The Foundations Catastrophe

The Beginnings

The origins in Analysis

1687 Newton's *Principles*

1684-99 Leibniz

The upshot

1734 Bishop Berkeley: *The Analyst, Or a Discourse Addressed to An Infidel Mathematician* (Edmund Halley)

1784 Competition of Berlin Academy of Sciences, requesting "a clear and precise theory of what is called 'infinite' in mathematics."

Acceptance of novelties: Negative numbers, Imaginary numbers, Complex numbers, Quaternions

Non-Euclidean Geometry: Gauss, Bólyai, Lobachevsky (hyperbolic, >1 parallel) Riemann (spherical, no parallels)

1829 Cauchy's Convergence Criterion (ε-δ) puts analysis on a firm axiomatic foundat

1872 Weierstrass publishes example showing that Continuity and Differentiability are not identical properties of a function

1888 Dedekind establishes Axiomatization of Rational Numbers

1889 Peano publishes Axioms for the Integers

The Debacle

1900 Hilbert's 23 problems (<u>Formalism</u>), including:

The Axiom of Choice

The Continuum Hypothesis

10th problem (Diophantine equations) → the Halting Problem

1893 (vol 1; 1903 vol 2) Frege, *Grundzüge der Arithmetik* (<u>Set Theory</u>)

1902 Russell's Paradox → Frege

1910 Russell & Whitehead, *Principia Mathematica* (<u>Logicism</u>)

1931 Godel's first Proof: "On Formally Undecidable Sentences in *Principia Mathematica* and Related Systems, I"; two parts:

(1) any axiomatic theory at least adequate to account for the integers is incomplete.

(2) the consistency of any such theory cannot be proven by logical or set-theoretic means.

1933 Löwenheim-Skolem theorem shows that one can always find interpretations of axiomatic systems for infinite number systems that vary cardinality; i.e, axioms do not provide enough information in themselves to characterize all of a number system.

1936 Church proves that the Halting Problem is insoluble.

1940 Godel's second Proof: "The Consistency of the Axiom of Choice and the Generalized Continuum Hypothesis with the Axioms of Set Theory" - adding either hypothesis as an axiom to Set Theory provides no contradiction; i.e, they are **consistent** with Set Theory, and thus can't be disproved.

1963 Cohen proves, to everyone's surprise, that adding the **negation** of either hypothesis as an axiom to Set Theory also provides no contradiction; i.e, they are **independent** of Set Theory, and thus can't be proved, either.