Problem Set 4
Physics 520

Fall 2005
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10/13/05 due 10/20/05

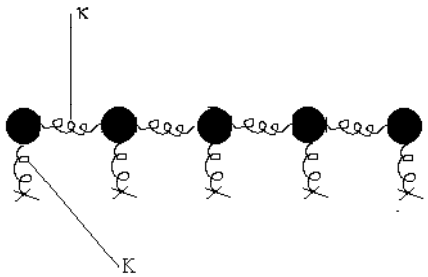
1. Consider a linear chain of atoms of alternating mass interacting via the Lennard-Jones nearest neighbor potential.
a.) Solve for the dispersion relation and exhibit the optical modes as in class. b.) Show that at long wavelength the optical modes correspond to atoms in the unit cell moving opposite to one another, but that for the acoustic modes they move together.

2, In this problem you will consider elastic scattering of thermal neutrons. a.) Model the potential between neutrons and nuclei by $v(\mathbf{r}) = 2\pi h^2 b\delta(\mathbf{r})/M$, where b is a constant (called the *scattering length)*. Show that b^2 is the cross-section for elastic scattering from a single nucleus.
b.) Consider a crystal with more than one isotope so that the b's vary randomly. Show that, in the rigid crystal approximation (no thermal vibration, and elastic scattering):

$$d^2\sigma/d\Omega = [\, N^2 |\langle b\rangle|^2 \Sigma_G \delta_{\mathbf{q},\mathbf{G}} + N\{\langle b^2\rangle - |\langle b\rangle|^2\}\,]$$

Sketch as a function of angle what the two terms do. Here <b> = average over isotopes, and $\langle b^2\rangle$ = average square of b.

3. Consider a linear chain of atoms interacting with springs of spring constant κ but also with *fixed* lattice points with lattice constant **a**, with another spring constant K.



Find and plot the dispersion relation. You should find that ω_k does not go to zero as k goes to 0. Explain.

4. a.) Show that the low-T lattice specific heat of a d-dimensional crystal behaves as T^d. Is this result exact in the harmonic theory, or does it depend on the Debye approximation? b.) Magnons are excitations of magnetic crystals with dispersion relation $\omega = Dk^2$. Show that for low T the magnon heat capacity goes as $T^{d/2}$ in d-dimensions.