Relating Physical Superpartner Masses to the MSSM Lagrangian

LHC Inverse Workshop

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Based in part on:

`hep-ph/0501132` (`TSIL`) with Dave Robertson

`hep-ph/0509115`

`hep-ph/0605???`

- Strategy
  - Masses as the key observables at LHC
  - The need for precision and flexibility
- Tactics
  - Differential equations method for multi-loop integrals
  - TSIL (Two-loop Self-energy Integral Library), a computer program for evaluating general two-loop self-energy and vacuum integrals
- Results
  - 2-loop, and partial 3-loop, gluino pole mass
  - 2-loop squark pole masses

Recent history confirms that hadron colliders can and will succeed at precision mass measurements:

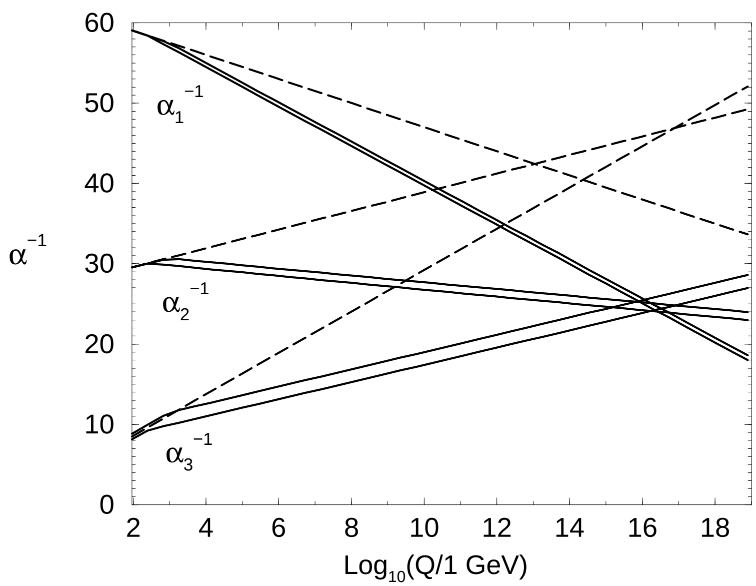
$$\begin{aligned} M_{\text{top}} &= 172.5 \pm 2.3 \text{ GeV} \\ M_W &= 80.454 \pm 0.059 \text{ GeV} \quad (p\bar{p} \text{ data only}) \end{aligned}$$

This is encouraging, because masses are the most important observables in new physics models, notably supersymmetry.

Masses are the key observables in SUSY

Most of what we do not already know about supersymmetric extensions of the Standard Model involves the soft SUSY-breaking terms with positive mass dimension.

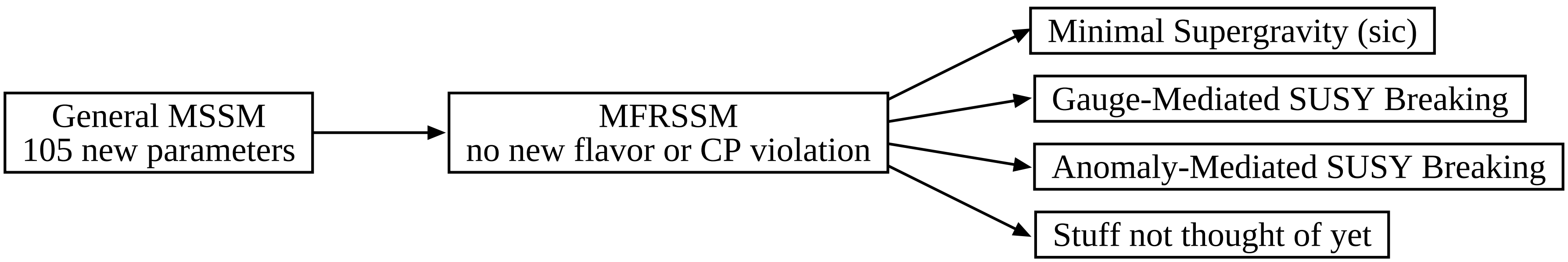
Predictions of specific models (Minimal Supergravity, Gauge Mediation, Anomaly Mediation, Extra-dimensional Mediation, ...) allow/require precise calculations.



The apparent unification of gauge couplings in the MSSM invites us to extrapolate the soft masses up to high scales, to see if they obey some Organizing Principle.

What is the Organizing Principle behind SUSY breaking?

A reasonable working hypothesis is the **Minimal Flavor-Respecting Supersymmetric Standard Model**. It is neither too painfully general, nor too naively specific:



MFRSSM parameter count:

3 gaugino masses	$M_1,\ M_2,\ M_3$
5 sfermion (mass)^2	$m^2_{\tilde{Q}},\ m^2_{\tilde{u}},\ m^2_{\tilde{d}},\ m^2_{\tilde{L}},\ m^2_{\tilde{e}}$
3 (scalar)^3 couplings	$A_{u0},\ A_{d0},\ A_{e0}$
3 Higgs mass parameters	$\mu,\ b,\ m^2_{H_u},\ m^2_{H_d}$ (but M_Z known)
1 input RG scale	Q_0

Total: 15 new parameters beyond the Standard Model

Gaugino Mass Unification is a popular and recurring theme.

$$M_1(Q) = M_2(Q) = M_3(Q) \equiv m_{1/2} \qquad \text{at } Q \approx 2 \times 10^{16} \text{ GeV},$$

resulting in

$$M_1 : M_2 : M_3 = 1 : 2 : 6$$

for Q near the TeV scale. To test this, we have to relate physical masses to running masses.

Obstacles at the LHC:

- Neutralinos, Charginos mix
- Not all states observed (Higgsinos, squarks may be out of reach)
- Overall mass scale (M_{LSP}) may be tough to get accurately
- Corrections to the gluino mass are big

Despite the obstacles, the LHC necessarily has a unique role:

- Can see the gluino. (ILC probably can't.)
- Will probably happen, and soon. (ILC ?)

The determination of the gluino mass is absolutely crucial. It feeds strongly into any attempt to connect TeV scale physics with high-scale Organizing Principles in SUSY. Under almost any set of assumptions, the uncertainty in the gluino mass will dominate the error in this effort, in the long run.

Goal: reduce purely theoretical sources of uncertainty to a negligible level, if possible.

TeV-scale SUSY will provide an interesting laboratory for quantum field theory:

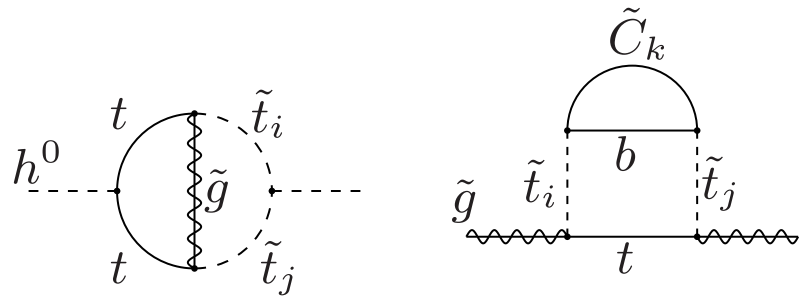
- Fundamental scalar particles
- No new dimensionless couplings in Lagrangian
- QCD coupling is perturbative at the TeV scale, but still strong enough to require multi-loop calculations
  - Corrections to Higgs masses at 1 loop go like y_t^2, at 2 loops go like $y_t^2 g_3^2$.
  - Corrections to gluino mass involve Cg_3^2 with $C = 3$, rather than $C = 4/3$ for quarks. So, the QCD coupling for the gluino is effectively $9/4$ larger.
  - Corrections to squark, quark masses get large effects from the strongly-coupled, heavy gluino.

Two-loop corrections to masses will be mandatory if SUSY is correct.

Some key features of the problem:

1) Two-loop diagrams involve many different mass scales simultaneously.

For example:



Large, diverse, and numerous hierarchies of ratios of squared masses will enter. Some of these hierarchies can be anticipated in advance, some can't.

This is a qualitative difficulty generally avoided in multiloop calculations in the Standard Model, where one knows in advance that

$$m_s^2 \ll m_c^2 \ll m_b^2 \ll m_t^2,$$

and calculations are organized around exploiting these hierarchies when doing multi-loop integrals.

2) To explore Organizing Principles, work in non-decoupled SUSY with mass-independent renormalization scheme

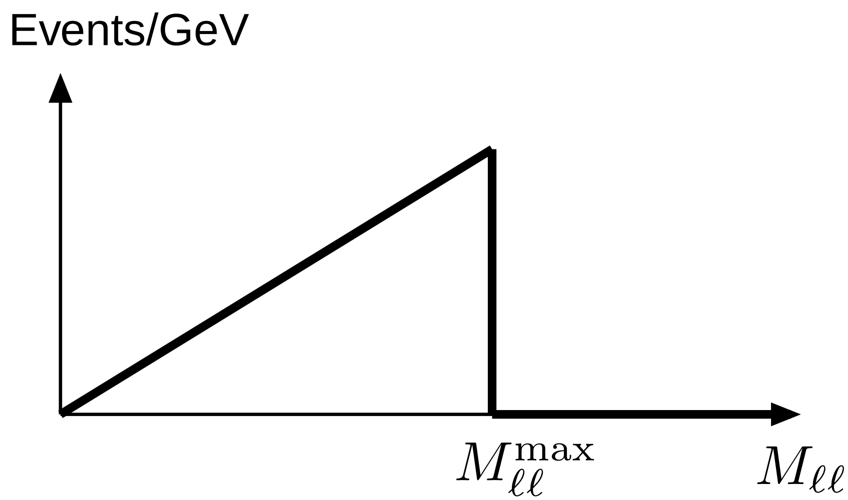
On-shell schemes are useful, as are effective theories in which some heavier superpartners are integrated out. Some problems are easier in those schemes. However:

- For the goal of running up to higher renormalization scales, we will want to know the running parameters in the full theory.
- Global fits can relate the directly measured observables to running $\overline{DR}$ input parameters.
- It is not so clear in advance what the best on-shell scheme input parameters will be. (For example, in the Higgs sector, A^0 mass or $H^\pm$ mass? For neutralinos and charginos, should the input parameters be masses, or mass differences, or some even more complicated kinematic function?)

For example, several LHC studies have remarked on the precision available from

$$\tilde{N}_2 \to \ell\tilde{\ell} \to \ell^+\ell^-\tilde{N}_1.$$

The dilepton invariant mass distribution looks like:



The edgepoint is at

$$M_{\ell\ell}^{\mathrm{max}} = m_{\tilde{N}_2}(1 - m_{\tilde{\ell}}^2/m_{\tilde{N}_2}^2)^{1/2}(1 - m_{\tilde{N}_1}^2/m_{\tilde{\ell}}^2)^{1/2}.$$

Building an on-shell scheme around this measurement as an input doesn't seem pleasant to me.

3) Methods should be generic, reuseable from start to finish.

To avoid wasted effort, do calculations for scalars, fermions, vectors in a general perturbative field theory. Then apply to Higgs, squarks, sleptons, and to quarks, gluino, charginos, neutralinos, etc., or, ???

After all, SUSY might not be the correct answer, or it might be an incomplete answer.

To calculate physical masses

Evaluate self-energy = sum of 1-particle irreducible Feynman diagrams:

$$\Pi(s) = \Pi^{(1)}(s) + \Pi^{(2)}(s) + \ldots$$

where $s =$ the external momentum invariant.

The complex pole mass

$$s_{\text{pole}} = M^2 - i\Gamma M$$

is the solution for complex s of:

$$\begin{aligned} s_{\text{pole}} &= m_{\text{tree}}^2 + \Pi(s_{\text{pole}}) \\ &= m_{\text{tree}}^2 + \Pi^{(1)}(m_{\text{tree}}^2)\left[1 + \Pi^{(1)\prime}(m_{\text{tree}}^2)\right] + \Pi^{(2)}(m_{\text{tree}}^2) + \ldots \end{aligned}$$

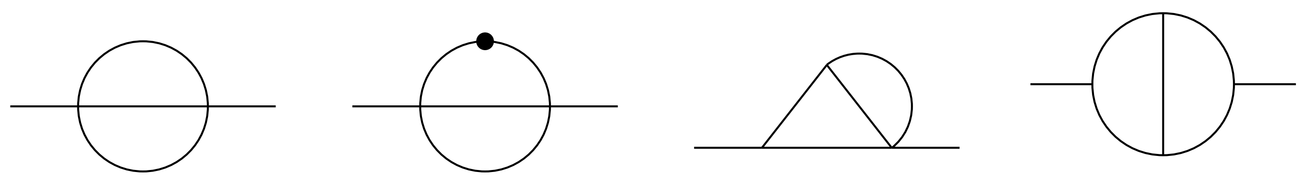
The pole mass is gauge invariant at each order in perturbation theory, can be related to kinematic masses as measured at colliders.

There are a large but finite number of 2-loop, two-point Feynman diagrams. Why not just do them once, for a general theory, and get it over with?

Method:

- Reduce all self-energies in general theory to a few basis integrals
- Basis integrals contain $\overline{\text{DR}}$ (or $\overline{\text{MS}}$) counter-terms, so finite.
- Numerically evaluate basis integrals quickly and reliably for arbitrary masses.

Tarasov's basis and recurrence relations:



S T U M ("Master integral")

Can always reduce 2-loop self-energies to a linear combination of these, with coefficients rational functions of:

$s = p^2 =$ external momentum invariant

$x, y, z, \ldots$ = internal propagator masses

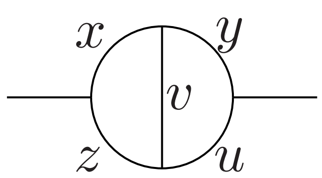
To evaluate basis integrals:

Values at $s = 0$ are known analytically, in terms of logs, polylogs.

$$\begin{aligned} \frac{\partial}{\partial s}\text{(basis integral)} &= \text{(another self-energy integral)} \\ &= \text{(linear combination of basis integrals)} \end{aligned}$$

So, we have a set of coupled, first-order, linear differential equations.

Consider the Master integral $M(x, y, z, u, v)$:



and the basis integrals obtained from it by removing propagators:

$$\begin{gathered} U(x, z, u, v),\ \ U(y, u, z, v),\ \ U(z, x, y, v),\ \ U(u, y, x, v), \\ S(x, u, v),\ \ T(x, u, v),\ \ T(u, x, v),\ \ T(v, x, u), \\ S(y, z, v),\ \ T(y, z, v),\ \ T(z, y, v),\ \ T(v, y, z) \end{gathered}$$

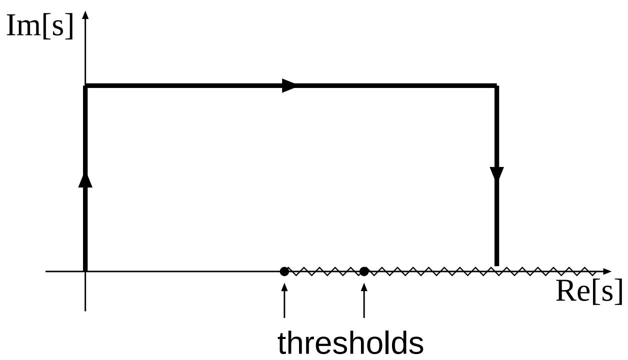
Call these 13 integrals I_n, $(n = 1, \ldots, 13)$.

Differential equations method for basis integrals

$$\frac{d}{ds} I_n = \sum_m K_{nm} I_m + C_n$$

Here K_{nm} are rational functions of s and $x, y, z \ldots$, and C_n are one-loop integrals. These are obtained by using Tarasov's recursion relations.

Solve for basis integrals I_n using Runge-Kutta integration in the complex s-plane, starting from known values at $s = 0$.



Method implemented for S, T, U type integrals by Caffo, Czyz, Laporta, Remiddi.

Dave Robertson and I have extended the method to also work for M:

$\mathtt{TSIL}$ = **T**wo-Loop **S**elf-energy **I**ntegral **L**ibrary

D.G. Robertson, SPM, `hep-ph/0501132`

Program written in C, callable from C++, Fortran

In the Hopi culture native to the American southwest, Tsil is the Chili Pepper Kachina. The Kachina are supernatural spirits, represented by masked figurines and impersonated by ceremonial dancers. They communicate between the tribe and their gods, who live in the San Francisco mountains and are never seen directly.

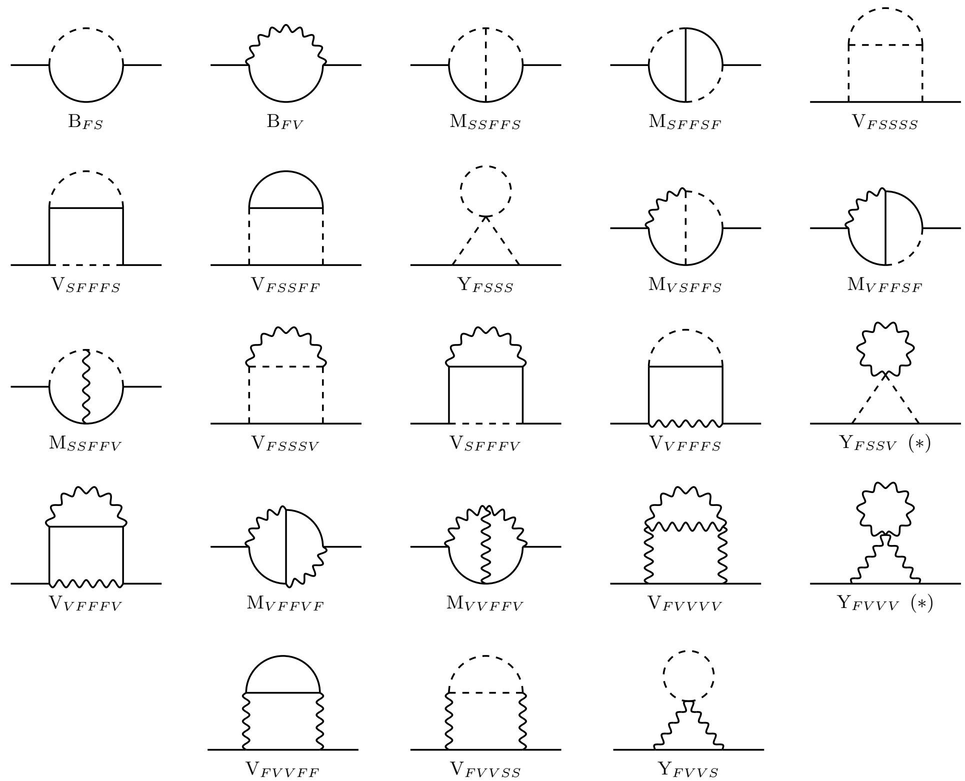
- Basis integrals computed for any values of all masses and s.
- All integrals from a given master integral obtained simultaneously in a single numerical computation.
- Checks on the numerical accuracy follow from changing choice of contour.
- Computation times generically $\ll$ 1 second on modern hardware.
- $\mathtt{TSIL}$ knows all special cases that have been done analytically in terms of polylogarithms

Using these methods, I've computed the 2-loop fermion pole masses in a general renormalizable theory with massless gauge bosons (hep-ph/0509115).

Each diagram is reduced to a linear combination of basis integrals, ready to be computed numerically using TSIL.

In favorable cases with only one or two distinct mass scales, the results are analytical in terms of polylogarithms.

Special case applications within the MSSM include the top quark mass, neutralino and chargino masses, and the gluino.



+ fermion mass insertions

+ ghost diagrams

+ counterterms

Checks on the calculation of 2-loop fermion pole masses:

- Independent of gauge-fixing parameter

  Individual diagrams depend on ξ; cancels in pole mass

- Pole mass is renormalization group invariant

  Checked analytically at 2-loop order; numerical check below

- Absence of divergent logs on shell

  Individual diagrams have $\log(1 - p^2/m^2)$, divergent as $p^2 \rightarrow m^2$;

  must and do cancel in pole mass

- Checks in (unphysical) supersymmetric limit

  Agrees with earlier calculation of scalar pole mass (SPM hep-ph/0502168)

Gluino pole mass at 2-loop order

(Y. Yamada, `hep-ph/0506262`; SPM, `hep-ph/0509115`)

The full formulas are a little too complicated to be presented in a talk, but are in the second paper. A C program based on TSIL can be obtained at:

`zippy.physics.niu.edu/gluinopole/`

Instead, I'll just show some simple special approximations.

In the following, squarks are always assumed to be degenerate and quarks to be massless, for simplicity. Also,

$$\alpha_s, \quad M_3, \quad \text{and} \quad m_{\text{squark}}$$

refer to running parameters in the $\overline{\text{DR}}$ scheme, evaluated at a renormalization scale $Q = M_3(Q)$.

The pole mass $M_{\tilde{g}}^{\rm pole}$ is computed in terms of these.

Example: In the special case of degenerate running masses, $M_3 = m_{\text{squark}}$, the result for the pole mass simplifies and can be written analytically:

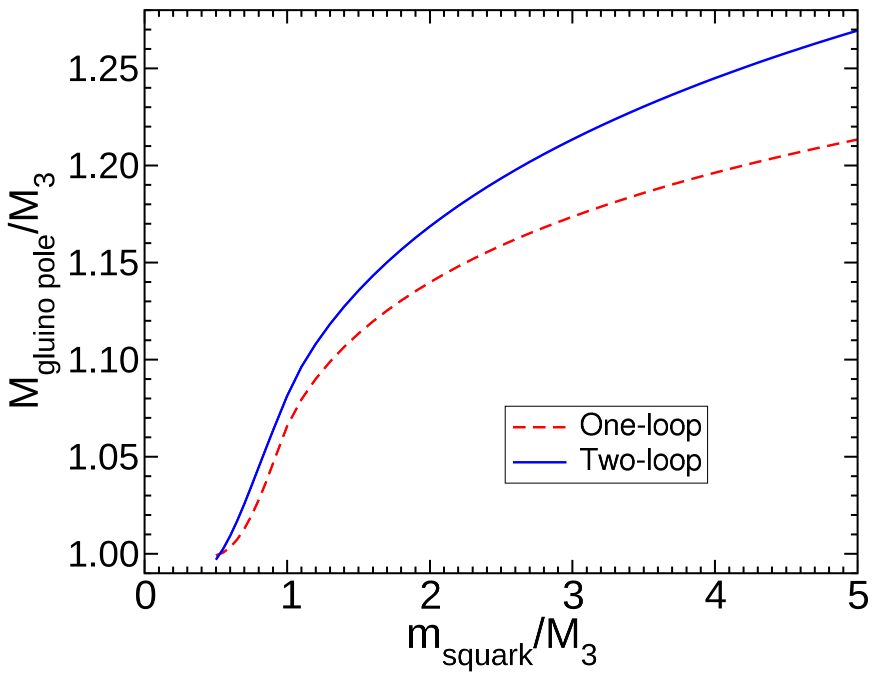
$$M_{\tilde{g}}^{\text{pole}} = M_3 \left[1 + \frac{\alpha_s}{4\pi} 9 + \left(\frac{\alpha_s}{4\pi} \right)^2 \left\{ 54\zeta(3) + \pi^2 (53 - 36 \ln 2) - 90 \right\} + \ldots \right]$$

$$= M_3 \left[1 \; + \; 0.716\, \alpha_s \; + \; 1.59\, \alpha_s^2 + \ldots \right]$$

(M_3 and α_s are running parameters evaluated at $Q = M_3$ in non-decoupled theory.)

However, the corrections for heavier squarks are quite large...

Dependence of gluino pole mass correction on the squark masses



For heavier squarks, part of the large corrections come from large logarithms that can be resummed using the renormalization group.

For $m_{\rm squark} \gg M_3$:

$$M_{\tilde g}^{\rm pole} \;=\; M_3\Big[1 \;+\; 0.955(L+1)\alpha_s \;+\; (0.46L^2 + 1.53L + 0.90)\alpha_s^2 + \ldots\Big]$$

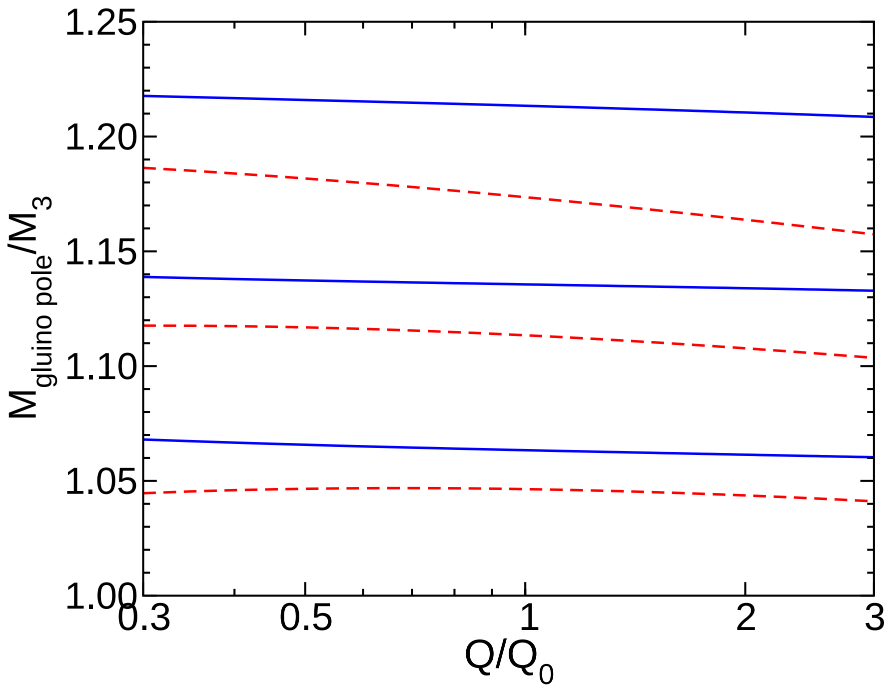
where $L \equiv \ln(m_{\rm squark}/M_3)$.

Obvious Questions: How big is the theoretical error? Can we estimate the 3-loop corrections? Is perturbation theory under control?

How NOT to estimate theoretical error: RG scale dependence

Run α_S, M_3 from Q_0 to a new RG scale Q, recompute pole mass:

Red $=$ 1-loop, Blue $=$ 2-loop



$\leftarrow m_{\rm squark}/M_3 = 3$

$\leftarrow m_{\rm squark}/M_3 = 1.5$

$\leftarrow m_{\rm squark}/M_3 = 0.9$

Scale dependence of 2-loop result is < 1%.

But, the 2-loop correction is much larger than the 1-loop scale dependence!

Dependence of the computation on the choice of RG scale significantly underestimates the true theoretical error.

A trivial estimate of theoretical error due to 3-loop correction:

$$(M_{\tilde{g}} \text{ to order } \alpha_s^2)^2 - (M_{\tilde{g}}^2 \text{ to order } \alpha_s^2) \neq 0$$

There is an unavoidable ambiguity in the pole mass at any given order in perturbation theory, of order the next order in perturbation theory.

In this case, it can be interpreted as a theoretical error in $M_{\tilde{g}}$, corresponding to:

$$\frac{\Delta M_{\tilde{g}}}{M_{\tilde{g}}} \sim \alpha_s^3 \begin{cases} 0.8 + 2.3L + 1.9L^2 + 0.5L^3 & (\text{for large } L) \\ 1.2 & (\text{for } L = 0) \end{cases}$$

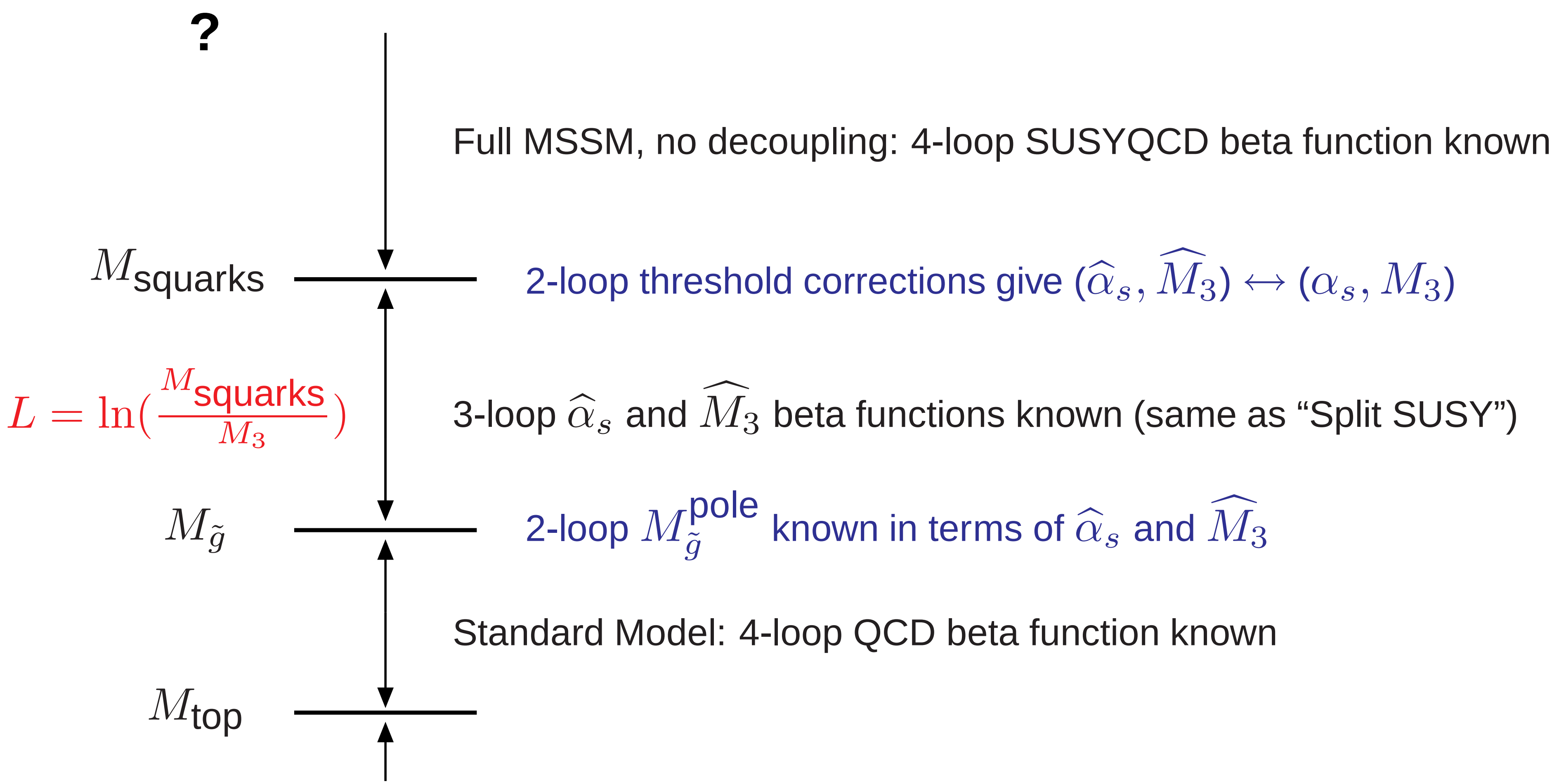
of order a few tenths of a percent for reasonable $L = \ln(m_{\rm squark}/M_3)$.

However, when L is not small, we don't have to accept this error.

We can do better…

Three-loop gluino mass corrections for heavy squarks

Exploit the fact that beta functions are easier to compute, known to $\geq$3-loop order. Let the running parameters in the full MSSM be α_s, M_3, and in the effective theory with squarks decoupled, $\widehat{\alpha}_s$, $\widehat{M}_3$.



Using the effective field theory matching and RG running technique, one obtains all terms of order

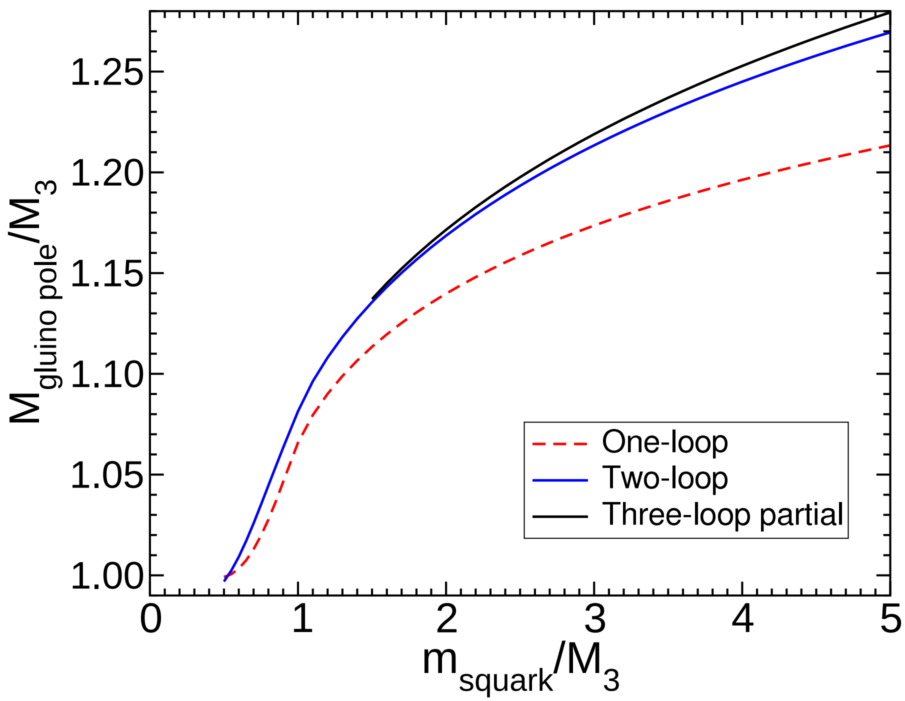
$$\alpha_s^n L^n, \quad \alpha_s^n L^{n-1}, \quad \alpha_s^n L^{n-2}$$

for all n. The 3-loop pole mass for the gluino is:

$$\begin{aligned} M_{\tilde{g}}^{\text{pole}} = M_3 \Big[1 &+ 0.955\,(L+1)\,\alpha_s \\ &+ (0.46L^2 + 1.53L + 0.90)\,\alpha_s^2 \\ &+ (0.19L^3 + 0.32L^2 + 1.61L + \textbf{???})\,\alpha_s^3 \\ &+ \mathcal{O}(M_3^2/m_{\tilde{Q}}^2) + \mathcal{O}(\alpha_s^4) \Big] \end{aligned}$$

- The “leading log” approximation is bad unless L is VERY large.
- Only a real 3-loop pole mass calculation (with at least two distinct non-zero mass scales, so hard) can tell us what **???** is.
- Circumstantial evidence leads me to wager that **???** is not much larger in magnitude than 1 (but I can’t even tell you the sign).

Three-loop log-enhanced effects on the gluino pole mass



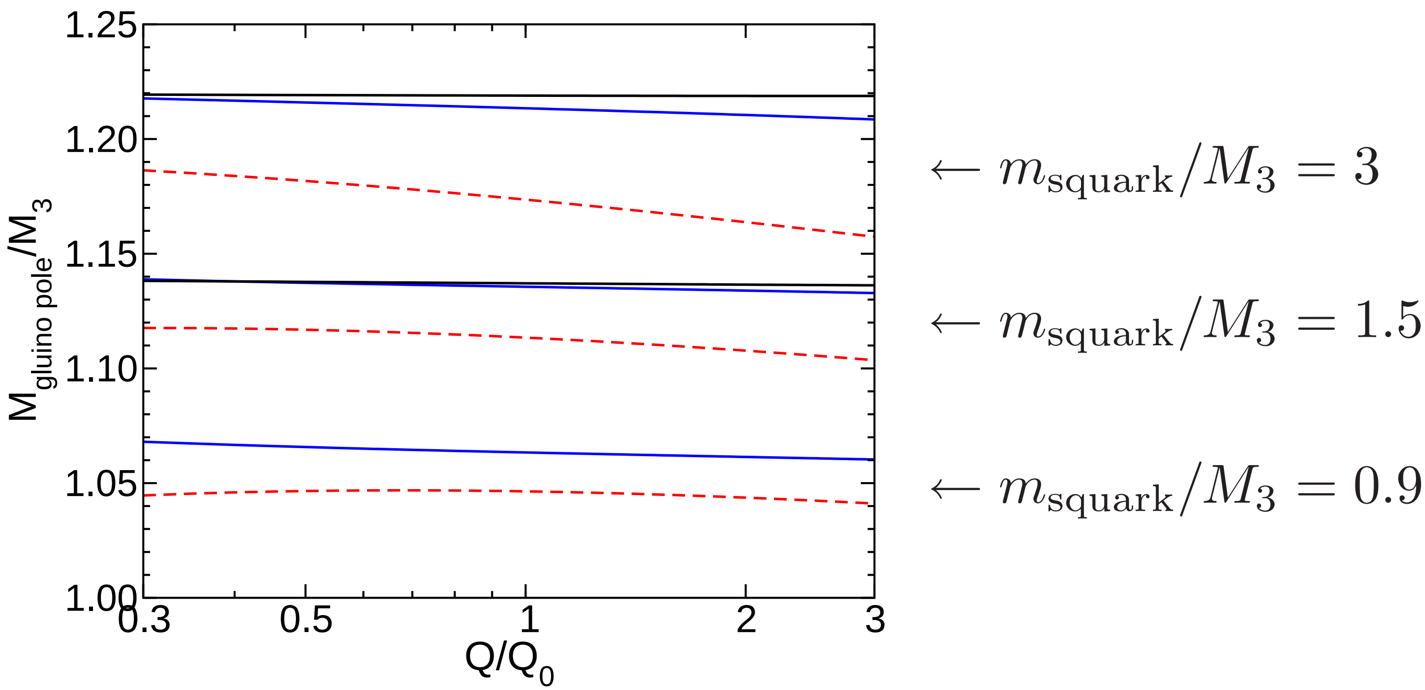
The three-loop log corrections are only shown for $m_{\text{squarks}}/M_3 > 1.5$, where the approximation starts to become meaningful.

The actual 3-loop correction involves a non-log-enhanced piece, not captured in this analysis. However, circumstantially, this seems likely to be well under 1%.

There are inevitable experimental uncertainties in $M_{\tilde{g}}^{\text{pole}}$ and α_S and the individual squark masses and mixing angles.

RG scale dependence revisited:

Red = 1-loop, Blue = 2-loop, Black = partial 3-loop



As expected, the 3-loop logs greatly reduce the scale dependence of the calculated pole mass.

But, as we have seen, this proves little.

Is it worth pursuing further perturbative corrections for the gluino pole mass?

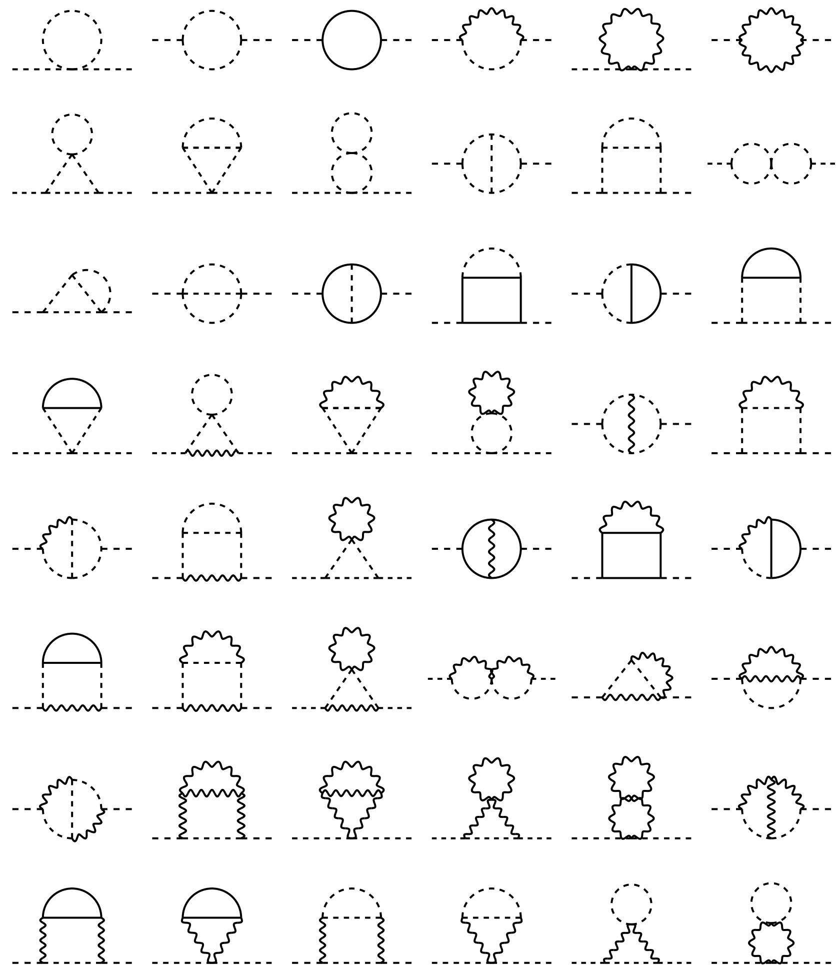
I have my doubts. The full 2-loop plus the 3-loop log-enhanced terms should be enough. . .

(Anyway, why risk this?)

2-loop corrections to scalar self-energies and pole masses in a general renormalizable theory (hep-ph/0502168)

(Approximation: vector boson masses neglected in digrams with more than one vector propagator.)

Applications to Higgs masses, slepton masses and squark masses in the MSSM.



+ fermion mass insertions + ghosts

+ counterterms

SUSYQCD corrections to squark masses in MSSM

Example: In the special case of degenerate running masses, $m_{\tilde{Q}} = m_{\tilde{g}} = Q$, the result for the pole mass simplifies:

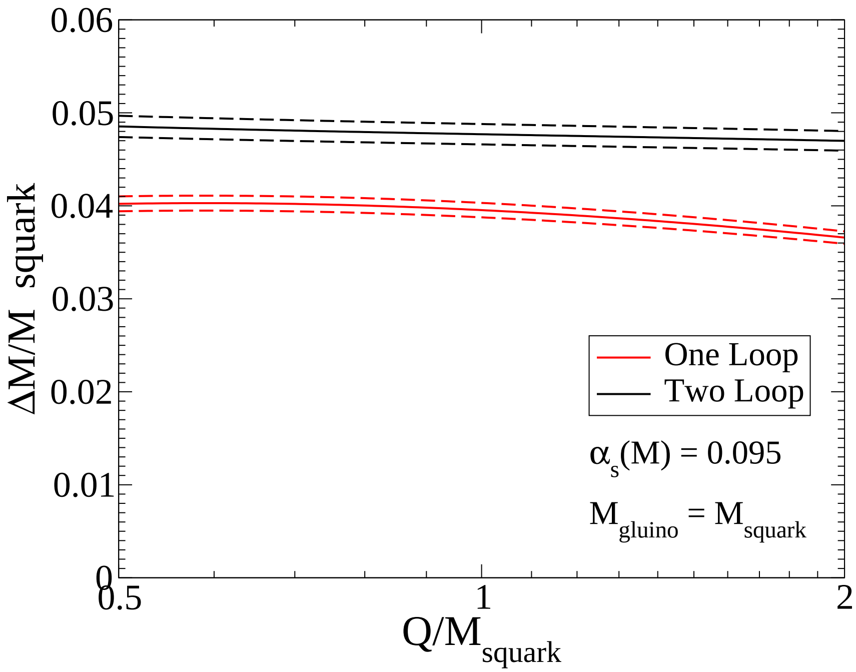
$$M^2_{\tilde{Q}} = m^2_{\tilde{Q}} \left[1 + \frac{\alpha_s}{4\pi} \left(\frac{32}{3} \right) + \left(\frac{\alpha_s}{4\pi} \right)^2 \left\{ \frac{112}{3} + \frac{664\pi^2}{27} + \frac{32\pi^2 \ln 2}{9} - \frac{16\zeta(3)}{3} \right\} \right]$$

$$= m^2_{\tilde{Q}} \left[1 + 0.849\, \alpha_s + 1.89\, \alpha_s^2 \right]$$

There are no large logs here (only one mass scale!), so this illustrates the intrinsic size of typical SUSYQCD 1-loop (~ 4%) and 2-loop (~ 1%) corrections to the squark masses.

 Here, it is not so clear that 3-loop effects are negligible for light top squarks measured at an ILC. But that is a different Workshop. . .

Renormalization scale (Q) dependence of calculated squark pole mass



Squark mixing, quark masses, and electroweak effects neglected; all squarks taken degenerate with each other and gluino at tree level.

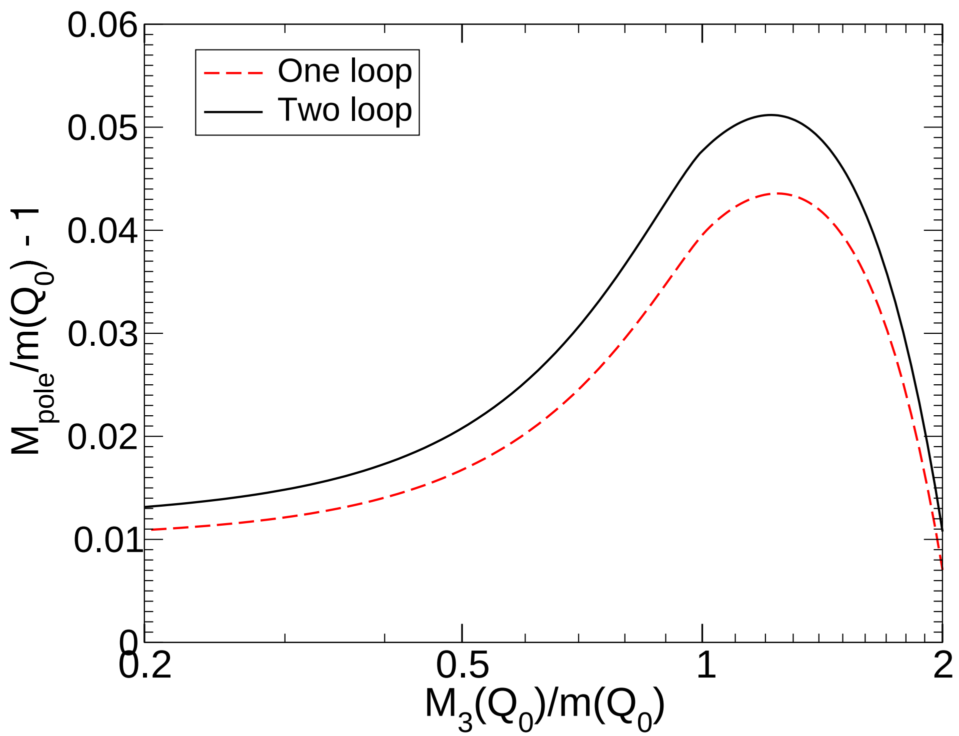
Dashed lines are $\pm 2\%$ variation of α_s.

Remaining scale dependence (from 3 loops and beyond) is small.

However, 2-loop correction is much larger than 1-loop scale dependence.

So, as usual, this proves little.

Dependence of squark mass correction on the gluino mass



A large part of the squark mass correction is due to the gluino mass.

In realistic models, effects due to variation in squark masses, top and bottom Yukawa effects, electroweak effects are significant, too. The general formulas (not shown here) take care of that.

Outlook

- Two-loop calculations for masses in supersymmetry are necessary, possible
- I favor a Strategy based on:
  - non-decoupled MSSM, using mass-independent $\overline{\text{DR}}$ scheme (complementary to on-shell scheme and effective field theory calculations)
  - Reusable, generic calculations
  - Efficient computations of basis two-loop integrals
- 2-loop corrections to gluino, squark masses are now known, typical remaining uncertainties should be $\lesssim 1$% in most cases
- For large mass hierarchies m_{squark}/M_3, new 3-loop corrections are worthwhile
- Some further 3-loop calculations (probably for h^0, maybe for gluino, squarks) might still be necessary