

Problem Set 5 Physics 520

Fall 2005
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11/08/05 due 11/15/05

1.) a.) Calculate the density of states for electrons:

$$D(\epsilon) = 2 \sum_{\mathbf{k}} \delta(\epsilon - \epsilon_{\mathbf{k}}), \quad \epsilon_{\mathbf{k}} = \hbar^2 k^2 / 2m$$

b.) Find $D(\epsilon)$ for the 1 and 2 dimensional electron gases.

2.) In statistical mechanics you learned about the Fermi distribution function:

$$f(\epsilon) = 1 / (\exp(\beta(\epsilon - \mu)) + 1).$$

Here, μ is the chemical potential which is determined by the equation:

$$N = 2 \sum_{\mathbf{k}} f(\epsilon_{\mathbf{k}}).$$

Figure out an exact expression for $\mu(T)$ in 2 dimensions. Make a rough plot of μ as a function of T in 2d.

3.) a.) For electrons in Na estimate E_F and k_F . You will have to look something up.

b.) Write down formulas for E_F and k_F for highly relativistic electrons for which $\epsilon = pc$.

4.) Check the numerical factors and the order of the operators in the expression I wrote down for the interaction energy. Do the following test: set $V(r_1, r_2) = 1$. Then the contribution to the Hamiltonian is $N(N-1)/2$ (tell me why). Now show that the second quantized version gives

$$V_2 = (1/2) (N^2 - N)$$

where $N = \sum_j c_j^\dagger c_j$, is the operator for the total number. Then show that

$$V_2 |A\rangle = [N(N-1)/2] |A\rangle.$$