# Quiz Solutions

(1) $\lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{1+\cos 2x} = \frac{1-1}{1+-1} = \frac{0}{0}$

**Invoke L'Hospital**

$\lim_{x\to\frac{\pi}{2}} \frac{-\cos x}{-2\sin 2x} = \frac{0}{0}$

**Invoke L'Hospital**

$\lim_{x\to\frac{\pi}{2}} \frac{\sin x}{-4\cos 2x} = \frac{1}{4}$

2 cycles got you 2 points if you substituted incorrectly.

**(2) (b) is correct. (b) uses direct substitution to evaluate the limit while (a) uses L'Hospital's rule, which cannot be used unless the limit is indeterminate of form $\frac{0}{0}$ or $\frac{\infty}{\infty}$.**

**(3) Given:**

$\frac{dV}{dt} = 36$

**Find $\frac{dA}{dt}$ when the edge of the cube is 2. One possible solution is to let $s$ be the side length.**

**So, $V = s^3$ and taking the derivative with respect to time we reach $\frac{dV}{dt} = 3s^2\frac{ds}{dt}$.**

**Using our givens $\frac{dV}{dt} = 36$ and $s = 2$ we obtain $\frac{ds}{dt} = 3$.**

**Now are able to determine $\frac{dA}{dt}$. $A = s^2$**

**$\frac{dA}{dt} = 2s\frac{ds}{st}$ and substituting our givens we find $\frac{dA}{dt} = 12$. Units of measure would be square inches per second. Some of you did some nice things in solving this a different way.**

**(4) (a) To find the <u>absolute maximum</u> of a function, take the derivative of the function, set it equal to zero and then solve. Once you have these points, create a sign chart to determine which points are local maximums and minimums by observing where the derivative changes sign. Finally, place these critical values into the original function to determine which has the greatest value.**

$y = xe^{-kx}$

$y' = e^{-kx} - kxe^{-kx} = e^{-kx}(1-kx)$

**(b)** $0 = (1-kx)$

$x = \frac{1}{k}$

$f'(x)$ ←+++++—— $\frac{1}{k}$ ——-----→

**I needed to see you verify it was a maximum value. You know it must be since k is a fixed <u>positive</u> number so any value of x greater than 1 / k would give you something that would be greater than 1, making the derivative negative.**

**Plugging back into f(x) we obtain coordinates $\left(\frac{1}{k}, \frac{1}{ke}\right)$.**

**(5)**

$f(x) = (2x+1)^5(x^2-4)^3$

$f'(x) = 5(2)(2x+1)^4(x^2-4)^3 + 3(2x)(2x+1)^5(x^2-4)^2$

**Don't forget the chain rule!!!**

**Now factor so that there are no sums left…**

$f'(x) = 10(2x+1)^4(x^2-4)^3 + 6x(2x+1)^5(x^2-4)^2$

$f'(x) = 2(2x+1)^4(x^2-4)^2\left[5(x^2-4) + 3x(2x+1)\right]$

$f'(x) = 2(2x+1)^4(x^2-4)^2(11x^2+3x-20)$

**This can now be solved for zero quickly.**

# Quiz Solutions

**(6) You two numbers could have been defined as $x$ and $y = 20 - x$.**

$$x^2 - (20-x)^2 = S(x)$$
$$S(x) = x^2 - 40x + 40 + x^2 = 2x^2 - 40x + 40$$
$$S'(x) = 4x - 40$$
$$x = 10$$
$$y = 10$$

**BONUS**

**Regardless of how you orientated your triangle, the hypotenuse was $\sqrt{3}$. Use two other variables two identify the other two sides ( I used $x$ and $y$.) By the Pythagorean Theorem $x^2 + y^2 = 3$. Eliminate one variable (doesn't matter which) so everything is in terms of a single variable. For example:**

$$y = \sqrt{3 - x^2}$$

**Now stand up your right triangle so that it is the cross section of a cone. The height is one of your sides and the radius is the other. Substitute the appropriate values into $V = \frac{1}{3}\pi r^2 h$.**

$$V(x) = \frac{1}{3}\pi\left(\sqrt{3-x^2}\right)^2 x = \frac{\pi}{3}\left(3x - x^3\right)$$
$$V'(x) = \frac{\pi}{3}\left(3 - 3x^2\right)$$
$$x = \pm 1$$

**Reject negative one. This makes the other side $\sqrt{2}$ and the volume $\frac{2}{3}\pi$.**