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Angular Correlations among the Leptons in Polarized $ZZ \to 4l$ Decays

Edward Timko[1]

[1] *Department of Physics, University of Dayton, Dayton, OH 45469*
Michigan Center for Theoretical Physics (MCTP)
Department of Physics, University of Michigan, Ann Arbor, MI 48109

Abstract

Angular correlations arising from $X \to ZZ \to 4l$ decays are investigated, where X is a general spin resonance. It is shown that a judiciously chosen function of the angles among the final four leptons can help distinguish the spin nature of the present resonance.

1 Introduction

One of the most important signals of a heavy Higgs boson is $gg \to H \to ZZ \to 4l$. This is the so-called gold-plated mode for Higgs discovery. However, the Higgs boson is not the only new way that four leptons could be produced at the LHC. There is also the possibility of an exotic resonant state that subsequently decays to $ZZ \to 4l$.

In this paper, we shall be concerned with a resonant state decaying into ZZ and then decaying into $4l$. We will call this resonant state X, which could be the Higgs boson or it could be a new vector boson, or even a spin-two particle (graviton KK state, for example). The Z bosons in the decay will be polarized differently depending on the nature of the originating resonance. For example, one may have

$$\begin{aligned} H &\to Z_S Z_S, Z_L Z_L, Z_R Z_R \\ Z' &\to Z_S Z_L, Z_L Z_R \\ G &\to Z_L Z_R \end{aligned}$$

where $Z_{L/R}$ are left and right polarized Z's, and Z_S is the longitudinal (or scalar) polarized Z. Depending on the polarization of the Z's, one can have different angular corrections among the leptons that are produced in their decays.

The main task of this note is to define observables that can distinguish between various polarized states of the Z bosons via opening angles among the four final-state leptons. These observables will be deined in the rest frame of the resonance, since the rest frame can be reconstructed from the four letons. We shall look at all permutations of Z pair polarizations. Other groups, [1] and [2], have complementary studies of lepton angular correlations in resonant decays through Z bosons.

2 $Z \to l\bar{l}$ Decay Distributions

2.1 Origin of the Distributions

The invariant amplitudes for $Z \to l\bar{l}$ decay are given in [3] by

$$\begin{aligned} |\mathfrak{M}_R|^2 &= \tfrac{g^2 M_Z^2}{2\cos\theta_W}[(C_R^2 + C_L^2)(1+\cos^2\theta) + 2(C_R^2 - C_L^2)\cos\theta] \\ |\mathfrak{M}_L|^2 &= \tfrac{g^2 M_Z^2}{2\cos\theta_W}[(C_R^2 + C_L^2)(1+\cos^2\theta) - 2(C_R^2 - C_L^2)\cos\theta] \\ |\mathfrak{M}_S|^2 &= \tfrac{g^2 M_Z^2}{\cos\theta_W}(C_R^2 + C_L^2)\sin^2\theta \end{aligned} \quad (1)$$

where θ is taken to be the angle between the lepton of the $l\bar{l}$ pair and the path of the incident Z. The angular distribution of decays is related to the invariant amplitude by

$$\frac{dN}{d\Omega} \propto |\mathfrak{M}|^2 \tag{2}$$

where $\frac{dN}{d\Omega}$ is the distribution of the lepton over the solid angle Ω. Since the invariant amplitudes given in (1) contain no azimuthal terms, we will average over the azimuthal angle ϕ to yield

$$\frac{dN}{d\theta} \propto |\mathfrak{M}|^2 \sin\theta \tag{3}$$

Thus, after normalizing the distributions, we find that

$$\begin{aligned} \frac{dN_{R/L}}{d\theta} &= \frac{3}{8}\left[1+\cos^2\theta \pm 2\frac{C_R^2-C_L^2}{C_R^2+C_L^2}\cos\theta\right]\sin\theta \\ \frac{dN_S}{d\theta} &= \frac{3}{4}\sin^3\theta \end{aligned} \tag{4}$$

where $\frac{C_R^2-C_L^2}{C_R^2+C_L^2} \approx -0.5193$. The R distribution takes on the $+$ sign while the L distribution takes on the $-$ sign. Plots of these distributions are shown in Figure 1. The mean, standard deviation, and fractional deviation of these distributions are given in Table 1.

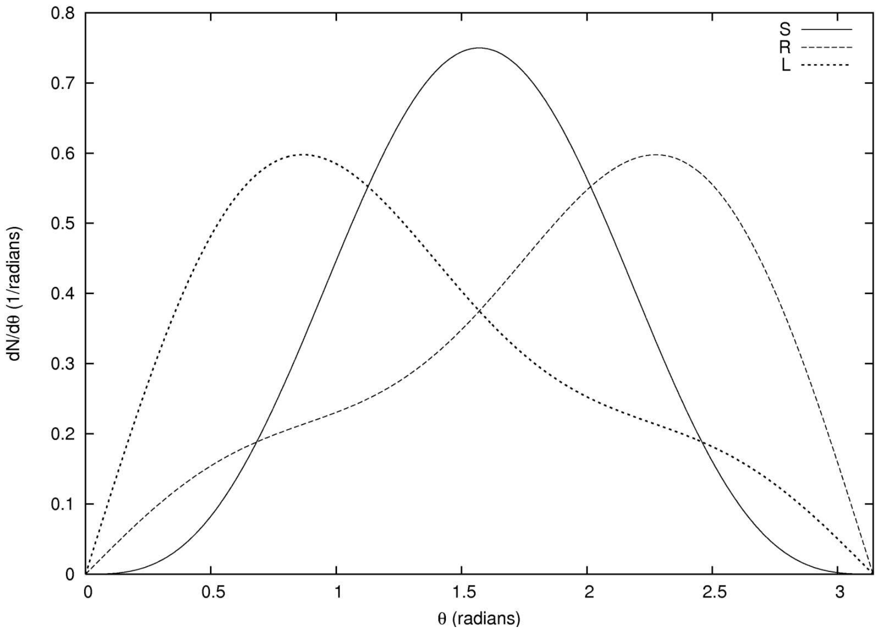

Figure 1: Distributions of leptons from S, R, and L polarized Zs.

	$\langle\theta\rangle$	σ_θ	$\sigma_\theta/\langle\theta\rangle$
R	1.8767	0.6964	0.3711
L	1.2649	0.6964	0.5505
S	1.5708	0.4952	0.3155

Table 1: Statistical values for transverse and longitudinal Z decays. Here $\langle\theta\rangle$ indicates the expectation value of θ for the indicated distribution, while σ_θ indicates the standard deviation in θ for the indicated distribution.

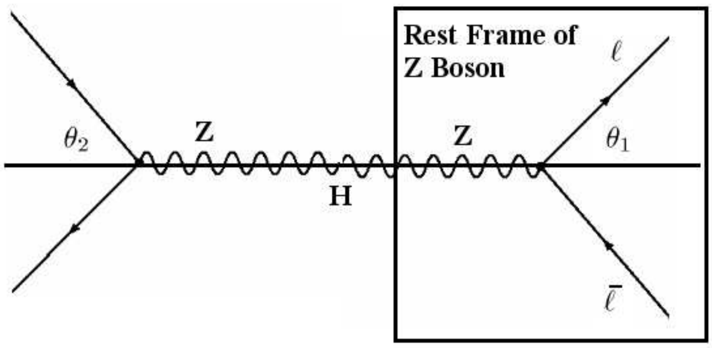

Figure 2: θ_1 and θ_2 are defined as the angle between a given incident Z and the path of its respective lepton decay product in the rest-frame of the Z.

2.2 Bivariate Distributions

Now consider the following decay, shown in Figure 2. We are interested in the joint distributions of θ_1 and θ_2. Since the assignment of θ_1 and θ_2 to any particular decay is arbitrary, and indeterminable from the outset, the occurance of two Z-bosons, say Z_1 and Z_2 in polarization X and Y respectively, is equally as likely as the occurance of Z_1 in polarization Y and Z_2 in polarization X. This implies bivariate distributions of the form

$$\frac{d^2 N_{XY}}{d\theta_1 d\theta_2} = \frac{1}{2}\left[\frac{dN_X}{d\theta_1}\frac{dN_Y}{d\theta_2} + \frac{dN_Y}{d\theta_1}\frac{dN_X}{d\theta_2}\right] \tag{5}$$

where X and Y may take on the values of R, L, and S. This results in six possible distributions for each of the combinations given in the introduction. These distributions are plotted in Figure 3. Associated with these distributions are a set of statistical quantities, given in Table 2.

In Table 2, $\langle\theta\rangle$ is the expection value of θ for the indicated distribution, while σ_θ denotes the standard devitation in θ for the indicated distribution.

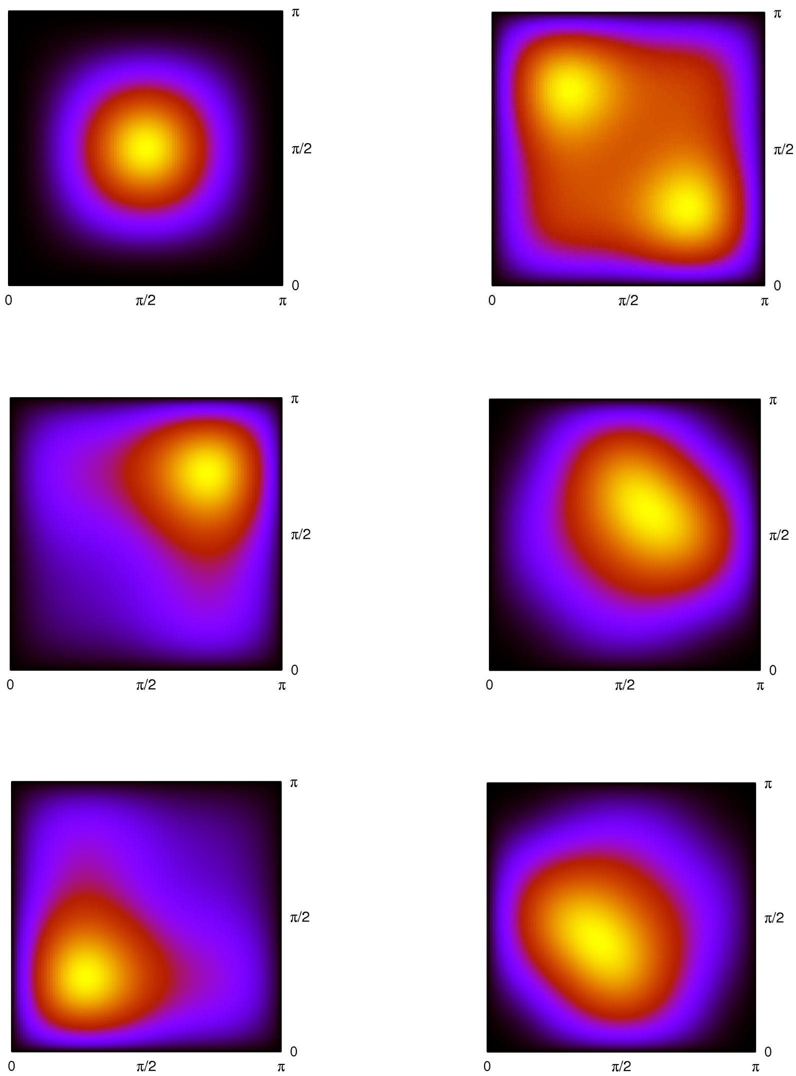

Figure 3: Counter-clockwise from the top: The SS, RR, LL, SL, SR, and RL distributions in the $\theta_1\theta_2$ plane.

XY	$\langle\theta_1\rangle = \langle\theta_2\rangle$	σ_θ	$\sigma_\theta/\langle\theta\rangle$
SS	1.5708	0.7003	0.4458
RR	1.8767	0.9849	0.5248
LL	1.2649	0.9849	0.7786
RL	1.5708	1.9039	1.2121
SR	1.7236	1.9360	1.1232
SL	1.4179	1.6695	1.1774

Table 2: Statiscal quantities for the various polarization pairings. Here $\langle\theta\rangle$ is the expection value of θ for the indicated distribution, while σ_θ denotes the standard devitation in θ for the indicated distribution.

Examining the values in Table 2, it is worth while to note that for the typical parings (SS, RR, LL), $\sigma_\theta/\langle\theta\rangle < 1$, while for atypical pairings (RL, SR, SL), $\sigma_\theta/\langle\theta\rangle > 1$.

3 The Lambda Variable

3.1 Definition

Consider the function of the variables θ_1 and θ_2 given by

$$\Lambda = \frac{\theta_2 - \frac{\pi}{2}}{\theta_1 - \frac{\pi}{2}} \tag{6}$$

In the $\theta_1\theta_2$-plane, this is corrisponds to the line

$$\theta_2 = \Lambda\left(\theta_1 - \frac{\pi}{2}\right) + \frac{\pi}{2} \tag{7}$$

Changing the value of Λ then corrisponds to rotations of this line about the point $(\pi/2, \pi/2)$. Now consider the regions of the $\theta_1\theta_2$-plane shown in Figure 4. Region I corresponds to $-1 \leq \Lambda \leq 1$, while Region II corresponds to $-\infty < \Lambda \leq -1$, $1 \leq \Lambda < \infty$ with many of its distinguishing features occuring near the infinite. Thus, Region I will be used exclusively.

In terms of the angles θ_1 and θ_2, Region I is described by the inequalities

$$\theta_1 \leq \theta_2 \leq \pi - \theta_1 \text{ if } 0 \leq \theta_1 \leq \tfrac{\pi}{2}$$
$$\pi - \theta_1 \leq \theta_2 \leq \theta_1 \text{ if } \tfrac{\pi}{2} \leq \theta_1 \leq \pi$$

3.2 Distribution of Lambda

It can be shown from methods in [4] that

$$\frac{dN_{XY}}{d\Lambda} = A_{XY}\left[\int_{-\frac{\pi}{2}}^{0} f(u,\Lambda)du - \int_{0}^{\frac{\pi}{2}} f(u,\Lambda)du\right] \tag{8}$$

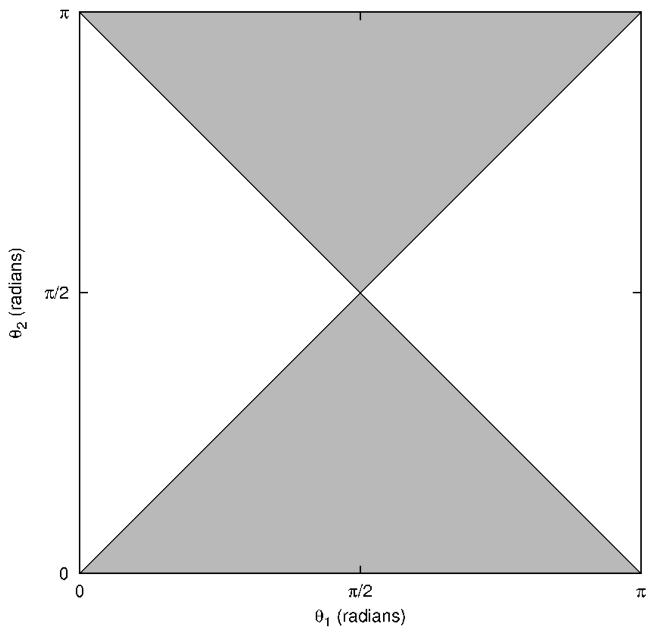

Figure 4: Region I consists of the white regions where $-1 \leq \Lambda \leq 1$, while Region II consists of the grey regions where $-\infty < \Lambda \leq -1$, $1 \leq \Lambda < \infty$.

where A_{XY} is the normalization constant, and

$$f(u,\Lambda) = -u\left[N'_X\left(u+\frac{\pi}{2}\right)N'_Y\left(\frac{\pi}{2}-\Lambda u\right)+N'_Y\left(u+\frac{\pi}{2}\right)N'_X\left(\frac{\pi}{2}-\Lambda u\right)\right] \quad (9)$$

with the prime denoting a derivative. From this, one may exactly determine a set of four functions discribing the distribution of Λ for the SS, RR/LL, SR/SL, and RL pairings. However, the equations are cumbersome and, for all practical purposes, the distributions can be just as well described by an approximate. The distributions of Λ are plotted in Figure 5.

Notice in Figure 5 that there are only four lines. Since the Λ variable does not distinguish between the left and right halfs of Region I, the RR distribution is equivalent to the LL, and the SR is equivalent to the SL.

3.3 Approximate Formulations of the Λ Distributions

In the following, each of the approximate functions were derived by using GNU-plot's equation fitting features to fit a series of sine and cosine functions to values resulting from the numerical evaluation of (8). The data was generated using 10^5 points for integration and 201 points for the fit. The uncertainty in a given coefficient, say c, is denoted by $\pm\Delta c$. These values are given in a series of tables.

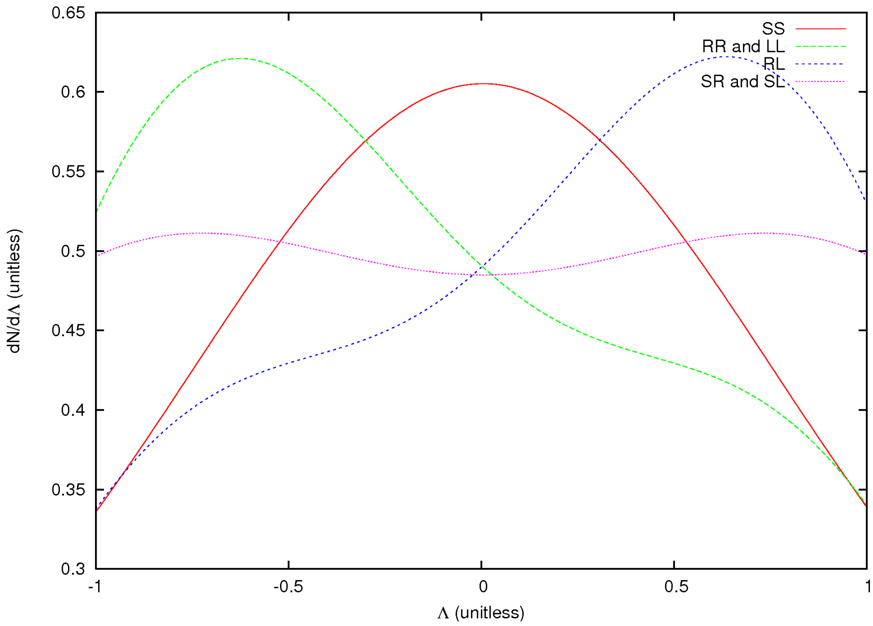

Figure 5: The various Λ distributions as produced by numerical evaluation of (8).

SS Distribution. Assume the function

$$\frac{dN_{SS}}{d\Lambda} \cong a_0 + a_1 \cos(\pi\Lambda/2) + a_2 \cos(\pi\Lambda) \quad (10)$$

The fit to the computed distribution yields the coefficients

j	a_j	$\pm\Delta a_j$
0	0.366583	0.0009475
1	0.209573	0.001482
2	0.0289469	0.0006486

RR and LL Distribution. Assume the function

$$\frac{dN_{RR}}{d\Lambda} \cong b_0 + b_1 \cos(\pi\Lambda/2) + b_2 \sin(\pi\Lambda/2) + b_3 \cos(\pi\Lambda) + b_4 \sin(\pi\Lambda) \quad (11)$$

The fit to the computed distribution yields the coefficients

j	b_j	$\pm\Delta b_j$
0	0.318623	0.0003943
1	0.284922	0.006167
2	-0.0930653	0.0001132
3	-0.112103	0.0002699
4	-0.0253914	0.0001138

RL Distribution. Assume the function

$$\frac{dN_{RR}}{d\Lambda} \cong c_0 + c_1 \cos(\pi\Lambda/2) + c_2 \sin(\pi\Lambda/2) + c_3 \cos(\pi\Lambda) + c_4 \sin(\pi\Lambda) \quad (12)$$

The fit to the computed distribution yields the coefficients

j	c_j	$\pm\Delta c_j$
0	0.319648	0.0004163
1	0.283311	0.0006511
2	0.0976107	0.0001195
3	−0.112811	0.000285
4	0.0219853	0.0001201

SR and SL Distributions. Assume the function

$$\frac{dN_{SR}}{d\Lambda} \cong d_0 + d_1 \cos(3\pi\Lambda/2) + d_2 \cos(5\pi\Lambda/2) \quad (13)$$

The fit to the computed distribution yields the coefficients

j	d_j	$\pm\Delta d_j$
0	0.49688	$5.285 \cdot 10^{-5}$
1	−0.0133902	$7.371 \cdot 10^{-5}$
2	0.00219806	$7.149 \cdot 10^{-5}$

Conclusion

The angular distributions for lepton generation from $X \to ZZ \to 4l$ decays were determined. From these a set of six unique distributions were found relating the opening angles of the leptons resulting from either Z in the $X \to ZZ \to 4l$ decay. A variable, called Λ, defined as a function of these angles, was established. Four distributions, one for the SS, the RR/LL, the SR/SL, and the RL pairings, were developed in terms of Λ. Each of these distributions was found to have a unique shape, permitting one to readily distinguish the four said pairings.

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References

References are presented in the order they appear in the article.

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