

Physics 520 Problem Set 8

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Fall 2005. 12/06/05 due 12/13/05

1. a.) Work out the commutation rules for the operators γ and $\bar{\gamma}$, where

$$\begin{aligned} \gamma_k &= u_k c_{k\uparrow} - v_k c_{-k\downarrow}^{\dagger} & c_{k\uparrow} &= u_k^* \gamma_k + v_k \bar{\gamma}_k^{\dagger} \\ \bar{\gamma}_k^{\dagger} &= v_k^* c_{k\uparrow}^{\dagger} + u_k c_{-k\downarrow} & c_{-k\downarrow}^{\dagger} &= -v_k^* \gamma_k + u_k \bar{\gamma}_k^{\dagger} \end{aligned}$$

Show under what conditions they are fermions.

b.) The pair creation operator, $P^{\dagger} = c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger}$ and its adjoint make pairs of fermions. Are they boson operators? Why, or why not?

c.) Show that the average number of particles in the BCS state is:

$$\langle N \rangle = \sum_k 2v_k^2$$

where N is the number operator.

(Hint: You could express

$$N = \sum \left[c_{k\uparrow}^{\dagger} c_{k\uparrow} + c_{-k\downarrow}^{\dagger} c_{-k\downarrow} \right]$$

in terms of the γ operators and use the fact that γ_k or $\bar{\gamma}_k$ give zero when applied to the ground state. Or you could look directly at the BCS wavefunction.)

2. Consider a flat superconducting slab of thickness d in a magnetic field. Set $h=H_a$ at $x=\pm d/2$. Show that the field at $x=0$ is

$$h=H_a/\cosh(d/2\lambda).$$