S-duality in N=2 superconformal gauge theories

P.C. Argyres and N. Seiberg

hep-th/0604 . . .

Outline

I Review of N=4 S-duality

II N=2 S-duality: the problem

III N=2 S-duality: the solution

IV Implications for isolated SCFTs

V An SU(3) example

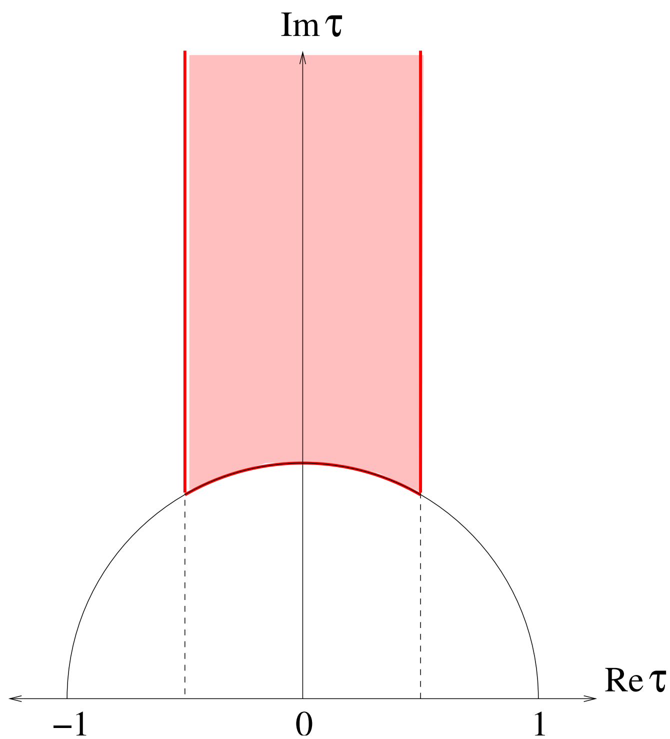
VI Further directions

I Review of N=4 S-duality (Olive-Montonen duality)

What happens as the gauge coupling constant becomes infinite?
S-duality for N=4 (d=4) superYM answer:

- The theory becomes a weakly coupled gauge theory again (though not necessarily with the same gauge group).

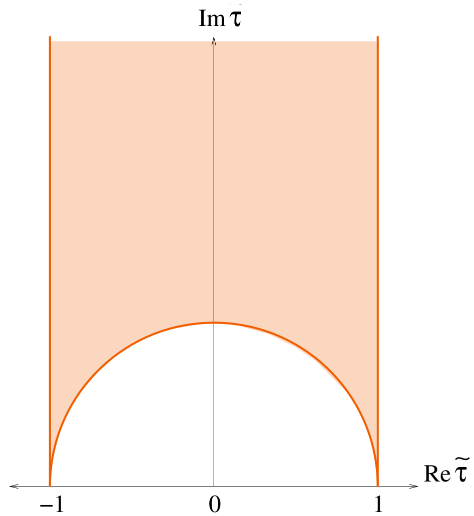
- For simply-laced groups, the complex coupling $\tau = \theta/2\pi + 4\pi i/g^2$ is identified under an SL$(2,\mathbb{Z})$ action generated by $\tau \simeq \tau + 1 \simeq -1/\tau$:



II N=2 S-duality: the problem

But this is not always the answer to the infinite coupling question in scale-invariant gauge theories with less supersymmetry.

- E.g., for SU(3) with six massless fundamentals, the S-duality group is $\Gamma^0(2) \subset SL(2,\mathbb{Z})$ generated by $\tilde{\tau} \simeq \tilde{\tau} + 2 \simeq -1/\tilde{\tau}$.

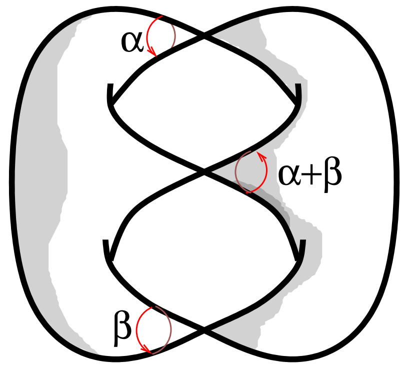


- The fundamental domain of this group contains the infinite coupling point $\tilde{\tau} = 1$.

The problem: What is the physics at the infinite coupling point?

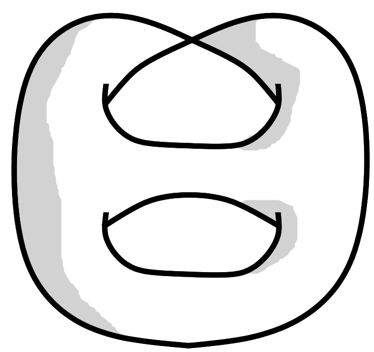
- Not a different weakly-coupled gauge theory:

  – SW curve: as $\mathrm{Im}\,\tilde{\tau} \to \infty$ in the SU(3) theory, genus 2 curve pinches to 2 genus 0 curves everywhere on moduli space.



  – Pinching cycle $\leftrightarrow$ charged $W^{\pm}$ bosons becoming massless, corresponding to expected SU(3) $\to$ U(1) $\times$ U(1) Higgsing.

  – But as $\tilde{\tau} \to 1$, only one cycle vanishes everywhere on moduli space. So, not enough W-bosons for a weakly-coupled Higgs mechanism.



- So, some new "phase" is indicated for this N=2 SU(3) gauge theory at $\widetilde{\tau} = 1$.

- This type of behavior is typical of the infinite-coupling points of (infinitely) many scale-invariant N=2 theories.

- Note: the numerical value of $\mathrm{Im}\,\tau$ is not significant, since it can be changed by a holomorphic, non-pert. redefinition of the coupling.
  "Infinite coupling" is really only a convenient phrase to describe those points in coupling space where the effective action has the singular (but not weak-coupling) behavior at generic points on the Coulomb branch described in the previous paragraph.

III N=2 S-duality: the (conjectural) answer

- The physics at the infinite coupling limit of a scale-invariant N=2 gauge theory with gauge group G, with rank$(G) = r$, is a weakly coupled scale-invariant gauge theory with gauge group H with smaller rank, rank$(H) = s < r$, which is coupled to an isolated rank-$(r-s)$ N=2 superconformal field theory:

	G	$\simeq$	H	$+$	SCFT
coupling:	g		$1/g$		—
rank:	r		s		$r-s$

  - "Isolated SCFT" $\equiv$ SCFT has no marginal coupling of its own.
  - "Rank of an isolated N=2 SCFT" $\equiv$ complex dimension of its Coulomb branch (number of U(1) gauge factors unbroken in IR).

- The coupling between H and the SCFT is the standard gauge coupling: H gauges a subgroup of the global symmetry group of the SCFT.

  Thus the SCFT provides "matter fields" charged under H.

- Generalizes S-duality from N=4 to N=2 scale-invariant field theories.

  Including strongly-coupled SCFTs as factors in the duals of non-abelian gauge theories is a natural generalization of the N=4 case, given the existence of isolated N=2 conformal gauge theories.

- We cannot prove this duality conjecture, since it relates strong coupling to "weak+strong" coupling.

  Instead we adduce evidence from:

  — from exact IR effective actions (SW curves),

  — global symmetries.

  These fix H, the SCFT, and their coupling, given the scale-invariant theory G.

- Can be made much more precise.

  E.g., if $G = \mathrm{SU}(3) + 6 \cdot \mathbf{3}$ (*i.e.*, with 6 fund. hyper-plets):

  — the dual is $H = \mathrm{SU}(2) + 1 \cdot \mathbf{2}$,

  — coupled to the isolated rank 1 SCFT with global symmetry group E_6; and

  — H couples to the E_6 SCFT by gauging the SU(2) factor in the maximal subgroup $\mathrm{SU}(2) \times \mathrm{SU}(6) \subset E_6$.

- $H \not\subset G$.

  E.g., the $H = \mathrm{SU}(2)$ W-bosons are magnetically charged under $G = \mathrm{SU}(3)$ in the above example.

IV Implications for isolated SCFTs

1. Isolated rank 1 SCFTs are subsectors of lagrangian theories.

E.g., the E_6 SCFT is a subsector of the SU(3)$+6 \cdot \mathbf{3}$ lagrangian field theory, which decouples in the infinite coupling limit. Others as well: *e.g.*, the E_7 SCFT $\subset$ Sp(2) $+ 6 \cdot \mathbf{4}$ theory.

The rank 1 N=2 SCFTs with E_n global symmetry were not constructed as 4d field theories; instead, were constructed by compactification of 6d LSTs.

2. The Kodaira classification of the isolated rank 1 SCFTs is incomplete.

Predicts that the Coulomb branch singularities of the isolated rank 1 SCFTs have more than one mass deformation consistent with N=2 supersymmetry.

E.g., $\lim_{g\to\infty}[\mathrm{SO}(5) + 3 \cdot \mathbf{5}] \Rightarrow$ the E_7 SCFT has a mass deformation with Sp(4) global symmetry (in addition to the E_7 mass deformation). This is thus a new rank 1 SCFT.

3. A new exact quantity can be calculated in N=2 SCFTs.

By weakly gauging the global symmetries of the G theory both at weak and infinite gauge coupling, and comparing its beta function to the one computed in the dual description allows us to compute the contribution of the SCFT "matter" to the H gauge coupling beta function.

This is governed by the central charge, k, of the global current algebra of the SCFT:

$$\text{``}J^a(x)J^b(0) \sim \frac{k\delta^{ab}}{x^6} + \cdots + \frac{f_c^{ab}J^c(0)}{x^3} + \cdots\text{''}.$$

k is thus a new exactly computed observable of isolated N=2 SCFTs.

V Example: SU(3)+6·**3**

- Ingredients:

  Coulomb branch moduli: $\{u, v\}$
  Coupling constant: $f \sim e^{2\pi i\tau}$. ($f \to 0$ is weak, $f \to 1$ is infinite coupling.)
  SW curve: $y^2 = (x^3 - ux - v)^2 - f^2 x^6$.
  Holomorphic 1-forms: $\omega_u{=}x\,dx/y$, $\omega_v{=}dx/y$.

- Limits:

  Set $u{=}0$ **and** $f{=}1$**:** degenerates to genus 1.
  Change variables: $\left(\tilde{y}^2 = \tilde{x}^3 - \tilde{v}^4,\ \omega_{\tilde{v}} = d\tilde{x}/\tilde{y}\right)$
  This is the E_6 SCFT. (Minahan,Nemeschansky)

  Set $v{=}0$ **and** $f{\sim}1$**:** degenerates to

$$y^2 = \underbrace{x^2}_{\substack{\text{pinch}\\ \text{cycle}}} \cdot \underbrace{\left[(x^2 - u)^2 - f^2 x^4\right]}_{\substack{\equiv \widehat{y}^2 \\ \text{weak SU(2) SCFT}}}, \qquad \underbrace{\omega_u = x\,dx/y}\,. \ \Updownarrow\ \overbrace{\omega_u = dx/\widehat{y}}\,.$$

- Global symmetries $\Leftrightarrow$ mass deformations:

E$_6$ **masses:** $M_n \sim m^n$ are explicit basis of E$_6$ Casimirs.

$$\tilde{y}^2 = \tilde{x}^3 - (\tilde{v}^2 M_2 + \tilde{v} M_5 + M_8)\tilde{x} - (\tilde{v}^4 + \tilde{v}^2 M_6 + \tilde{v} M_9 + M_{12}).$$

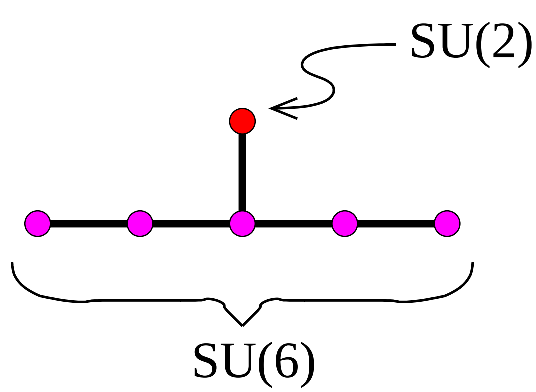
Turn on u at $f = 1$: same c.o.v. gives

$$\tilde{y}^2 = \tilde{x}^3 - (\tilde{v}^2\, u + \tfrac{1}{48} u^4)\tilde{x} - (\tilde{v}^4 + \tilde{v}^2\, \tfrac{1}{12} u^3 + \tfrac{1}{864} u^6).$$

Implies: u is mass deformation of E$_6$ SCFT with

$$\{M_2{=}u,\, M_5{=}0,\, M_6{=}\tfrac{1}{12}u^3,\, M_8{=}\tfrac{1}{48}u^4,\, M_9{=}0,\, M_{12}{=}\tfrac{1}{864}u^6\}.$$

Since $u \neq 0$ higgses SU(2) $\to$ U(1), some **group theory** implies that the SU(2) gauges the $\bullet$ subgroup of E$_6$:



The unbroken SU(6) is part of the **global symmetry** of the SU(3)+ $6 \cdot \mathbf{3}$ theory. But the **full global symmetry** of the theory is

$$\mathrm{U}(1)_B \times \mathrm{SU}(6) \times \mathrm{U}(2)_R.$$

— The R symmetry is realized at infinite coupling as a certain combination of the R symmetries of the rank 1 SU(2) and E_6 SCFTs.

— The SU(6) flavor symmetry was identified above as the "ungauged" part of the E_6 group.

— Which leaves the $\mathrm{U}(1)_B$ "baryon number" unaccounted for.

The solution can only be that there is a single **SU(2)-doublet hypermultiplet** at infinite coupling:

$$\boxed{\mathrm{SU}(3)_g + 6 \cdot \mathbf{3} \sim \mathrm{SU}(2)_{1/g} + 1 \cdot \mathbf{2} + (\mathrm{E}_6\text{-CFT}).}$$

- Further checks: anomaly matching, beta functions ...

VI Further directions

1. Theory of N=2 SCFT mass deformations.

Even at rank 1 there are many new theories, but it seems computationally difficult to construct them all. (J. Wittig)

2. Construction of string duals.

Novel type IIA branes construction gives non-singular description of infinite-coupling points. Also gives self-dual examples of gauge theories with no weak-coupling limit, only "infinite-coupling" limits. (J. Vazquez-Poritz)

3. Theory of N=2 SCFT central charges?

Holds promise of calculating "a", and possible algebraic constructions of N=2 SCFTs. (M. Edalati, J. Vazquez-Poritz, J. Wittig)