  | $\theta=\left(48.2^{\circ}, 71.7^{\circ}\right)^{\mathrm{a}}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $i=45^{\circ}$ | $i=60^{\circ}$ | $i=45^{\circ}$ | $i=60^{\circ}$ | $i=45^{\circ}$ | $i=60^{\circ}$ | $i=45^{\circ}$ | $i=60^{\circ}$ |
| 2 | 0.34 | 0.19 | 0.0063 | 0.0034 | 0.013 | 0.0072 | 0.018 | 0.0098 |
| 3 | 0.28 | 0.15 | 0.0051 | 0.0027 | 0.011 | 0.0058 | 0.015 | 0.0079 |
| $11^{\text {b }}$ | 0.24 | 0.21 | 0.0044 | 0.0039 | 0.009 | 0.0084 | 0.013 | 0.011 |
| 12 | 0.36 | 0.31 | 0.0066 | 0.0057 | 0.014 | 0.012 | 0.019 | 0.017 |
| 13 | 0.39 | 0.29 | 0.0072 | 0.0052 | 0.015 | 0.011 | 0.021 | 0.015 |
| 14 | 0.42 | 0.42 | 0.0076 | 0.0076 | 0.016 | 0.016 | 0.022 | 0.022 |
|  |  | Mean | 0.0062 | 0.0048 | 0.013 | 0.010 | 0.018 | 0.014 |
|  |  | St. Dev. | 0.0012 | 0.0018 | 0.003 | 0.004 | 0.004 | 0.005 |

${ }^{\text {a }}$ Values taken from Siwak et al. (2010).
${ }^{\mathrm{b}}$ Data Sets 31-33 excluded.

