Problem Set 3
Physics 520

Fall 2005
L. Sander
10/4/05 due 10/13/05

1. a.) In a fog water droplets have a radius of about 1μm. Water at 20°C has a surface tension of 72.8 dynes/cm. What is the pressure inside the droplets in atmospheres? b.) A small Ge crystal has a radius of 10 Angstroms. It is in contact with its melt. The interfacial tension is = 250 erg/cm^2, the entropy of melting is 30:55 J/K mol, and v_c = 2.3 10^{23} cm^3. Find the depression of the normal melting point (1200K).

2. Consider a Kossel crystal with nearest neighbor interaction ε_1 and next nearest neighbor interaction ε_2 and lattice constant a. Show that:

$$\gamma_{100} = (\varepsilon_1 + 4\varepsilon_2)/(2a^2) \qquad \gamma_{110} = (n\varepsilon_1 + m\varepsilon_2)/(2\sqrt{2}a^2)$$

Here n, m are integers which you must find.

3. Another form of the Wulff construction is available if the crystal has only facetted faces. Suppose the facets are labeled 1, 2., ..n. The surface tensions are γ_n, and the distance of the facet from the center of mass of the crystal (the Wulff point) is h_n. Show that γ_n / h_n is the same for each facet – that is low energy facets are close to the origin.

Hints;
The crystal may be considered to be made up of pyramids whose base is the facet and whose altitude is h_n. Then the total volume of the crystal is $V = \Sigma_n h_n S_n/3$, where S_n is the area of the nth facet.
Use a work argument like the one I used in class: $(P_{in} – P_{out})\, dV = \Sigma_n \gamma_n dS_n$. The volume of the crystal changes by $dV = \Sigma_n dh_n S_n$. Now use the fact that the volume is the sum of pyramids to get an expression for dV in terms of dS_n.

4. Consider a step which wanders thermally by detaching and attaching atoms.
a). Write the shape of the step as

$$x(y) = \Sigma_k x_k e^{iky}.$$

What k's are allowed if we use periodic boundary conditions for length L? Prove that

$$\langle|x_k|^2\rangle = k_BT/(L\beta k^2)$$

where β is the step energy /unit length.
b.) Show that $\langle x^2\rangle = \Sigma_k \langle|x_k|^2\rangle = Lk_BT/(12\beta)$

Hints; The energy can be written in a coarse-grained way as

$$E= (\text{energy/unit length}) \bullet (\text{arclength}) = \int \beta\, (1+[dx/dy]^2)^{1/2}\, dy.$$

Expand the square root.. If you prefer the more physical case of a step with fixed ends L apart, then for part (b) you should average over y.For (b) also note that the sum cannot be converted to an integral. The sum

$$\Sigma_n(1/n^2) = \pi^2/6$$

may be useful.